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The Survey of the Buckling Effect of Laminated Plate under the Thermal Load using Complex Finite Strip Method

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***Abstract*—**This article considers the positional buckling of composite thick plates under thermal loading . For this purpose , the complex finite strip method is used . In analysis of complex finite strip, harmonic complex function in longitudinal direction , cubic functions in transversal direction and parabola distribution of transverse shear strain in thickness of thick plate based on higher-order shear deformation theory are used . In given examples , the effect of angles of stratification , number of layers , dimensions ratio and length – to – thick ratio across critical temperature are considered**.**

***Keywords*—**Thermal buckling , Thick plate , Complex finite strip , Higher – order shear deformation theory**.**

I. Introduction

THE ever – increasing use of composite plates in various industries has made many researches on the stability of these factors under mechanical loading. But, in spite of special importance of heat loads , has been paid attention to them a little . The composite plates used in air – space and automobile industries on effect of radiation and other environ mental effects, are put under thermal loading. Hence, a comprehensive investigation of thermal buckling behavior of factorial parts and estimation of buckling critical temperature is an important research to make desirable engineering designs.

The first creditable research on the thermal buckling of plates was made perhaps by Gossard in 1952. [1]

To He considered the thermal buckling of isotropic oblong plates simply supported conditions using Ritz rail method. After him, the other researchers used different methods to solve the thermal buckling of plates under more complex conditions. Among them, whitney and Ashton in 1971 considered the environmental effects on the composite plates. [2] In 1986, Tauchert and Hung presented same results of analysis of buckling behavior of composite plates based on classic theory of plate. [3] In 1990, Sun and Hsu [4] investigated the solution of thermal buckling of composite plates considered shear strain. Chen and sued the finite element method based on the first – order shear theory to analyze the thermal buckling of composite plates. [5] For this purpose, Cheung , Akhras and Li used the finite strip method. [6] In present paper, the thermal buckling analysis composite thick plates has done the complex. Finite strip based on the higher – order shear deformation theory. The classic plates theory which is based on the assumption of being vertical and upright the vertical plates on the middle plate after deformation, will give many good results of foils. However, if this theory uses to solve the thick plates, because of disregarding the transverse shear strain in classic theory, quantities of displacements will be less than actual quantities and the quantities of the normal frequencies and buckling loads will be more. Hence, For thick plates analysis ware extended more higher – order theories Among them are the first – order and the higher – order shear deformation theories. In the higher – order theory which has been presented by reddy (1984), not only transverse shear strains is included, but also they contain the parabolic variation in plate thickness. The shear stresses quantity is assumed zero, there for in addition to getting the better results in relation to the first – order theories, there will be no need to the shear correction coefficients to account transverse shear hardness. This theory is suitable for analysis of the kinds of factorial behaviors like deformation, vibrations and stability analysis. The present theory, in analysis of high thick plates, in spite of facility in comparison with three – dimensions theories has many satisfactory results. [7]

Most of the researchers have used the finite element method to solve thermal buckling of plates while with regard to the special geometry of plates the optimal complex finite strip method, which particularly has been extended for plates analyses, is usable. Kassai [8] extended the complex finite strip method, extended by blank and wittrick [9], to analyses the thick plates based on higher – order theory of Reddy. In this paper, the extended complex finite strip method is employed to analyze the thermal buckling of thick laminated plates. The advantage of this approach in comparison with the source [6] is facility in contact to shear stress.

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II. THEORY

A. Material properties

Material properties in one layer the laminated plates consist of some continuous orthotropic plates. Because of thermal stress, the overall strain of the plate is according to this formula

:

$$\varepsilon = \varepsilon_e + \varepsilon_1 \quad (1)$$

Witch ε_e is the elastic strain and ε_1 is thermal strain. Hence, with disregarding σ_z, stress – strain formula from layer k is:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = Q \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \varepsilon_{xy} - \alpha_{xy} \Delta T \\ \gamma_{yz} \\ \gamma_{yz} \end{Bmatrix} \quad (2)$$

Jones has explained the way of finding the Q matrix. [9]

B. Theory

Higher – order shear deformation theory as mentioned, higher – order shear deformation theory consists of these characteristics:

- taking in to account the transverse shear strains.
- Parabolic variation of shear strain in plate thickness.
-taking into account the shear strain of zero on the surface of plate. These characteristics lead to the following displacement field:

$$u = u_\circ + z\psi_x - \frac{4}{3h^2} z^3 (\psi_x + \frac{\delta w}{\delta x})$$

$$v = v_\circ + z\psi_y - \frac{4}{3h^2} z^3 (\psi_y + \frac{\delta w}{\delta y}) \quad (3)$$

$$w = w(x, y)$$

Which u and v are the displacements in interior of plate, in the points (x,y,z) u_0 and v_0 are the displacements of the middle of plate and w is displacement of the plate in z direction which takes fix in to account in thickness of plate. ψ_x and ψ_y are vertical rotations on the middle of plate around the x and y axis's and h is the thickness of plate. These properties are shown in figure (1). The number of degrees of freedom in this theory is the same as the first – order shear theory, nevertheless, leads to more exact results. Incidentally, for thick plates there is no need to correction coefficients of shear hardness.

