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Retaining Structural System Active Vibration Control

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Abstract**—**This study presents an active vibration control technique to reduce the earthquake responses of a retained structural system. The proposed technique is a synthesis of the adaptive input estimation method (AIEM) and linear quadratic Gaussian (LQG) controller. The AIEM can estimate an unknown system input online. The LQG controller offers optimal control forces to suppress wall-structural system vibration. The numerical results show robust performance in the active vibration control technique.

Keywords**—**Active vibration control, AIEM, LQG, Optimal control

I. INTRODUCTION

MANY recent earthquake disasters have occurred around the world, causing innumerable losses in human lives and property. Earthquake-resistant design has an important role in structure design. Seismic action is a load that structures must be able to resist, accepting a certain level of structural degradation. Control technique applications, including passive, semi-active and active control systems have been developed for civil engineering structures. The passive control technique examples include using tuned-mass-dampers, base isolations, friction and viscous dampers and so on [1]. In semi-active techniques the vibration is attenuated through an indirect manner by changing the structural parameters of the machine, such as damping and stiffness. Recently, various active control technique methods have been developed. The optimal control methods, such as linear quadratic regulator (LQR) and LQG, are popular with many structural engineers. Optimal LQR controllers have been developed and used in practical implementations [2-4]. Yang [5] applied the optimal control theorem to control the vibrations of civil engineering structures under stochastic dynamic loads such as earthquakes and wind loads. He used the instantaneous optimal control method, which minimizes the quadratic performance index at every time instant, to overcome the deficiency of neglecting the earthquake input [6]. The external load influence is not considered in the optimal controller design because the external load disturbances are immeasurable or inestimable in the control force calculation. Therefore, the traditional LQG controller has difficulty maintaining robust control performance with the random dynamic input condition.

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To solve the above problems, this study first investigates active control technique application to reduce the retaining structure response when subjected to earthquake excitations. The active control technique is a synthesis algorithm of the AIEM and LQG controller. The AIEM is frequently employed in structural dynamic problems. The input estimation method has been successfully used to inversely solve 1-D, 2-D heat conduction problems [7, 8]. Ji et al. [9] used the Kalman filter with the recursive least square method to estimate the input force of a plate. Lee et al. [10] utilized the adaptive weighted input estimation method to inversely solve the burst load of a truss structure system. Chen et al. [11, 12] investigated the adaptive input estimation method applied to the inverse estimation of load input in a multi-layer shearing stress structure and moving load identification in a bridge structure system. The AIEM can estimate on-line dynamic loads. An active LQG controller can apply the same inverse control forces on a structural system. The control results of the proposed method are effective in suppressing vibration in a retaining structural system.

II. DYNAMIC MODELING AND ANALYSIS OF THE RETAINING STRUCTURAL SYSTEM

The geometry and coordinates of a soil-wall system are shown in figure 1(a). The semi-infinite, homogeneous and visco-elastic soil medium is retained by a vertical rigid retaining wall along one of its vertical boundaries, connected to a rigid base. The soil layer base is excited by the ground motion accelerations of the 9/21 Chi-Chi earthquake in Taiwan. The soil-wall system is modeled using a simple two-degree of freedom (2-DOF) mass spring dashpot dynamic model, as shown in figure 1(b). Considering the dynamic equilibrium of these two masses using D'Alemberl's principle, the soil-wall system under active control, the basic dynamic equation can be written in matrix form [13]:

$$M\ddot{X} + C\dot{X} + KX = M\ddot{x}_g(t) + DF(t) \tag{1}$$

where $M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ is the diagonal mass matrix,

$C = \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & c_2 \end{bmatrix}$ is the damping matrix and

$K = \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix}$ is the restoring force vector.

$\ddot{x}_g(t)$ is the ground motion acceleration. D is the control force distribution matrix, $F(t)=[f_1(t) \quad f_2(t)]^T$ is the control force vector. $X=[x_1 \quad x_2]^T$, $\dot{X}=[\dot{x}_1 \quad \dot{x}_2]^T$ and $\ddot{X}=[\ddot{x}_1 \quad \ddot{x}_2]^T$ is the displacements, velocities and accelerations of the masses m_1 and m_2, respectively.

