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Kinematic Parameter-Independent Modeling and Measuring of Three-Axis Machine Tools

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Abstract**—**The primary objective of this paper was to construct a "kinematic parameter-independent modeling of three-axis machine tools for geometric error measurement" technique. Improving the accuracy of the geometric error for three-axis machine tools is one of the machine tools' core techniques. This paper first applied the traditional method of HTM to deduce the geometric error model for three-axis machine tools. This geometric error model was related to the three-axis kinematic parameters where the overall errors was relative to the machine reference coordinate system. Given that the measurement of the linear axis in this model should be on the ideal motion axis, there were practical difficulties. Through a measurement method consolidating translational errors and rotational errors in the geometric error model, we simplified the three-axis geometric error model to a kinematic parameter-independent model. Finally, based on the new measurement method corresponding to this error model, we established a truly practical and more accurate error measuring technique for three-axis machine tools.

Keywords**—**Three-axis machine tool, Geometric error, HTM, Error measuring

I. Introduction

ENHANCING the accuracy of CNC machine tools is an important task in the area of machine tools. Errors which influence a machine tool's accuracy primarily originate from three categories: structurally-induced errors, driver-induced errors, and quasi-static errors. According to relevant research reports, quasi-static errors account for 70% of volume errors in CNC machine tools. This kind of error includes both geometric and thermal errors.

This paper researched geometric errors in quasi-static errors. The technique of building machine tool's geometric error model is well developed in the past few years [1]–[5]. The error model describes the position and orientation errors of tool relative to workpiece at specific machine position, whereby inaccurate influential factors come from kinematic link parameters and individual error sources. It is well known that the inaccurate motion of a linearly driven axis is associated with six motional errors, including one linear error, two straightness errors, and three rotational errors. With modern measurement devices such as the 6D laser interferometer [6], all six motional errors of the linearly driven axis can be measured rapidly. Based on the error model, the accuracy of three-axis machine tools can be dramatically improved through the error compensation [7]-[8].

Since 2008 a total volumetric compensation by Siemens for the controller 840 D and Heidenhain iTNC 530 in 2009. These functions allow for increasing the accuracy of machining centers if the volumetric errors were initially determined using suitable measuring technology. With the LaserTRACER [9] offers an efficient and high-precision measurement system for volumetric calibration.

Currently, geometric error modeling depends on the three-axis machine kinematic chain to create a geometric error model of three-axis machine tools, and the home position for which each motion axis is regarded as the motion axis's reference coordinate system. For this reason kinematic parameters between the coordinate systems for the linear axes and the rotary axes are needed to effective describe their relationship of motion. However, the ideal motion axis line and the center of revolution of the linear motion slide is difficult to define precisely, and therefore the kinematic parameter value cannot be defined. Furthermore, the fact that geometric errors defined on the ideal axis line of the linear motion slide must be measured by placing the measurement device on this axis line to avoid Abbe's error creates practical measuring difficulties when the linear motion slide is at a high position or when there is interference. The overall errors on the tool end in the geometric error model with kinematic parameters constructed based on the machine reference coordinate system. In actual machining, however, a certain point on the workpiece will be set as the origin of the workpiece coordinate system, which will be the error-free position. The errors will then correspond to this point rather than corresponding to the machine reference coordinate system.

For this reason, current errors modeling methods face the following three practical issues:

(1) The kinematic parameters in the model are unable to be accurately obtained.
(2) Avoiding causing the Abbe error during geometric error measurement creates practical operational difficulties with the applied measuring device.
(3) The largest problem with using traditional modeling and measurement methods is that the error model includes kinematic parameters which have a bearing on the contribution of rotational errors to overall errors: rotational errors measuring inaccuracy will magnify uncertainty of machine tools accuracy with overall errors, thus increasing the uncertainty in the error model.

Therefore it is necessary to establish a new modeling, measurement method for geometric errors of three-axis machine tools, which is more practical, convenient and accurate.

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II. DEFINING GEOMETRIC ERRORS FOR LINEAR AXES

Definitions in ISO230 related to error inspection standards for CNC machine tools include the definition for geometric errors and the method for test. A single linear motion axis is defined to possess six component errors(three translational errors and three rotational errors), and a location (perpendicularity) error exists between two linear motion axes. According to the above definitions, a three-axis machine tool with three linear axes would have a total of 21 geometric errors.