C. Complex finite strip method

In present analysis, the composite laminated oblong plate is designed with some finite strip which each strip has nodular lines. This is shown in figure (1).

$$P=\{ p_{x1}, ip_{y1}, p_{z1}, m_1, T_{x1}, iT_{y1}, p_{x2}, ip_{y2}, T_{x2}, iT_{y2}, p_{x3}, ip_{y3}, p_{z3}, m_3, T_{x3}, iT_{y3} \} \quad (4)$$

$$D=\{ u_1, iv_1, w_1, (\frac{\delta w}{\delta x})_1, \psi_{x1}, i\psi_{y1}, u_2, iv_2, \psi_{x2}, i\psi_{y2}, u_3, iv_3, w_3, (\frac{\delta w}{\delta x})_3, \psi_{x3}, i\psi_{y3} \} \quad (5)$$

If the plate buckles, under effect of disturbing, powers, will put in nodular lines which have sinusoidal distribution with half λ the wave length in longitudinal direction. Disturbing powers in unit of length of strip and corresponding disturbing displacements are defined as followings.

P , ***m*** and ***T*** are power, moment and torsion , the indexes of ***x*** , ***y*** and **z** show directions and the numbers 1 to 3 are the numbers of nodular lines. ***U, V*** and ***W*** are displacements in ***x***, ***y*** and ***z*** directions ψ_x and ψ_y and are vertical rotations on the middle of plate around the ***x*** and ***y*** axis's.

In general, ***p*** and ***d*** are complex. The ***i*** = √-1 coefficient makes automatically a 90 differential phase between ***u*** , ***v*** rotations , powers P_x, P_y and moments T_x, T_y. The formulas (6) and (7) explain the deformations in the middle of plate and vertical rotations on it.

$$u_0 = \mathrm{Re}\{XJde^{i\eta}\}, v_0 = \mathrm{Re}\{YJde^{i\eta}\}, w = \mathrm{Re}\{ZJde^{i\eta}\} \quad (6)$$

$$\psi_x = \mathrm{Re}\{R_x Jde^{i\eta}\}, \psi_y = \mathrm{Re}\{R_y Jde^{i\eta}\} \quad (7)$$

The ***Re { }*** is the actual part of phrase in the parenthesis . The ***X , Y***, R_x, R_y are square interpolation matrixes and ***z*** is Hermitian interpolation . The ***J*** is a *16* × *16* diametrical matrix which is shown in formula (8) . In all of the formulas,

$\eta = \frac{\pi y}{\lambda}$ and $\xi = \frac{2x}{b}$ that λ and b, sequentially, are half the buckling wavelength and width of strip.

$$J = \lceil 1 - i 1 1 1 - i 1 - i 1 - i 1 - i 1 1 1 - i \rceil \tag{8}$$

Now , the formulas (6) and (7) are replaced in formula (3). There for the displacement field of plate according to higher – order theory of Reddy, finds by complex finite method. The elastic strains consist of two linear and nonlinear parts witch each of them are shown in the formulas (9) and (10)

$$\varepsilon_{\mathrm{L}} = \left\langle \frac{\delta \mathrm{u}}{\delta \mathrm{x}}; \frac{\delta \mathrm{v}}{\delta \mathrm{y}}; \frac{\delta \mathrm{u}}{\delta \mathrm{y}} + \frac{\delta \mathrm{v}}{\delta \mathrm{x}}; \frac{\delta \mathrm{w}}{\delta \mathrm{y}} + \frac{\delta \mathrm{v}}{\delta \mathrm{z}}; \frac{\delta \mathrm{w}}{\delta \mathrm{x}} + \frac{\delta \mathrm{u}}{\delta \mathrm{z}} \right\rangle^{\mathrm{T}} \tag{9}$$

$$\varepsilon_{XL} = \left\langle \begin{matrix} \frac{1}{2}\left[\left(\frac{\delta u}{\delta x}\right)^2 + \left(\frac{\delta v}{\delta x}\right)^2 + \left(\frac{\delta w}{\delta x}\right)^2\right]; \\ \frac{1}{2}\left[\left(\frac{\delta u}{\delta y}\right)^2 + \left(\frac{\delta v}{\delta y}\right)^2 + \left(\frac{\delta w}{\delta y}\right)^2\right]; \\ \left(\frac{\delta u}{\delta y}\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y}\frac{\delta v}{\delta x} + \frac{\delta w}{\delta y}\frac{\delta w}{\delta x}\right) \end{matrix} \right\rangle^{T} \tag{10}$$

