World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:1, No:6, 2007

Anti-Synchronization of two Different Chaotic Systems via Active Control

Amir Abbas Emadzadeh, Mohammad Haeri, *Member, IEEE*

***Abstract*—** This paper presents anti-synchronization of chaos between two different chaotic systems using active control method. The proposed technique is applied to achieve chaos anti-synchronization for the Lü and Rössler dynamical systems. Numerical simulations are implemented to verify the results.

***Keywords*—**Active control, Anti-Synchronization, Chaos, Lü system, Rössler system.

Open Science Index, Electrical and Computer Engineering Vol:1, No:6, 2007 publications.waset.org/10099/pdf

I. Introduction

ONE of the most striking discoveries in the study of chaos is that chaotic systems can be made to synchronize with each other [1]. This discovery by Pecora and Carroll in 1990 was both theoretically surprising and practically significant. Theoretically, chaos stipulates that nearby trajectories diverge exponentially in time and, thus, synchronization of chaotic systems seems unlikely in the presence of inevitable small differences in parameters of the systems, and noise.

The most familiar synchronization phenomenon is that the difference of states of synchronized systems converges to zero, and is called complete synchronization (CS). Almost all the research reports on chaotic synchronization are relevant to complete Synchronization [2].

On the other hand, anti-synchronization (AS) is a phenomenon that the state vectors of synchronized systems have the same absolute values but opposite signs. We say that anti-synchronization of two systems S_1 and S_2 is achieved if the following equation holds:

$$\lim_{t\to\infty}\|x_1(t)+x_2(t)\|=0 \tag{1}$$

where $x_1(t)$, $x_2(t)$ are the state vectors of the systems S_1, S_2.

It is well known that the first observation of synchronization of two oscillators by Huygens in the seventeenth century was, in fact, AS between two pendulum clocks [3]. Recent reinvestigation of Huygens' experiment by Blekhman [4] shows that either synchronization or AS can appear depending on the initial conditions of the coupled pendulua. Here, AS can also be interpreted as *anti-phase synchronization* (APS) [5]. That is to say, there is no difference between AS and APS for oscillators with identical amplitudes [6]. AS phenomena have been observed experimentally in the context of self-synchronization, e.g., in salt-water oscillators [7], and some biological systems where a non-chaotic signal is generated. Another potential usage of the phenomenon of anti-phase synchronism lies in nonlinear digital communication, which has become a field of recent interest [8]. Moreover, it has been reported that APS can theoretically occur in a subsystem of hyper-chaotic systems with symmetry [8].

In this paper, we apply active control theory to anti-synchronize two different chaotic systems. We demonstrate the technique capability on the anti-synchronization of Lü and Rössler systems.

The rest of the paper is organized as follows: In Section II the Lü and Rössler chaotic systems are introduced. In Section III, the theory of active control is adopted to anti-synchronize the systems. In Section IV numerical simulations are presented to verify the effectiveness of the proposed method and finally the concluding remarks are given in Section V.

II. Systems Description

Lü system as a typical transaction system, found by Lü and Chen, which connects the Lorenz and Chen attractors and represents the transition from one to the other [9]. The Lü system is described by;

$$\begin{cases}\dot{x}=\alpha(y-x)\\ \dot{y}=-xz+\beta y\\ \dot{z}=xy-\gamma z\end{cases} \tag{2}$$

which has a chaotic attractor as shown in Fig. 1(a) when $\alpha=36$, $\beta=20$, $\gamma=3$. The so called Rössler system is credited to Otto Rössler and arose from work in chemical kinetics. The system is described with 3 coupled non-linear differential equations [10].

$$\begin{cases}\dot{x}=-y-z\\ \dot{y}=x+a\,y\\ \dot{z}=b+z(x-c)\end{cases} \tag{3}$$

This work was supported by the Iranian Telecommunication Research Center (ITRC) under Grant T500-10177.