To describe three-axis machine tool geometric overall errors, it is necessary to establish a geometric error model for the target machine. Assuming the structure of the machine tool is a rigid body, then a 4x4 HTM could be used to show the relationship between each kinematic and servo control axis, and the machine error model could go through an individual kinematic and driver components HTM to obtain the order of products, depending on the machine kinematic chain [1].

Fig. 1 displays a case study for the X-axis linear motion slide. The geometric error model for kinematic parameters, location errors, and component errors in X-axis linear slide, showing the relationship of the x coordinate system with respect to the reference coordinate system ${}^{r}T_{x}$, is shown in the formula below.

$$
{}^{r}T_{x} = \begin{bmatrix} 1 & 0 & 0 & X_x \\ 0 & 1 & 0 & Y_x \\ 0 & 0 & 1 & Z_x \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -COX & 0 & 0 \\ COX & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -ECX & EBX & X_m + EXX \\ ECX & 1 & -EAX & EYX \\ -EBX & EAX & 1 & EZX \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)
$$

where X_x, Y_x, Z_x are the constant offset which the x home position with respect to the reference coordinate system in the x,y,z direction respectively, or the kinematic parameter for X-axes linear slide. *COX* is the location error between linear X axis and an ideal linear axis (in this example, Y-axis of the reference coordinate system) which will cause a small angular rotation at between two coordinate systems at the Z axial direction. *EXX*, *EYX*, *EZX*, *EAX*, *EBX* and *ECX* are the six component errors for linear X axis, and X_m is the servo-controlled position of the X-axis slide.

The order of products for the kinematic parameter matrix, the location (perpendicularity) error matrix, and the 6D component error matrix in the above formula is dependent upon the pattern arrangement in linear X axis's kinematic chain. First the 6D component errors matrix for the X axis linear slide. And assuming that when the X-axis slide goes home position the Z-axis of the X coordinate system is identical with the Z-axis of the reference coordinate system, then perpendicular error *COX* exists between the ideal motion axis (the X-axis of the X coordinate system) and the Y-axis of the reference coordinate system, and so does the perpendicularity error matrix. When X axis slide moves to the X home position, the X axis slide having the kinematic parameter matrix for the origin coordinate offsets.

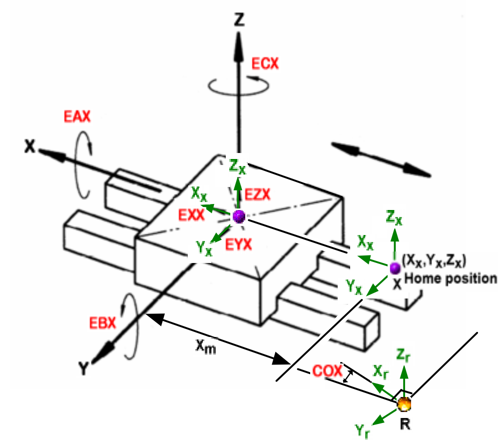

Fig. 1 X linear axis geometric error definition

III. MODELING AND MEASUREMENT WITH KINEMATIC PARAMETER-INDEPENDENCE

A. Geometric Error Modeling

For an ideal three-axis machine tool, each tool position (X_w, Y_w, Z_w) and orientation (I_w, J_w, K_w) on the workpiece coordinate system for the three machine motion axes has a corresponding drive position to cut the needed work pieces and the tool orientation can only be defined on the (0,0,1) direction. Fig. 2 is the three-axis machine tool (Coordinate Measuring Machine, CMM) and its coordinating system definition. The machine's kinematic chain is linked by several link components and three linear motion axes. One end of the chain is a tool holder and the holder should hold the tool. The spindle block is mounted on the Z-slide. The Z-slide moves vertically with a prismatic joint. The Z-slide is bolted on the X-slide and the X-slide is then stacked on the Y-slide, making the three linear axes (x,y,z) perpendicular to each other. Y-slide is then moves on the beds with a prismatic joint. Finally, based on the ISO230 definition and this machine's kinematic chain sequence, the location errors are *COX*, *BOZ*, and *AOZ*.

Based on Fig. 3, the relationship of the tool (T) coordinate system with respect to the holder (H) coordinate system, ${}^{h}T_{t}$, is shown in the below.

$$
{}^{h}T_{t} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & Z_t \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)
$$

where Z_t is the length of the tool (probe).