Using of formulas (3) , (6) and (7) , the ε_L linear strain vector is explained with formula (11) .

$$\varepsilon_L = \mathrm{Re}\left(\Gamma J d e^{i\eta}\right) \tag{11}$$

Which Γ is a *5×16* matrix.

D. Finding stiffness matrixes

According to formula (2) in elastic section, virtual work for a 2λ wavelength subjected to virtual displacement δd is explained with formula (12).

$$\delta W_1 = \frac{\lambda b}{2\pi} \int_{\circ}^{2\pi} \int_{-1}^{1} \int_{-\frac{h}{2}}^{\frac{h}{2}} \delta \varepsilon_L^T Q \varepsilon_L \, d\xi \, d\eta \, dz \tag{12}$$

With replacement of formula (11) in (12) , integrating in η direction and doing mathematical process , the formula (12) is summarized in to formula (13)The mark (-) means the conjugate of phrase .

$$\delta \mathrm{W}_1 = \lambda \, \mathrm{Re}\left(\delta \overline{\mathrm{d}}^{\mathrm{T}} \mathrm{Ad}\right) \tag{13}$$

$$\mathrm{A} = \frac{\mathrm{b}}{2} \overline{\mathrm{J}} \left(\int_{-1}^{1} \int_{-\frac{\mathrm{h}}{2}}^{\frac{\mathrm{h}}{2}} \overline{\Gamma}^{\mathrm{T}} \mathrm{Q} \Gamma \, \mathrm{d}\xi \mathrm{dz} \right) \mathrm{J} \tag{14}$$

For finding membranous virtual work must find the membranous powers resulted of temperature variation – thermal power vector for an strip is as Followed

$$f_1 = \int_{\circ}^{2\pi} \int_{-1}^{1} \int_{-\frac{h}{2}}^{\frac{h}{2}} \Gamma^T Q \varepsilon_L \, d\xi \, d\eta \, dz \tag{15}$$

After assembling the strips, power vector of full plate and then with the use of formula (16), the ε_L elastic strains will find

$$\mathrm{Ad} = \mathrm{F}_1 \tag{16}$$

With replacement in formula (2) , the membranous stresses is found , now , having membranous stresses, the membranous virtual work will find according to formula (17)

$$\delta W_m = \delta \iiint \left\langle \sigma_x ; \sigma_y ; \tau \right\rangle \varepsilon_{NL} \, dxdydz \tag{17}$$

With replacing formulas (3) , (6) and (7) , the membranous virtual work will summarized into formula (18)

$$\delta W_m = \lambda \, \mathrm{Re}\left(\delta \overline{d}^T \left(\sum_{r-1}^{6} \left(B_r + iC_r\right) \right) d \right) \tag{18}$$

Details of the $\mathbf{C_r}$ and $\mathbf{B_r}$ matrixes have been brought in the source. [8] If stiffness matrix **A** and stability matrixes $\left(\sum_{r-1}^{6}\left(B_r + iC_r\right)\right)$ find for each strip, with Junction of these matrixes, the stiffness matrixes and total stability will find. Solving a question, special value of critical temperature of plate buckling will fin.

E. Numerical results

A If a thick laminated composite plate with symmetric stratification is puts under increasing heat with uniform distribution, no deformation will happen before buckling. Increasing the heat, the thermal stresses increase up to buckling limit and the plate will be deformed. The thermal buckling happens in anti – symmetric stratification under kinds of heat distribution. In this paper, thermal buckling of plates with symmetric. And anti – symmetric stratification considered under simple boundary condition. In given examples , the effect of the number of layers, dimensions ratio and length – to – thickness ratio across critical temperature are considered .The properties of layers are the same and as following:

$$\mathrm{E}_1 = 181.0\mathrm{E}_0, \mathrm{E2} = 10.3\mathrm{E}_0$$

$$G_{12} = 7.17E_0, G_{23} = 2.39E_0, G_{31} = 5.98E_0$$

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The membranous stresses are the result of temperature variation there for, thermal stress will be proportional with $\alpha_0 T_{cr}$ for thick plates, considering the transverse shear strain causes decrease in hardness. Therefore, the critical temperature without dimension $\frac{\alpha_0 T_{cr} b^2}{h^2}$, will decrease with, increase in thickness. This is clear in examples.