Amir Abbas Emadzadeh is with the Advanced Control Systems Lab, Electrical Engineering Department, Sharif University of Technology, Azadi Ave., Tehran, Iran (e-mail: emadzadeh@mehr.sharif.edu).

Dr. M. Haeri is with the Electrical Engineering Department, Sharif University of Technology, and the head of the Advanced Control Systems Lab., Iran (phone: +98-21-616-5964, fax: +98-21-602-3261, e-mail: haeri@sina.sharif.edu).

Open Science Index, Electrical and Computer Engineering Vol:1, No:6, 2007 publications.waset.org/10099/pdf

which has a chaotic attractor as shown in Fig. 2 (b) when $a = 0.2$, $b = 0.2$, $c = 5.7$.

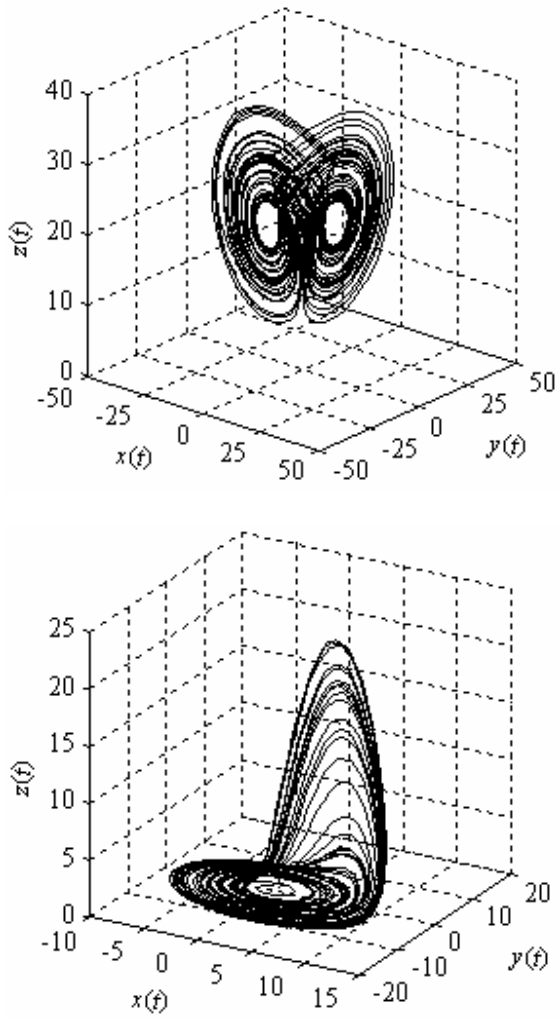


Fig. 1 (a) Lü chaotic attractor, (b) Rössler chaotic attractor

III. Anti-Synchronization Between Lü and Rössler Systems

To observe the anti-synchronization behavior in Lü and Rössler systems we assume that Lü system drives the Rössler system. Therefore, we define the *master* and *slave* systems as follows.

$$\begin{cases} \dot{x}_1 = \alpha(y_1 - x_1) \\ \dot{y}_1 = -x_1 z_1 + \beta y_1 \\ \dot{z}_1 = x_1 y_1 - \gamma z_1 \end{cases} \tag{4}$$

and

$$\begin{cases} \dot{x}_2 = -y_2 - z_2 + u_1(t) \\ \dot{y}_2 = x_2 + a y_2 + u_2(t) \\ \dot{z}_2 = b + z_2(x_2 - c) + u_3(t) \end{cases} \tag{5}$$

We have introduced three control functions $u_1(t)$, $u_2(t)$ and $u_3(t)$ in (5). Our goal is to determine the mentioned functions. In order to estimate the control functions, we add (4) to (5). We define the AS error system as the summation of the Lü system (4) and the controlled Rössler system (5). Let us define the states of the AS errors for the *slave* system (5) that is to be controlled and the controlling system (4) as;

$$\begin{cases} s_1 = x_1 + x_2 \\ s_2 = y_1 + y_2 \\ s_3 = z_1 + z_2 \end{cases} \tag{6}$$