The holder coordinate system with respect to the Z coordinate system, ${}^{z}T_{h}$, is expressed in the formula below.

$$
{}^{z}T_{h} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & Z_h \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)
$$

where Z_h is the Z directional offset of the holder origin in relation to the origin of the Z axis coordinate system.

The Z axis coordinate system with respect to X axis coordinate system, ${}^{x}T_z$, is express in the formula below.

$$
{}^{x}T_z = \begin{bmatrix} 1 & 0 & 0 & X_z \\ 0 & 1 & 0 & Y_z \\ 0 & 0 & 1 & Z_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & BOZ & 0 \\ 0 & 1 & -AOZ & 0 \\ -BOZ & AOZ & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -ECZ & EBZ & EXZ \\ ECZ & 1 & -EAZ & EYZ \\ -EBZ & EAZ & 1 & Z_m + EZZ \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}
$$

where X_z, Y_z, Z_z are the offsets for Z home position in relation to X home position. *AOZ* and *BOZ* are location (perpendicular) errors for Z linear motion axis in relation to Y and X axis, respectively. *EXZ*, *EYZ*, *EZZ*, *EAZ*, *EBZ* and *ECZ* are the six component errors for Z linear axis, and Z_m is the servo-controlled position of the Z servo-axis.

The X axis coordinate system with respect to the Y coordinate system, ${}^{y}T_x$, is expressed in the formula below.

$$
{}^{y}T_x = \begin{bmatrix} 1 & 0 & 0 & X_x \\ 0 & 1 & 0 & Y_x \\ 0 & 0 & 1 & Z_x \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -COX & 0 & 0 \\ COX & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -ECX & EBX & X_m + EXX \\ ECX & 1 & -EAX & EYX \\ -EBX & EAX & 1 & EZX \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}
$$

where X_x, Y_x, Z_x are offsets for X home position in relation to Y home position. *COX* is the location (perpendicular) error for X linear motion axis in relation to Y axis. *EXX*, *EYX*, *EZX*, *EAX*, *EBX* and *ECX* are the six component errors for X linear axis, and X_m is the servo-controlled position of the X servo-axis.

The Y axis coordinate system with respect to the reference coordinate system, ${}^{r}T_y$, is expressed in the formula below.

$$
{}^{r}T_y = \begin{bmatrix} 1 & -ECY & EBY & EXY \\ ECY & 1 & -EAY & Y_m + EYY \\ -EBY & EAY & 1 & EZY \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}
$$

where *EXY*, *EYY*, *EZY*, *EAY*, *EBY* and *ECY* are the six component errors for Y linear axis, and Y_m is the servo-controlled position of the Y servo-axis. In the above equation, the Y linear motion axis 6D error matrix follows the errors created by the ideal axis movement. In the process of deducing the entire error model, assuming that when Y motion axis goes to the Y home position the Y coordinate system is identical to the reference coordinate system, then the ideal axis line should also be identical to the Y-axis in the reference coordinate system and no perpendicular error exists between the Y coordinate system and the reference coordinate system.

Deducing another kinematic chain, Fig. 3 indicates that the end of the three-axis machine tool aligns with the end of the workpiece. For this reason, the workpiece coordinate system is defined on the end of the machine tool and the workpiece coordinate system (w) with respect to the workpiece origin coordinate system, ${}^{wo}T_w$ is expressed in the formula below.

$$
{}^{wo}T_w = \begin{bmatrix} 1 & 0 & 0 & X_w \\ 0 & 1 & 0 & Y_w \\ 0 & 0 & 1 & Z_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}
$$

where X_w, Y_w, Z_w is the translational offset for the workpiece coordinate system (w) in respect to the workpiece origin coordinate system (wo), which can be accurately defined through measurement tools.

The workpiece origin coordinate system (wo) with respect to the reference coordinate system (r), ${}^{r}T_{wo}$, without geometric errors is expressed in the formula below.

$$
{}^{r}T_{wo} = \begin{bmatrix} 1 & 0 & 0 & X_{wo} \\ 0 & 1 & 0 & Y_{wo} \\ 0 & 0 & 1 & Z_{wo} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}
$$

where X_{wo} , Y_{wo} and Z_{wo} are the translational offset for the workpiece origin coordinate system (w) in respect to reference coordinate system (r).

