Feedrate optimization for ball-end milling of sculptured surfaces using fuzzy logic controller

Njiri J.G., Ikua B.W. and Nyakoe G.N.

Abstract-Optimization of cutting parameters important in precision machining in regards to efficiency and surface integrity of the machined part. Usually productivity and precision in machining is limited by the forces emanating from the cutting process. Due to the inherent varying nature of the workpiece in terms of geometry and material composition, the peak cutting forces vary from point to point during machining process. In order to increase productivity without compromising on machining accuracy, it is important to control these cutting forces. In this paper a fuzzy logic control algorithm is developed that can be applied in the control of peak cutting forces in milling of spherical surfaces using ball end mills. The controller can adaptively vary the feedrate to maintain allowable cutting force on the tool. This control algorithm is implemented in a computer numerical control (CNC) machine. It has been demonstrated that the controller can provide stable machining and improve the performance of the CNC milling process by varying feedrate.

Keywords—Ball-end mill, feedrate, fuzzy logic controller, machining optimization, spherical surface.

I. INTRODUCTION

MANY products are designed with sculptured surfaces to meet specific functional well as aesthetical requirements, which are important factors for customer satisfaction. Examples of include automobile body parts, aircraft structural components and household electronic gadgets. The manufacture of such parts normally involves use of dies and moulds. Machining of such moulds presents a challenge owing to geometrical complexity and requirement of close dimensional and form tolerances, as well as high surface integrity.

Use of improper machining parameters can result in many detrimental effects such as excessive cutting forces, severe vibrations, poor dimensional and geometric accuracy and poor surface integrity of machined parts, tool breakage, low productivity, and high manufacturing costs. In order to optimally carry out a machining operation, it is important to have an idea of how the process would behave under various cutting conditions, and hence, there is need to develop accurate and reliable simulation models for the prediction of machining process performance.

The problem of determining the optimum machining parameters in sculptured surface machining is complex as

there are many different and contradictory parameters must be simultaneously adjusted. One important such parameter is the cutting force generated during the machining process. Cutting forces are the main factor governing machining accuracy, surface quality, machine tool vibration, power requirements and tool life and hence the ability to predict them is useful for the design of machine tool structure and cutting tools as well as for the control and optimization of machining processes, [1]. Predicted cutting forces may give information about cutter deflections which lead to dimensional errors, machine tool chatter and tool breakage. Thus tool life and surface integrity can be optimized by selecting appropriate cutting conditions. The other factors that govern machining output include; cutting time, cutting tool cost, quality of surface of the machined part and the machining errors, [2].

When a CNC machine programmer is writing part program, machining parameters such as feedrate, spindle speed and cutting depth are programmed depending on his/her experience or stored machine database, [3]. Even with machining data in database, variations in material structure such as different hardness make it necessary to set these parameters extremely conservatively to avoid tool breakage or excessive tool wear, and as a result, the machine is usually operated far below the optimum operating conditions. To ensure quality surface integrity of products, reduce the machining cost and increase the machining efficiency, it is necessary to adjust the machining parameters during machining process in real-time, [3].

During the machining of sculptured surfaces, cutting forces change instantaneously because tool feed direction and uncut chip thickness vary with time, hence when modelling such forces it becomes necessary to consider true kinematics of the cutting edges, Ikua et al (2001). An adaptive controller is used in machining process in manufacturing industries to take care of these variables and unpredictable cutting conditions and to optimize machining process by adjusting the cutting parameters such as feedrate and spindle speed.

Classical and modern control theories have successfully been used in areas where the systems are well defined either deterministically or stochastically, but they cannot cope with the needs of manufacturing industries because of the complexity and vagueness of practical processes, [4]. Fuzzy control and fuzzy systems have the benefit of replicating all desired features of human input, while maintaining all the advantages of classical and modern control theories. Control rules are

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presented in a form of IF-THEN, statements, [5]. In this paper, an adaptive fuzzy logic controller for regulation of feedrate, based on the emanating cutting forces in ball-end milling of spherical surfaces is developed.

A. Modeling of ball-end milling

1) Geometry of ball-end mill: Ball-end mills are widely used in machining of sculptured surfaces. Figure 1 shows a ball end mill with helical cutting edges and the profile of the cutting edge at the ball section.