By adding (4) to (5) and using the notation (6) we can get;

$$\begin{cases} \dot{s}_1 = \alpha(s_2 - s_1) + \alpha x_2 - (1+\alpha) y_2 - z_2 + u_1(t) \\ \dot{s}_2 = \beta s_2 - s_1 s_3 + s_1 z_2 + s_3 x_2 + x_2 + (a - \beta) y_2 \\ \qquad - x_2 z_2 + u_2(t) \\ \dot{s}_3 = -\gamma s_3 + s_1 s_2 - s_1 y_2 - s_2 x_2 + (\gamma - c)\, z_2 + x_2 y_2 \\ \qquad + x_2 z_2 + b + u_3(t) \end{cases} \tag{7}$$

Then, by defining the active control inputs $u_1(t)$, $u_2(t)$ and $u_3(t)$ as follows;

$$\begin{cases} u_1(t) = V_1(t) - \alpha x_2 + (1+\alpha) y_2 + z_2 \\ u_2(t) = V_2(t) - x_2 - (a - \beta) y_2 + x_2 z_2 - s_1 z_2 \\ \qquad - s_3 x_2 + s_1 s_3 \\ u_3(t) = V_3(t) + (c - \gamma) z_2 - x_2 y_2 - x_2 z_2 + s_1 y_2 \\ \qquad + s_2 x_2 - s_1 s_2 - b \end{cases} \tag{8}$$

this leads to;

$$\begin{cases} \dot{s}_1 = \alpha(s_2 - s_1) + V_1(t) \\ \dot{s}_2 = \beta s_2 + V_2(t) \\ \dot{s}_3 = -\gamma s_3 + V_3(t) \end{cases} . \tag{9}$$

The AS error system in (9) is a linear system with control inputs $V_1(t)$, $V_2(t)$ and $V_3(t)$. Design of an appropriate feedback control stabilizes the system so that s_1, s_2 and s_3 converge to zero as time t tends to infinity. This implies that Lü and Rössler systems are anti-synchronized with feedback control. There are many possible choices for the control inputs $V_1(t)$, $V_2(t)$ and $V_3(t)$. We choose;

$$\begin{bmatrix} V_1(t) \\ V_2(t) \\ V_3(t) \end{bmatrix} = A \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} \tag{10}$$

where A is a 3×3 constant matrix. In order to make the closed loop system stable, the matrix A should be selected in such a way that the feedback system has eigenvalues with

negative real part. Let the matrix A is chosen in the following form;

$$A=\begin{bmatrix} \alpha-1 & -\alpha & 0 \\ 0 & -(1+\beta) & 0 \\ 0 & 0 & \gamma-1 \end{bmatrix} \quad (11)$$

For this particular choice, the closed loop system (9) has all its three eigenvalues in -1. This choice will lead to the AS error states s_1, s_2 and s_3 converge to zero and hence the anti-synchronization between Lü and Rössler systems is achieved. We can also make non-zero numbers less than -1. If the eigenvalues get smaller, the convergence will become better.

IV. Simulation Results

In this section, numerical simulations are carried out using MATLAB. The fourth order Runge-Kutta integration method is used to solve two systems of differential equations (4) and (5). In addition, a time step size 0.001 is employed. We will select the parameters of Lü system as $\alpha=36$, $\beta=20$, $\gamma=3$ and the parameters of Rössler system as $a=0.2$, $b=0.2$, $c=5.7$. Therefore, both Lü and Rössler systems exhibit chaotic behavior. The initial values of the *master* and *slave* systems are $x_{10}=-10$, $y_{10}=-17$, $z_{10}=15$ and $x_{20}=0$, $y_{20}=0.5$, $z_{20}=1.5$, respectively. These choices result in initial errors of $s_{10}=-10$, $s_{20}=-16.5$, $s_{30}=16.5$. The diagram of the Rössler system controlled to be anti-synchronized with Lü system accompanied with the control functions $u_1(t)$, $u_2(t)$ and $u_3(t)$ is shown in Fig. 2. The dynamics of anti-synchronization errors for the *master* and *slave* systems is shown in Fig. 3. In order to show the eigenvalues effect on convergence, we will use standard deviation criterion ($SD=\sqrt{\sum_{k=1}^{N}(x(k)-<x(k)>)/N}$). Let m, n, p be the eigenvalues of the system (9), we make $m=n=p$, change the values from -1 to -50 and calculate SD for s_3. As shown in Fig. 4, when eigenvalues get smaller, the values of SD become smaller. That is to say, the convergence becomes better.