For this reason, the spatial relationship between the tool coordinate system and the reference coordinate system can be obtained through the formula below.

$$
{}^{r}T_t = {}^{r}T_y \; {}^{y}T_x \; {}^{x}T_z \; {}^{z}T_h \; {}^{h}T_t \tag{9}
$$

The spatial relationship between the workpiece coordinate system and the reference coordinate system can be obtained through the formula below.

$$
{}^{r}T_w = {}^{r}T_{wo} \; {}^{wo}T_w \tag{10}
$$

Fig. 3 illustrates that, when it is an ideal machine, the tool coordinate system should be an identical point with the workpiece coordinate system. However, actual machines have geometric errors, so the position of the origin of the tool coordinate system with respect to the reference coordinate system $\boldsymbol{P_t} = [X_t \;\; Y_t \;\; Z_t]$, can be obtained through the formula below.

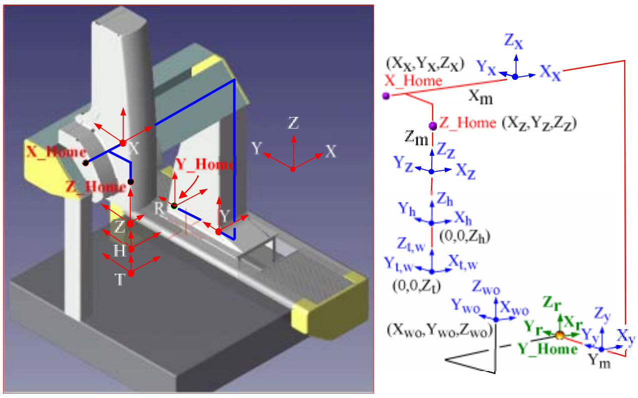

Fig. 2 Three-axis machine tools

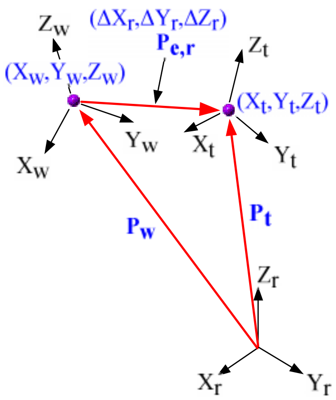

Fig. 3 Overall errors of the tool end

$$[\boldsymbol{P_t} \quad 1]^T = {}^rT_t[0 \quad 0 \quad 0 \quad 1]^T \tag{11}$$

The position of the origin of the workpiece coordinate system with respect to the reference coordinate system $\boldsymbol{P_w} = [X_w \quad Y_w \quad Z_w]$, can be obtained through the formula below.

$$[\boldsymbol{P_w} \quad 1]^T = {}^rT_w[0 \quad 0 \quad 0 \quad 1]^T \tag{12}$$

Now, the position error for the tool coordinate system with respect to the workpiece coordinate system in the reference coordinate system $\boldsymbol{P_{e,r}}(\Delta X_r, \Delta Y_r, \Delta Z_r)$ can be obtained through the formula below.

$$\boldsymbol{P_{e,r}} = \boldsymbol{P_t} - \boldsymbol{P_w} \tag{13}$$

The orientation error in the reference coordinate system $\boldsymbol{O_{e,r}}(\Delta I_r, \Delta J_r, \Delta K_r)$ can be obtained through the three formulas listed below.

$$[\boldsymbol{O_w} \quad 0]^T = ({}^rT_w - {}^rT_{w,ideal})\ [0 \quad 0 \quad 1 \quad 0]^T \tag{14}$$

$$[\boldsymbol{O_t} \quad 0]^T = ({}^rT_t - {}^rT_{t,ideal})\ [0 \quad 0 \quad 1 \quad 0]^T \tag{15}$$

$$\boldsymbol{O_{e,r}} = \boldsymbol{O_t} - \boldsymbol{O_w} \tag{16}$$

where ${}^rT_{w,ideal}$ and ${}^rT_{t,ideal}$ are the HTM for the workpiece coordinate system and tool coordinate system with respect, individually, to the reference coordinate system for ${}^rT_w$ and ${}^rT_t$, respectively, when geometric errors are not considered (the ideal machine).