The continuous-time measurement equation is shown below:

$$Z(t)=HX(t)+v(t), \tag{2}$$

where $Z(t)$ is the observation vector, $H=[1 \quad 0]$ is the measurement matrix and $v(t)$ is the measurement noise.

The continuous-time state equation of the structure system can be presented as follows [14]:

$$\dot{X}(t)=AX(t)+BG(t)+EF(t), \tag{3}$$

$$G(t)=[\ddot{x}_g(t) \quad \ddot{x}_g(t)]^T,$$

$$A=\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix},$$

$$B=\begin{bmatrix} 0 \\ M^{-1}M \end{bmatrix},$$

$$E=\begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix},$$

where $X(t)$ is the modal state vector. A is the coefficient matrix. $F(t)$ is the control force inputs. B and E are the coefficient vectors of $G(t)$ and $F(t)$, respectively. Constant matrices are composed of the *n*th natural frequency and the inertia moment of the structure system. Using the sampling time, Δt, to sample the continuous-time state equation, (3) and assuming that the system model error, $w(k)$ is Gaussian white noise with zero mean, the discrete-time equation can be obtained as follows [14].

$$X(k+1)=\Phi X(k)+\Gamma G(k)++\Lambda F(k)+w(k) \tag{4}$$

$$X(k)=[x_1(k) \quad x_2(k)]^T$$

$$\Phi=\exp(A\Delta t)$$

$$\Gamma=\int_{k\Delta t}^{(k+1)\Delta t}\exp\{A[(k+1)\Delta t-\tau]\}Bd\tau$$

$$\Lambda=\int_{k\Delta t}^{(k+1)\Delta t}\exp\{A[(k+1)\Delta t-\tau]\}Dd\tau$$

Where Φ is the state transition matrix. Γ and Λ are the coefficient matrices of $G(k)$ and $F(k)$, respectively. $G(k)$ is the certain input array. $F(k)$ is the control array. $w(k)$ is the processing error vector, which is assumed as Gaussian white noise with zero mean and variance, $E\{w(k)w^T(k)\}=Q\delta_{kj}$, $Q=Q_w\times I_{2n\times 2n}$, Q is the discrete-time processing noise covariance matrix. δ_{kj} is the Kronecker delta function. The discrete-time measurement equation of equation (2) is shown below:

$$Z(k)=HX(k)+v(k) \tag{5}$$

$Z(k)$ is the discrete observation vector. $v(k)$ represents the measurement noise vector and is assumed as the Gaussian white noise with zero mean and the variance, $E\{v(k)v^T(k)\}=R\delta_{kj}$, $R=R_v\times I_{2n\times 2n}$, R is the discrete-time measurement noise covariance matrix.

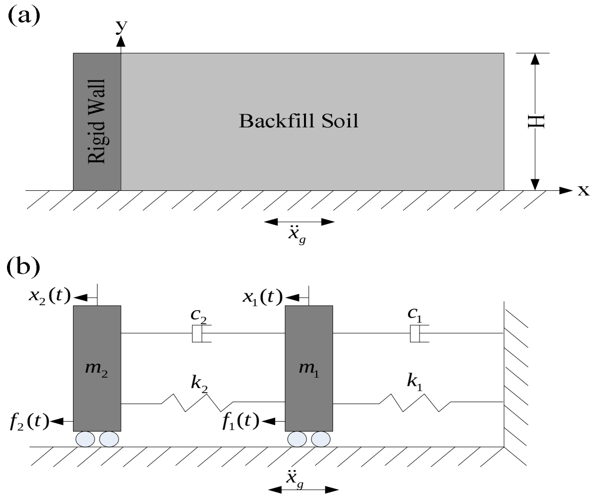

Fig. 1 (a) Considered soil-wall system. (b) Proposed 2-DOF mathematical model [15]