The radius of the ball part is equal to the radius of cylindrical part of the tool. The helix angle of the cutting edge varies along the cutting edge, from a minimum at the lowest point M, to a maximum at the spherical /cylindrical interface N. Point O is the center of the ball, and P is an arbitrary point on the cutting edge. The location of the cutting point P is described by an angle ϕ between line \overline{OP} and the z-axis, and a rotational angle ψ of the cutting point, from the y-axis, which is given by

$$\psi = \psi_M - \int_{\phi} \gamma d\phi, \tag{1}$$

Where ψ_M is the rotational angle between the y-axis and a tangent to the cutting edge at the lowest point M of the ball end mill, and γ is the helix angle at the point. This local helix angle is obtained through measurement as a function of the angle ϕ , that is $\gamma = f(\phi)$.

2) Model for ball-end milling: In this paper two cutting styles are considered i.e., contouring and ramping. In the contouring, the tool cuts along the latitude of the sphere while in the ramping; the cutting is along the longitude. These cutting styles are shown in Figure 2 (a) and (b).



Fig. 2 Various cutting styles and cutting modes

In this figure, f_p is the cross-feed and θ is the milling position angle. Different cutting modes, which are named with reference to the direction of cross-feed and tool-feed are considered. In contouring cutting style, the cross-feed is referred to as "Up cross-feed" if the z-component of the toolfeed is in the positive z-direction and "Down cross-feed" if it is in the negative z-direction. The feed is referred to as "Up-cut" if the direction of motion of cutting edge due to rotation is the same as that of the tool-feed, and "Down cut" if the direction of motion of the cutting edge due to rotation is opposite that of tool-feed. In ramping, the feed is referred to as "Upward cut" when the z-component of the tool feed is in positive z-direction and "Downward cut" when it is in negative z-direction.

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(b) ramping

Fig. 3 Cutting models

In this paper four cutting modes are considered that is. Up cross-feed, Up-cut (U-U), Up cross-feed, Down cut (U-D), Down cross-feed, Up-cut (D-U) and Down cross-feed, Down cut (D-D). Figure 3 shows two cutting models that are used in this paper. In contouring, a point P on the cutting edge follows a epitrochoidal path, which is defined by the following general parametric equations,

$$\xi_P = r(\phi) \sin\left(\psi - \frac{2\pi n n_t f + f n_t \psi}{2\pi R_{ef}}\right) + R_{ef} \sin\left(\frac{2\pi n n_t f + f n_t \psi}{2\pi R_{ef}}\right)$$
(2)

$$\mu_P = r(\phi) \cos\left(\psi - \frac{2\pi nn_t f + fn_t \psi}{2\pi R_{ef}}\right) + R_{ef}$$
$$-R_{ef} \cos\left(\frac{2\pi nn_t f + fn_t \psi}{2\pi R_{ef}}\right),$$
$$n = 1, 2, 3.....$$

where, f is the feed rate, n_t is the number of cutting edges, n is the number of revolutions, ψ and ϕ are the rotational and locational angles of the point, respectively, and $r(\phi)$ is the radius of the arc traced by the cutting point, $R_{ef} = R_{eff} \cos \theta$, where R_{eff} is the effective radius given by, $R_{eff} = R + r_b - h$, R is the radius of the workpiece, r_b is radius of the ball end mill and h is nominal depth of cut.

In "Down cut", f is taken as negative. By considering equations of loci traced in two consecutive cuts for a tool with one cutting edge, we obtain the equation

$$r(\phi) \left[\sin\left\{ \frac{2\pi R_{ef} - f}{2\pi R_{ef}} \cos^{-1}[G] \right\} - \sin(\psi - K) \right]$$
$$+ R_{ef} \left[\sin\left\{ \frac{f}{2\pi R_{ef}} \cos^{-1}[G] \right\} - \sin(K) \right]$$
(3)
$$- d\sin\left(\frac{2\pi f + f\psi}{2\pi R_{ef} - \psi} \right) = 0$$

Where,

$$G = \cos \psi - \frac{d^2}{2r(\phi)R_{ef}} + \frac{d}{R_{ef}} - \frac{d\cos \psi}{r(\phi)} \text{ and}$$
$$K = \left(\frac{2\pi f + f\psi}{2\pi R_{ef}}\right)$$

from which the chip thickness d in the horizontal plane can be determined using numerical methods.