Using small-angle approximations assumption and the second-order errors are negligible, and consolidating the geometric errors, the geometric error model for this three-axis machine tool is displayed in Table I. The overall error for the direction of X, ΔX_r, is the product of each error multiplied by each error's error gain. For example, the error contribution for the direction of X in *ECX* is $-ECX*Y_z$. This table, which is considered a geometric error sensitivity analysis table, indicates that linear errors (*EXX*, *EYX*, *EZX*, *EXY*, *EYY*, *EZY*, *EXZ*, *EYZ*, and *EZZ*) are machine kinematic parameter-independent, while rotary errors (*EAX*, *EBX*, *ECX*, *EAY*, *EBY*, *ECY*, *EAZ*, *EBZ*, *ECZ*, *COX*, *AOZ*, and *BOZ*) are machine kinematic parameter-dependent.

TABLE I
ERROR MODEL AND SENSITIVITY AND ANALYSIS

Error	⊿ X_r	⊿ Y_r	⊿ Z_r	⊿ I_r	⊿ J_r	⊿ K_r
EXX	1	0	0	0	0	0
EYX	0	1	0	0	0	0
EZX	0	0	1	0	0	0
EAX	0	$-Z_h-Z_t-Z_m-Z_z$	$+Y_z$	0	-1	0
EBX	$+Z_h+Z_t$ $+Z_m+Z_z$	0	$-X_z$	1	0	0
ECX	$-Y_z$	$+X_z$	0	0	0	0
EXY	1	0	0	0	0	0
EYY	0	1	0	0	0	0
EZY	0	0	1	0	0	0
EAY	0	$-Z_h-Z_t$ $-Z_m-Z_z-Z_x$	$+Y_z+Y_x$	0	-1	0
EBY	$+Z_h+Z_t+Z_m$ $+Z_z+Z_x$	0	$-X_z-X_m$ $-X_x$	1	0	0
ECY	$-Y_z$	$+X_z+X_m$ $+X_x$	0	0	0	0
EXZ	1	0	0	0	0	0
EYZ	0	1	0	0	0	0
EZZ	0	0	1	0	0	0
EAZ	0	$-Z_h-Z_t$	0	0	-1	0
EBZ	$+Z_h+Z_t$	0	0	1	0	0
ECZ	0	0	0	0	0	0
COX	$-Y_z$	$+X_z+X_m$	0	0	0	0
AOZ	0	$-Z_h-Z_t-Z_m$	0	0	-1	0
BOZ	$+Z_h+Z_t+Z_m$	0	0	1	0	0

B. Measurement for Kinematic Parameter-independent

In defining geometric errors and deducing formulas above, the three-axis machine tool linear axis was structured by kinematic stacking and each motion axis had a home position. For this reason, kinematic parameters were necessary between linear axis coordinate systems to effectively describe their movement relative to each other. However, in practice, the position of the ideal motion axis line for the linear motion slide was difficult to clearly define. Moreover, to avoid Abbe's error, the measurement device must be placed on this axis line when measuring. This requirement creates practical measurement difficulties if the linear motion slide is at a high position or there is interference. For this reason, it is necessary to establish a new measurement method for a geometric error model without kinematic parameter.

Ideally, the geometric error model coordinate system should be set up on the ideal motion axis line for the linear slide to effectively describe the spatial errors caused by Abbe's error. For example, measurement of the Y linear slide, displayed in Fig. 5, had three translational errors (*EXY*, *EYY* and *EZY*) and three rotational errors (*EAY*, *EBY* and *ECY*). If, when measuring geometric errors, directions x,y,z between measurement axis line (M) and ideal motion axis line (I) each have offset L_x, L_y, L_z, then the 6D component error model for the measurement construction method and the results of the measurement are:

$$EXY' = EXY + L_x*(1-\cos(EBY)) + L_x*(1-\cos(ECY)) + L_y*\sin(ECY) + L_z*\sin(EBY) \quad (17)$$

$$EYY' = EYY + L_x*\sin(ECY) + L_y*(1-\cos(EAY)) + L_y*(1-\cos(ECY)) + L_z*\sin(EAY) \quad (18)$$

$$EZY' = EZY + L_x*\sin(EBY) + L_y*\sin(EAY) + L_z*(1-\cos(EAY)) + L_z*(1-\cos(EBY)) \quad (19)$$

$$EAY' = EAY \quad (20)$$

$$EBY' = EBY \quad (21)$$

$$ECY' = ECY \quad (22)$$

When the rotational error slightly angled, then $\cos(EAY) \fallingdotseq 1$, $\cos(EBY) \fallingdotseq 1$, $\cos(ECY) \fallingdotseq 1$, and $\sin(EAY) \fallingdotseq EAY$, $\sin(EBY) \fallingdotseq EBY$, $\sin(ECY) \fallingdotseq ECY$. These three formulas can be simplified to:

$$EXY' \fallingdotseq EXY + L_y*ECY + L_z*EBY \quad (23)$$

$$EYY' \fallingdotseq EYY + L_x*ECY + L_z*EAY \quad (24)$$

$$EZY' \fallingdotseq EZY + L_x*EBY + L_y*EAY \quad (25)$$

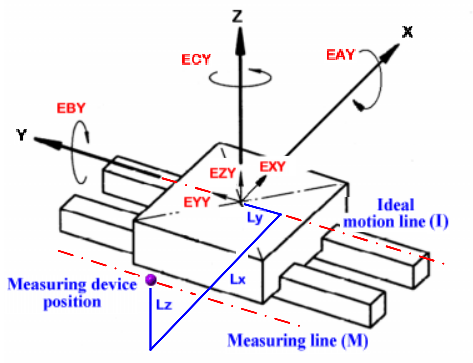

Fig. 4 Linear axis geometric error measuring method

As the above explanation indicates, when measuring rotational errors (*EAY*, *EBY* and *ECY*), the measuring line is independent of the position of the motion line so it is not necessary for the measuring device to be stacked on the ideal motion line (I). When measuring translational errors (*EXY*, *EYY* and *EZY*) however, the measurement position matters and therefore, the measurement device must be placed on the ideal motion line (I). If it is placed on line M from Fig. 5, then the spatial errors created by the rotational errors will be included with translational errors. Besides its own translational errors, the translational errors discovered by this method of measurement construction will also include errors which were created due to rotational errors. For this reason, the translational error measurement results obtained by this measurement method described above include the influence of rotational errors on the measurements. This result is explained in (23)-(25).

Additionally, when constructing this geometric error measuring, the kinematic parameter for L_x, L_y and L_z has a constant value. When the linear motion axis moves to a position, the spatial errors created by the rotational errors at that position (*EAY*, *EBY*, and *ECY*) will each be entered into the translational errors (*EXY*, *EYY*, and *EZY*) and the measuring line for this measurement device can be considered the ideal motion line for the linear motion axis, meaning rotational errors have no spatial errors for any position on this measuring line. Since the error gain of rotational errors is 0, the measuring position is the initial error position for rotational errors. Furthermore, in actual cutting and measuring, a certain position on the workpiece will be established as the origin of the workpiece coordinate system. Set up as an error-free position, all work position errors are no longer errors with respect to the geometric error model constructed by the machine ideal motion line but errors with respect to this point. For this reason, this measuring method has practical application value.

C. Error Model with Measurement Method

Using API 6D laser interferometer instrument as an example of applying the methods and principles of the measuring method described above to three-axis machine tools, we installed a reflect mirror to the tool holder on the spindle of the machine in Fig. 2 to individually measure the six component errors in a linear motion axis and the location (perpendicular) error for the three linear axes. When, for example, the 6D component errors were measured for Y linear motion axis, we first returned X, Y, and Z axes to their individual home positions, which were set as the zero error position, and then installed a reflect mirror to the tool holder on the machine's spindle to carry out measurements. At this point, because the measuring device's measurement position would react with Abbe's error, the Y axis 6D measurement results included all the errors created by the machine's kinematic parameter. Next, we measured the component and location (perpendicular) errors for the other two linear motion axes according to the principles described in the last section.

Applying the new measuring method to the three-axis CNC machine tool, we could simplify the original geometric error model containing kinematic parameters shown in Table I to the kinematic parameter-independent Table II. Considering, for instance, measuring the six component errors in X linear motion axis, there were three error contributions (*EZX*, *EAX* and *EBX*) to the tool end's overall errors, the contributing factors of which were 1, $Y_z, -X_z$. Under the premise that the machine possesses positioning repeatability, we can assume that when X axis slide is located at a specified position, the $Y_Z, -X_Z$ kinematic parameter will be a constant. Due to the fact that the reflection mirror was installed at the tool end of the spindle, the error contribution of $EAX * Y_Z$ and $EBX * (-X_Z)$ will be reflected in *EZX*. For this reason, these two kinematic parameters can be set to zero, and their other errors can be simplified in this way.