III. AIEM COMBINED LQG CONTROL TECHNIQUE DESIGN

The conventional LQG controller has a specific level of interference suppression. It is weak in maintaining high performance in the suppression of external loads; which are complex and arbitrary style disturbances. In other words, in equation (4), if a time-varying load, $G(k)$ exists, the optimal control method combining the Kalman filter and the LQG regulator will not be able to obtain the optimal control forces. To resolve this situation, this study proposes combining the AIEM with the LQG control technique for active vibration control of the retaining structural system. The AIEM can estimate the unknown dynamic inputs while the active LQG controller can apply the same inverse control forces on the structural system.

The AIEM is composed of a Kalman filter without the input term and the adaptive weighting recursive least square algorithm. The detailed formulation of this technique can be found in the research by Tuan *et al* [14]. The Kalman filter equations are as follows:

The optimal estimate of the state is

$$\hat{X}(k+1/k)=\Phi\hat{X}(k/k-1)+K(k)\left[Z(k)-H\hat{X}(k/k-1)\right] \tag{6}$$

The bias innovation produced by the measurement noise and input disturbance is expressed by

$$\bar{Z}(k)=Z(k)-H\hat{X}(k/k-1) \tag{7}$$

The Kalman gain is

$$K(k)=\Phi(k+1/k)P(k/k-1)H^T(k)\left[H(k)P(k/k-1)H^T(k)+R\right]^{-1} \quad (8)$$

The covariance of residual is $S(k)$

$$S(k)=HP(k/k-1)H^T+R \quad (9)$$

The prediction error covariance matrix is

$$\begin{aligned}P(k+1/k)&=\Phi(k+1/k)P(k/k-1)\Phi^T(k+1/k)\\&-\Phi(k+1/k)P(k/k-1)H^T(k)\\&\times\left[H(k)P(k/k-1)H^T(k)+R\right]^{-1}\\&H(k)P(k/k-1)\Phi^T(k+1/k)\\&+\Gamma(k+1/k)Q\Gamma^T(k+1/k)\end{aligned} \quad (10)$$

The recursive least square estimator equations are as follows: The sensitivity matrices are $B(k)$ and $M(k)$

$$B(k)=H[\Phi M(k-1)+I]\Gamma \quad (11)$$

$$M(k)=[I-K(k)H][\Phi M(k-1)+I] \quad (12)$$

The correction gain is expressed as

$$K_b(k)=\gamma^{-1}P_b(k-1)B^T(k)\left[B(k)\gamma^{-1}P_b(k-1)B^T(k)+S(k)\right]^{-1} \quad (13)$$

The error covariance of the input estimation process is

$$P_b(k)=\left[I-K_b(k)B(k)\right]\gamma^{-1}P_b(k-1) \quad (14)$$

The estimated earth motion acceleration is

$$\hat{G}(k)=\hat{G}(k-1)+K_b(k)\left[\bar{Z}(k)-B(k)\hat{G}(k-1)\right] \quad (15)$$

In equations (15-16), γ is a weighting factor using the adaptive weighting function, which is formulated in [15]. That is,

$$\gamma(k)=\begin{cases}1, & \left|\bar{Z}(k)\right|\le\sigma,\\ \sigma/\left|\bar{Z}(k)\right|, & \left|\bar{Z}(k)\right|\le\sigma.\end{cases} \quad (16)$$

In the optimal estimation portion of the LQG optimal control method, by substituting $\hat{G}(k)$ of equation (15) for $G(k)$ and substituting the control input in equation (6), the optimal state estimation equation can be rewritten as:

$$\begin{aligned}\hat{X}(k+1/k)&=\Phi\hat{X}(k/k-1)+K(k)\left[Z(k)-H\hat{X}(k/k-1)\right]\\&+\Gamma\hat{G}(k)+\Lambda F(k)\end{aligned} \quad (17)$$