V. Conclusion

By means of active control theory, we can achieve anti-synchronization between two different chaotic systems. Rössler system is controlled to be anti-synchronized with Lü system. The simulations confirm that AS of two systems operates satisfactorily in presence of the proposed control method.

Acknowledgment

The results presented in this paper come from a project that is supported in part by Iran Communication Research Centre.

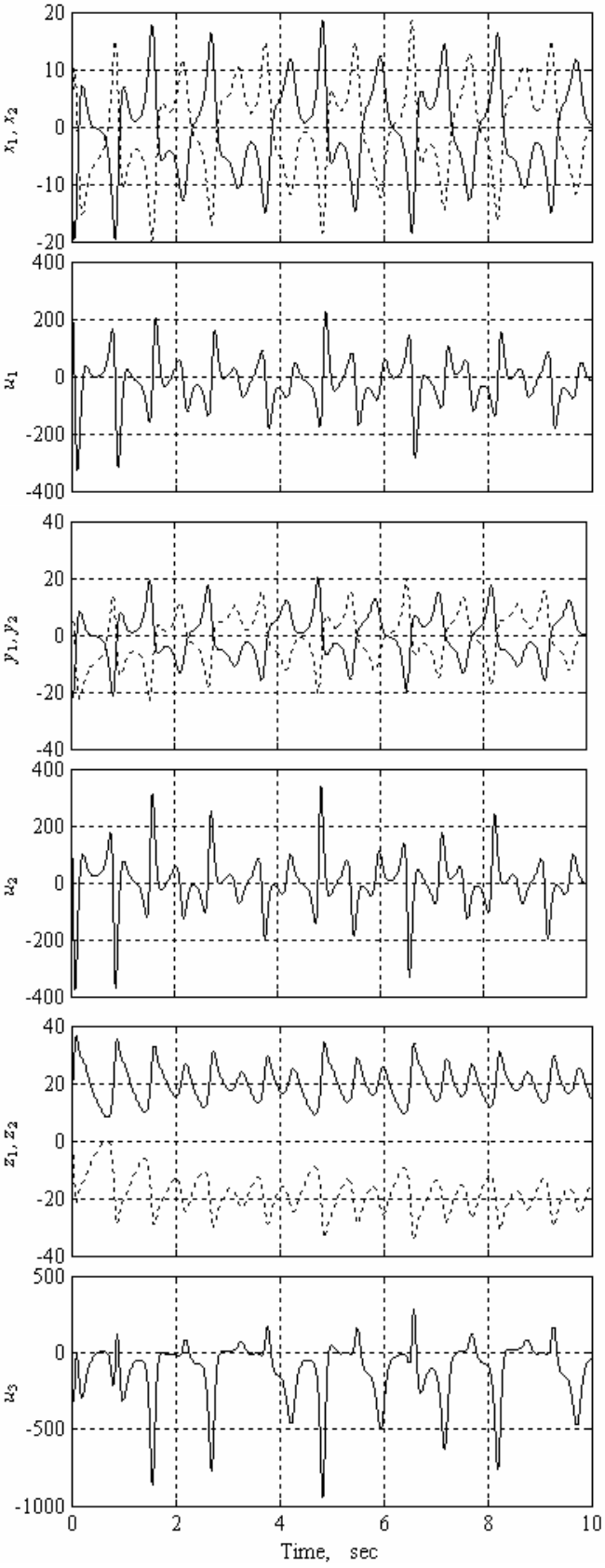


Fig. 2 The diagram of anti-synchronizing between the Lü and Rössler systems using active control method