III. Numerical examples

A symmetric , laminated plate consisted of four layers with the same thickness with (90°/0°/90°/0°) has put under uniform increase heat. The thickness of plate is h and its width is b. The boundary conditions in four sides are the same, hinge and: without deformation in inner direction of plate. The full plate is divided to four finite strips; the results are in table (I).

TABLE I
Oblong laminated plate (90°/0°/90°/0°)

$\frac{\alpha_0 T_{cr} b^2}{h^2}$					
$\frac{b}{h}$	$\frac{b}{\lambda}=1.0$	$\frac{b}{\lambda}=1.25$	$\frac{b}{\lambda}=1.5$	$\frac{b}{\lambda}=1.75$	$\frac{b}{\lambda}=2$
4	0.1841	0.1826	0.1859	0.1891	0.1841
10	0.4303	0.4458	0.4496	0.4322	0.4303
20	0.5553	0.5818	0.5716	0.5534	0.5553
50	0.6071	0.6382	0.6219	0.6035	0.6071

For comparison the kinds of theories, the anti symmetric laminated plate consisted of number of couple – layer with the same thickness and degrees (45°/0°/-45°/0°) has been put under uniform increase of heat. He result is shown in figure (2).

IV. Conclusion

In present study, thermal buckling analysis composite laminated plates considered by the complex finite strip method base on higher – order shear theory. In was shown:

1- In spite of facility of this method in relation to the lower – order theories, better results will gain, specially, for high thick plates which transverse shear strains cause to decrease the hardness, considerably. With variation $\frac{b}{h}$ from 50 up to 4, the critical temperature will decrease up to 70 percent.

2- For such plates, using the first – order theory. The critical temperature will get more than what it is.

3- The dimensions ratio doesn't have effect so much on the critical temperature variation.

4- Using the complex functions, to design the shear stresses which exist in certainly in case of the thermal buckling of laminated composite plates under every thermal distribution, is done easily.

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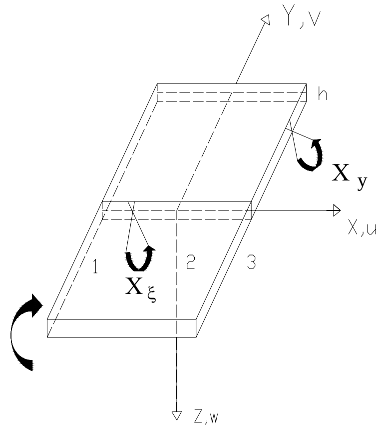

Fig. 1 A finite strip before buckling

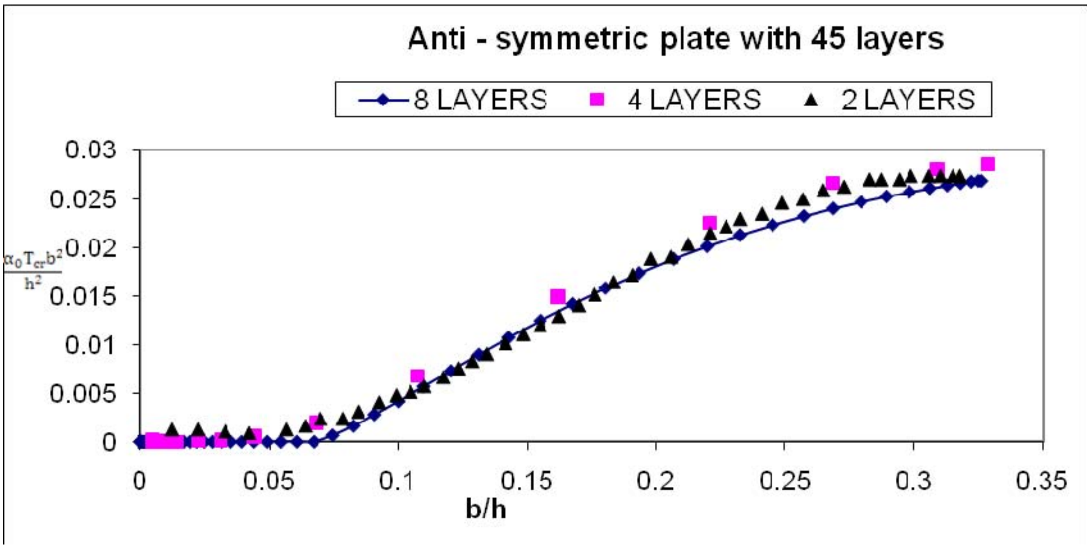

Fig. 2 anti – symmetric plate with 45 layers