B. Instantaneous cutting forces

The cutting process can be considered as an aggregation of small oblique cuttings along the helical cutting edge of the ball end mill tool. Therefore, oblique cutting theory is used to calculate the cutting forces. The radial and tangential components of differential cutting forces acting on an infinitesimal element of the cutting edge shown in Fig 4, are given by

where $s(\phi, \psi)$ is the instantaneous depth of cut, k_r and k_t are the radial and tangential edge force coefficients, which account for ploughing and rubbing, k_{rc} and k_{tc} are radial and tangential cutting coefficients given by [5].

$$k_{rc} = \frac{\tau_s \sin(\beta - \alpha_e)}{\sin \phi_s \cos(\phi_s + \beta - \alpha_e)}$$
$$k_{tc} = \frac{\tau_s \cos(\beta - \alpha_e)}{\sin \phi_s \cos(\phi_s + \beta - \alpha_e)}$$
(5)

where, τ_s is the shearing strength of the workpiece material, β is the friction angle, ϕ_s is the shear angle and α_e is the effective rake angle. Due to variation of the helix angle along the cutting edge of the tool, the effective rake angle is calculated for each element of the cutting edge. According to the Stabler's rule of chip flow [6], the chip flow direction angle is approximately equal to the helix angle. Then the effective local rake angle can be determined in terms of local helix angle γ and normal rake angle α_n as

$$\sin \alpha_e = \sin^2 \gamma + \cos^2 \gamma \sin \alpha_n \tag{6}$$



Fig. 4 Elemental cutting forces

The elemental cutting forces in Eq. (4) are projected to the cartesian coordinates system as follows

$$\left\{\begin{array}{c}
dF_x\\
dF_y\\
dF_z
\end{array}\right\} = \mathbf{T} \left\{\begin{array}{c}
dF_t\\
dF_r
\end{array}\right\}$$
(7)

Where the matrix T is given by

$$\mathbf{T} = \begin{bmatrix} -\cos\psi & -\sin\phi\sin\psi\\ \sin\psi & -\sin\phi\cos\psi\\ 0 & \cos\phi \end{bmatrix}$$
(8)

The cutting force components are obtained by performing numerical integration of the elemental forces in Eq. (7), for the engaged part of the cutting edge. Fig. 5 show a photo of experimental set up.



Fig. 5 Experimental set up

II. DESIGN OF FUZZY CONTROL SYSTEM FOR FEEDRATE OPTIMIZATION

A. FLC structure

The schematic diagram for the fuzzy control system is shown in Fig. 6. In this paper, feedrate is varied depending on the force generated during machining process in contour machining of spherical surfaces. Since during contour machining there is no tool feed in z-axis direction, the cutting force component in the z-direction is small and stable which in this case can be neglected for constant cutting force control. The cutting forces that are fed to the input of the FLC constitutes F_X and F_Y components measured from dynamometer. The controller has four inputs, namely, force error in x-axis ($\triangle X$), change in force error in x-axis ($\triangle^2 X$), force error in y-Axis ($\triangle Y$) and change in force error in y-axis ($\triangle^2 Y$). ΔX and ΔY are computed from the difference of crisp values of the reference and measured cutting forces. Thus, $\Delta X = F_{X_{Ref}} - F_X$ and $\Delta Y = F_{Y_{Ref}} - F_Y$, while $\Delta^2 X$ and $\Delta^2 Y$ are computed from the difference in errors for two consecutive sampling intervals

$$\Delta^2 X = \Delta X(k+1) - \Delta X(k)$$

$$\Delta^2 Y = \Delta Y(k+1) - \Delta Y(k)$$

where k is the sampling interval.