Open Science Index, Electrical and Computer Engineering Vol:1, No:6, 2007 publications.waset.org/10099/pdf

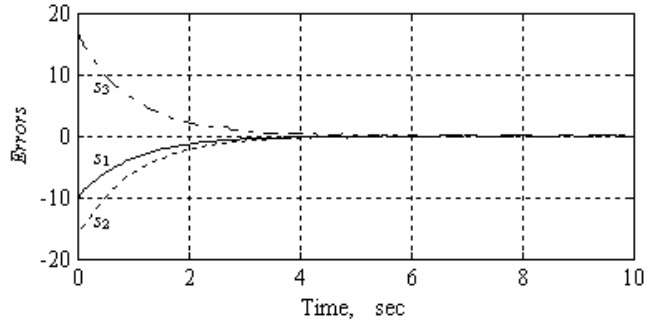


Fig. 3 Anti-synchronization errors (s_1, s_2, s_3) between the Lü and Rössler systems

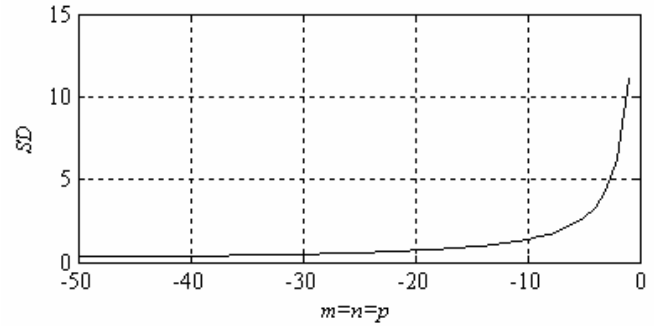


Fig. 4 The Standard deviation (SD) of the signal s_3 .

REFERENCES

[1] T.L. Carroll, L.M. Pecora, "Synchronization in chaotic systems", *Physical. Review Letters*, Vol. 64, pp. 821-824, 1990.

[2] Y. Zhang, J. Sun, "Chaotic synchronization and anti-synchronization based on suitable separation", *Physics Letters A*, Vol. 330, pp. 442-447, 2004.

[3] Ch-M et al., "Anti-synchronization of chaotic oscillators", *Physics Letters A*, Vol. 320, pp. 39-46, 2003.

[4] M. Bennett, M.F. Schatz, H. Rockwood, K. Wiesenfeld, "Huygens' clocks", *Proc. R. Soc. London A*, Vol. 458, pp. 563-579, 2002.

[5] J. Liu, C. Ye, S. Zhang, W. Song, "Anti-phase synchronization in coupled map lattices", *Physics Letters A*, Vol. 274, pp. 27-29, 2000.

[6] A. Uchida, Y. Liu, I. Fischer, P. Davis, T. Aida, "Chaotic antiphase dynamics and synchronization in multimode semiconductor lasers", *Physical Review. A*, Vol. 64, 023801, 2001.

[7] S. Nakata, T. Miyata, N. Ojima, K. Yoshikawa, "Self-synchronization in coupled salt-water oscillators", *Physica D*, Vol. 115, pp. 313-320, 1998.

[8] L-Y. Cao, Y-Ch. Lai, "Antiphase synchronism in chaotic systems", *Physical Review E*, Vol. 58, No. 1, pp. 382-386, 1998.

[9] J. Lü, G. Chen, S. Zhang, "Dynamical analysis of a new chaotic attractor", *Int. J. Bifurcation Chaos*, Vol. 12, pp. 1001, 2002.

[10] X. Yu, "Variable structure control approach for controlling chaos", *Chaos Solitons and Fractals,* Vol. 8, No. 9, pp. 1577-1586, 1997.

Open Science Index, Electrical and Computer Engineering Vol:1, No:6, 2007 publications.waset.org/10099/pdf