The performance index is defined as:

$$J_i(F)=E\left\{\frac{1}{2}X^T(N)Q_0X(N)+\frac{1}{2}\sum_{k=i}^{N-1}\begin{bmatrix}X^T(k)Q_1X(k)+\\u^T(k)Q_2X(k)\end{bmatrix}\right\} \quad (18)$$

where $Q_1\ge 0$, $Q_2\ge 0$ and $Q_0\ge 0$ are all symmetric weighting matrices. The optimal feedback control force vector can be obtained by using the separation theorem [16]:

$$F(k)=-K_r(k)\hat{X}(k/k-1) \quad (19)$$

Here the regular gain $K_r(k)$ is given by

$$K_r(k)=\left[\Lambda^TP_2(k+1)\Lambda+Q_2\right]^{-1}\Lambda^TP_2(k+1)\Phi \quad (20)$$

where $P_2(k)$ is the discrete-time Ricatti equation solution. The Ricatti equation is shown below:

$$P_2(k)=\Phi^T\begin{Bmatrix}P_2(k+1)\\-P_2(k+1)\Lambda\left[\Lambda^TP_2(k+1)\Lambda+Q_2\right]^{-1}\Lambda^TP_2(k+1)\end{Bmatrix}\Phi,\quad +Q_1$$

$$k\le N, \quad (21)$$

$$P_2(N)=Q_0 \quad (22)$$

According to equation (21), $P_2(k)$ can be obtained by inversely calculating from $k=N$ to $k=1,$. The method combining AIEM and the LQG active controller is presented using the AIEM to estimate $\hat{G}(k)$ and combining equation (6) to obtain the optimal state estimate, $\bar{X}(k+1/k)$, which can be further substituted in equation (19). The combination of AIEM and the LQG controller has been designed.

IV. Results and Discussion

To demonstrate the accuracy and efficiency of the controller design, several numerical retaining structure simulations were investigated. The soil-wall system considered is shown in figure 1(a). The system is modeled using a simple two-degree freedom (2-DOF) mass spring dashpot dynamic model as shown in figure 1(b) [13]. The wall material and soil layer is defined by the mass density, ρ, shear modulus of elasticity, G, Poisson's ratio, μ, and the material damping factor η of concrete and dense sand respectively. The material data and soil-wall system dimensions are shown in Table I.

TABLE I
THE MATERIAL DATA AND DIMENSIONS OF THE SOIL-WALL SYSTEM [13]

Wall height, H	5 (m)
Unit weight of the concrete wall, γ_{conc}	24 (KN/m^3)
Shear modulus of the concrete wall, G_{conc}	$9921\times10^3(KN/m^2)$
Unit weight of the backfill soil, γ_{soil}	$18(KN/m^3)$
Shear modulus of the backfill soil, G_{soil}	$5769(KN/m^2)$

For method estimating the stiffness value for both soil and wall was described by Veletsos and Younan [17]. It is determined such that the undamped natural frequency of the model equals the fundamental natural frequency of the medium idealized as a series of vertical shear-beams. The stiffness, k of a particular system can be estimated as

$$k=m\left(\pi^2/4H^2\right)\left(G/\rho\right) \quad (23)$$

where m is the mass of the system considered. This method is composed of the Kalman filter without the input term and the intelligent fuzzy weighted recursive least square estimator. The initial simulation conditions and other parameters are shown as follows: $p(0/0) = diag\left[10^4\right]$. $\hat{G}(0) = 0$. $p_b(0) = 10^8$. $M(0)$ is set as a zero matrix. The weighting factor, γ, is an intelligent fuzzy weighting function. The sampling interval, $\Delta t = 0.005$ s, and the total simulation time, $t_f = 70s$. The earth motion accelerations of the 921 Chi-Chi Earthquake in Taiwan were measured from the seismological station (TCU 056) at the Li-Ming elementary school [18]. The unknown earth motion acceleration can also be estimated from the dynamic responses of the wall.