As Table II illustrates, all nine translational errors (*EXX*, *EYX*, *EZX*, *EXY*, *EYY*, *EZY*, *EXZ*, *EYZ* and *EZZ)* contribute to the tool end overall errors, but only five of the rotational errors (*EAX*, *EBX*, *EAY*, *EBY* and *ECY*) contribute while four (*ECX*, *EAZ*, *EBZ* and *ECZ*) do not. Therefore, only 17 (21-4) geometric errors need to be measured in this model. Also in Table II, considering the home positions for X, Y and Z motion axes in this model, the error gains for *EAY*, *EBY*, and *ECY* require revision to properly express the total physical significance of kinematic parameter. X_s, Y_s, and Z_s represent the stroke for X, Y, and Z linear motion axes, respectively.

TABLE II
ERROR MODEL WITH PARAMETRIC-INDEPENDENT

Error	$\triangle X_r$	$\triangle Y_r$	$\triangle Z_r$	$\triangle I_r$	$\triangle J_r$	$\triangle K_r$
EXX	1	0	0	0	0	0
EYX	0	1	0	0	0	0
EZX	0	0	1	0	0	0
EAX	0	$-Z_m$	0	0	-1	0
EBX	Z_m	0	0	1	0	0
ECX	0	0	0	0	0	0
EXY	1	0	0	0	0	0
EYY	0	1	0	0	0	0
EZY	0	0	1	0	0	0
EAY	0	$-(Z_s+Z_m)$	0	0	-1	0
EBY	Z_s+Z_m	0	X_s-X_m	1	0	0
ECY	0	$-(X_s-X_m)$	0	0	0	0
EXZ	1	0	0	0	0	0
EYZ	0	1	0	0	0	0
EZZ	0	0	1	0	0	0
EAZ	0	0	0	0	-1	0
EBZ	0	0	0	1	0	0
ECZ	0	0	0	0	0	0
COX	0	X_m	0	0	0	0
AOZ	0	$-Z_m$	0	0	-1	0
BOZ	Z_m	0	0	1	0	0

Constructing a kinematic parameter-independent three-axis geometric error model and measurement method based on the above measuring method is both practical and accurate. Furthermore, compensating for persistent geometric errors can also be facilitated by using this geometric error model to establish a geometric error compensation model to effectively compensate for three-axis geometric errors. The three-axis machine tool geometric error compensation scheme is displayed in Fig. 5. First, a laser interferometer device based on the above measurement construction method was used to measure the 21 geometric errors in the three axes. The measurement data was used to carry out coordinate translational, aligning it with the error model coordinate system. The measurement results were then plugged into the three-axis kinematic parameter-independent error model. The results indicated that when the three-axis machine tool moved to **u**(x,y,z) and the tool end spatial errors are **du**, then the compensation applied by the kinematic parameter-independent error compensation model is –**du**. Finally, the x,y,z motion axis direction errors, compensated through a controller, were returned to their ideal position at $\boldsymbol{u_c}$.

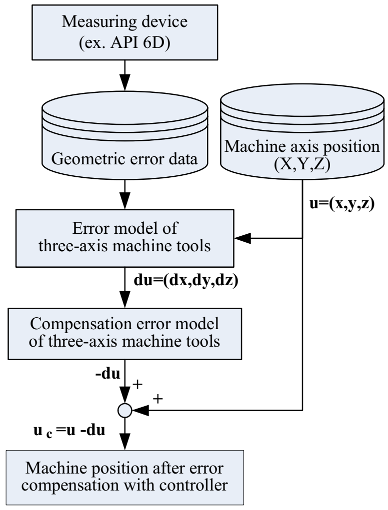

Fig. 5 Three-axis machine tools error compensation scheme

IV. CONCLUSION

The three-axis geometric error models derived by traditional methods all set the machine reference coordinate system at a fixed point on the machine's base and depend on the machine kinematic chain to derive a machine kinematic parameter-dependent model. For practical applications, this dependence makes kinematic parameters impossible to accurately obtain, measurement device operations inconvenient, and overall errors overvalued. For this reason, this paper created a measurement method-integrated "modeling for geometric error model of three-axis machine tools with kinematic parameter independent" technique. This technique, which integrated simple geometric error measuring methods, which constructed the corresponding three-axis geometric error model, and whose geometric error model is machine kinematic parameter-independent, is a practical, convenient, and accurate integrated three-axis geometric error modeling and measurement method.

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