Fig. 6 Schematic diagram of fuzzy logic controller for milling process

The design of the fuzzy logic controller (FLC) involves; identification of the inputs and outputs including their ranges, design of the fuzzy membership function for each input and output, construction of the knowledge base that contains the fuzzy rules used to operate the system, fuzzy decision making or inference mechanism that performs fuzzy reasoning and defuzzification to determine the crisp control output. The output of FLC is feed-rate-percentage-override (FRPO). In this paper FRPO varies from 0-150% depending on the error and change in error between the reference and measured instantaneous cutting force. The actual feedrate is computed by multiplying the preprogrammed feedrate by the FRPO.

B. Membership functions

To optimize the feedrate, the FLC is employed which controls the cutting force either by increasing or reducing the feedrate during the machining process. The FLC for cutting force control is a typical multiple-input-single-output (MISO) control system with four inputs and one output. The input linguistic variables to the FLC are, force error in X-Axis $(\triangle X)$, change in force error in X-axis $(\triangle^2 X)$, force error in Y-axis ($\triangle Y$) and change in force error in Y-axis ($\triangle^2 Y$). In the design of FLC, the machine operator's intuition and experience were used to define the fuzzy variables. For each input linguistic variable, there are three linguistic terms. These include Negative(Ne), Zero(Ze), and Positive (Po). In defining the fuzzy linguistic terms for the input fuzzy variables, triangular membership functions were considered. Since the aim of controller is to minimize the error between the set and measured force, the range for all input fuzzy variables was set at $(-1.5 \sim 1.5 \text{N})$. The output of FLC is comprised of feedrate percentage override (FRPO) which has five fuzzy linguistic terms i.e, Negative small (NS), Negative medium (NM), Zero (Ze), Positive medium (PM), Positive large (PL). In defining the linguistic terms for output linguistic variable, trapezoidal membership functions were considered. Figure 7 and 8 shows the membership functions for input and output linguistic variables respectively.

C. Rule base

The set of control rules defines the system behavior and replaces the mathematical modeling of the system. The fuzzy



Fig. 7 Membership function for force error in X-Axis



Fig. 8 Membership function for Feed rate percentage override

rules, which use fuzzy inputs to determine system actions, are obtained from the skilled operators, experiments, and prior knowledge of the end milling processes. The collection of fuzzy control rules that are expressed as fuzzy conditional statements forms the rule base or the rule set of an FLC. In the design of FLC rule base, multiple "IF-THEN" statement are joined by connective AND. In order to maintain cutting force to the optimum value, the error between reference and measured force is minimized as much as possible by varying table feed. The actual table feed is computed by multiplying the preprogrammed table feed by the FRPO. A total of 81 control rules were formulated as shown in the Table I.

TABLE I Fuzzy rules for cutting force control

					Feedrate
Rules	ΔX	$\Delta^2 X$	ΔY	$\Delta^2 Y$	percentage
					override
1.	Ne	Ne	Ne	Ne	Ns
2.	Ne	Ne	Ne	Ze	Ns
3.	Ne	Ne	Ne	Ро	Nm
4.	Ne	Ne	Ze	Ne	Ns
5.	Ne	Ne	Ze	Ze	Nm
6.	Ne	Ne	Ze	Ро	Ze
7.	Ne	Ne	Ро	Ne	Nm
8.	Ne	Ne	Ро	Ze	Ze
9.	Ne	Ne	Ро	Ро	Ze
:	:	:	:	:	:
81.	Po	Ро	Po	Ро	PL

D. Fuzzy Inference Mechanism

In this paper the Mamdani inference system is used because of its widespread acceptance. Since fuzzy rules used have multiple antecedents, the fuzzy operator (AND) is used to obtain the firing levels of the rules. The results of the antecedent evaluation is applied to the membership function of the consequent to obtain the overall control action. In order to evaluate the conjunction of the fuzzy rule antecedents, the AND fuzzy operation is used as follows.

$$\mu_{\triangle X} \cap \mu_{\triangle^2 X} \cap \ldots = \min\left(\mu_{\triangle X}, \mu_{\triangle^2 X}, \ldots\right) \tag{9}$$

The center of gravity (COG) defuzzification method is used to come up with the overall control action which varies the machine table feed depending on the forces generated during the machining process as shown in Fig. 9.



Fig. 9 Mamdani Inference system

III. RESULTS AND DISCUSSION

A. Cutting forces

In designing a fuzzy controller for cutting force control, optimal cutting force from the force prediction model is used as a reference to the FLC. The fuzzy logic controller works in a closed loop to control cutting force by minimizing the error between the measured and reference force.