The process noise and measurement noise are both considered in the simulation process. The process noise covariance matrix, $Q = Q_w \times I_{2n\times 2n}$, where $Q_w = 10^{-2}$. The measurement noise covariance matrix, $R = R_v \times I_{2n\times 2n}$, where $R_v = \sigma^2 = 10^{-14}$. σ is the standard deviation of noise. Figure 2 shows the displacement, velocity and acceleration-time history responses of the wall structure to the earth motion acceleration. Figure 3 shows the estimation result for the earth motion acceleration. The estimation results show that the estimator tracking performance is good enough and suitable in reducing the noise effect. The time histories of the responses for a wall-structural system with and without control are shown in Figures 4-7. The conventional LQG controller has the issue that the unknown input cannot be obtained and the control reaction is slower. The AIEM estimates the unknown input in on-line and combined with the LQG controller (which computes the optimal control force) can be used to obtain better results. Figure 8 shows the overall time histories of the control forces required for the proposed method and LQG method. The simulation results demonstrate that the proposed control method successfully reduces the wall-structure responses when subjected to seismic excitation.

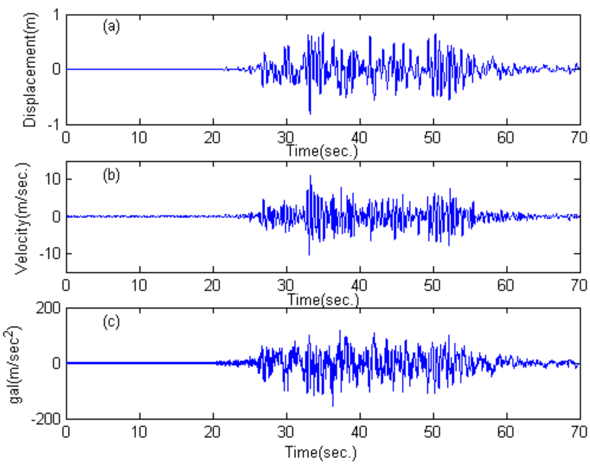

Fig. 2 The displacement, velocity and acceleration of the wall caused by the seismic force