The proposed design of the fuzzy logic controller was implemented and tested using a personal computer in LabVIEW environment. The controller was tested on a precision, light duty 3-axis CNC milling machine (DENFORD TRIAC PC) with FANUC Numerical controller.

The cutting force signal in X- and Y-axis components were captured in a computer via a PCI data acquisition card (NI PCI-6259) which was connected to the strain amplifiers via a 68-pin shielded connector block (NI SCB-68). The workpiece material was aluminium Al 6063 T4.

Typical waveforms of the predicted and measured instantaneous cutting forces of spherical surfaces for U-D and D-D cutting modes at milling position angle $\theta = 45^{\circ}$ are shown in Fig. 10 (a) and (b). The dotted line in these figures represent the predicted forces, and continuous line represent the measured one. It can be shown that in all cutting modes there is a close correspondence between the predicted and



Fig. 10 Typical waveforms of cutting forces (Milling position angle θ =45 °)

measured forces. A summary of cutting conditions is shown in Table II.

The measured forces were compared with reference optimum force from the force prediction model to get force error for both X- and Y-axis components. From the computed force errors, change in force error was calculated using consecutive sampling time.

The control algorithm was designed such that the machining process occurred at constant force. This approach is desirable because it increases the machining efficiency by considering the compromise between tool life and material removal rate. The peak cutting forces F_x and F_y were specified as 120 N and 40 N, respectively.

Ũ	
Spindle speed, N	300mm^{-1}
Feed rate, f	0.1mm/tooth
Milling position angle, θ	15° -75°
Cross-feed, f_p	1.0mm
Depth of cut, h	1.0mm
Tool overhang length, L	45mm
Work piece radius, R	40mm

B. Feedrate response

First, the effect of depth of cu on cutting forces was determined while keeping the feedrate constant. Figure 11

shows the influence of depth of cut on the three orthogonal components of cutting forces for U-U cutting mode. It can be seen that the x-component is the one largely affected by the depth of cut. For this reason, the x-component was used in establishing the feedrate response on varying the depth of cut. Figure 12 shows the cutting force response when there is no controller applied. When the radial depth of cut is step changed from 1.0 mm to 1.5 mm, the peak cutting force increased, just as expected. This sudden increase can be seen at after 1 second in the figure.







Fig. 12 Cutting at a constant feed without controller(30 mm/min)



Fig. 13 Feedrate response on change in radial depth of cut

Figure. 13 shows feedrate response on change in radial depth of cut. To demonstrate constant peak force control, the radial depth of cut was abruptly changed from 1.0 to 1.5 mm. As a result, the peak cutting forces increased and the FRPO from FLC's output was recorded. In order to counteract this increase and maintain a constant peak force, the preprogrammed table feed was decreased by assigning the appropriate override. For the above change in depth of cut, the FRPO was found to be about 78%, which is a drop of table feed from 30 mm/min to 23.4 mm/min as shown in Fig. 13.b. It can be seen that the peak cutting force was maintained at a constant value of about 120 N despite the change in radial depth of cut.

IV. CONCLUSION

In this paper a fuzzy logic control algorithm for optimizing the feedrate by controlling the peak cutting forces was developed and implemented in LabVIEW environment. The algorithm was such that the machining occurred at a constant peak force. This had a compromise between the tool life and MRR in that when cutting force exceeded the reference peak force, the controller reduced the table feed thus preventing tool damage. When the force was below the reference force, the controller increased the feedrate hence increasing MMR.

A step change in depth of cut from 1.0 mm to 1.5 mm, resulted in a change in table feed from preprogrammed 30 mm/min to about 23.4 mm/min. Conventionally when the NC program is being designed, it is usually based on the maximum depth to select a conservative feedrate for the whole machining process. With the introduction of constant cutting force control loop, feedrate is adjusted in real-time based on the feedback to increase the machining efficiency. The proposed controller has an advantage over the conventional controller such as PID in that one does not have to model the plant especially

when it come to milling of sculptured surfaces since in such cases most of the parameters do not relate linearly and sometimes are ill defined.

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