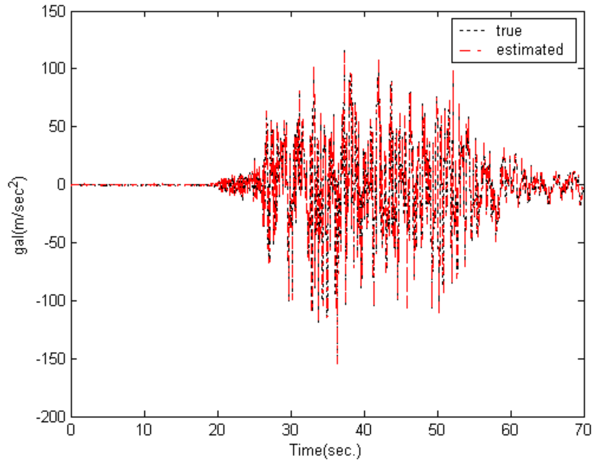

Fig. 3 the estimation results using the adaptive weighting function

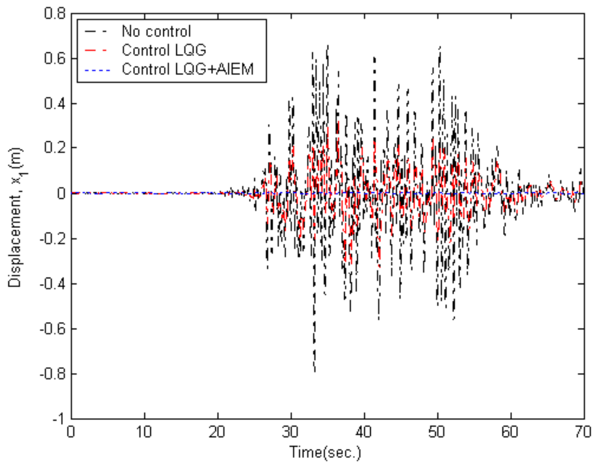

Fig. 4 Comparison of wall displacement control (x_1) caused by the seismic force

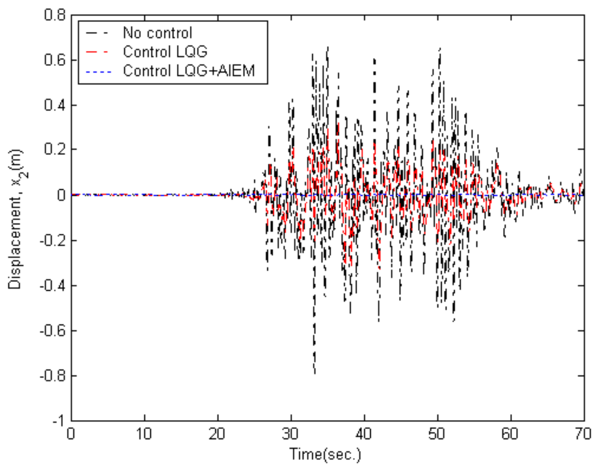

Fig. 5 Comparison of wall displacement control (x_2) caused by the seismic force

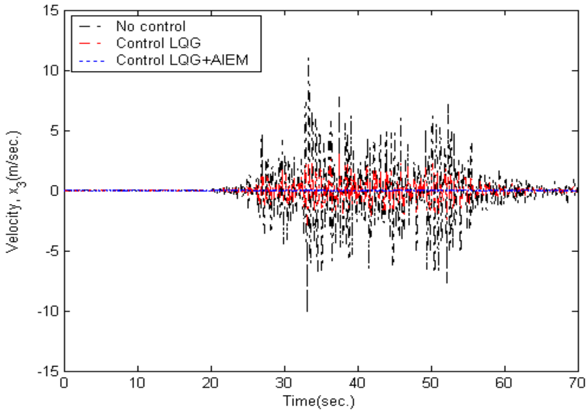

Fig. 6 Comparison of wall velocity control (x_3) caused by the seismic force

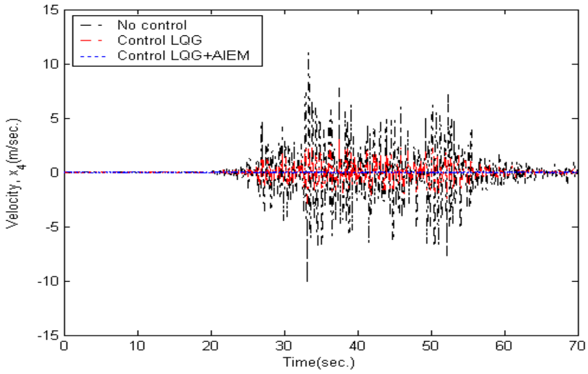

Fig. 7 Comparison of wall velocity control (x_4) caused by the seismic force

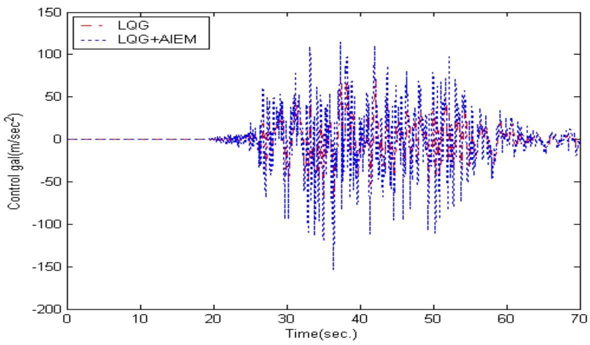

Fig. 8 Comparison of the wall-structural system control forces

V. Conclusions

The study proposed an active control technique for active ground motion acceleration control in a retained structural system. This research exploited an active control technique based on the excellent AIEM and LQG controller. Because this active control technique requires external forces information the AIEM is proposed for on-line excitation estimation. This active control technique demonstrated excellent performance by solving the earthquake-excitation control problem. The results demonstrate that this method has better active vibration control than the conventional LQG controller. Future work is being conducted to extend this application to a nonlinear structural system.

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