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Extension of a Smart Piezoelectric Ceramic Rod

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***Abstract*—**This paper presents an exact solution and a finite element method (FEM) for a Piezoceramic Rod under static load. The cylindrical rod is made from polarized ceramics (piezoceramics) with axial poling. The lateral surface of the rod is traction-free and is unelectroded. The two end faces are under a uniform normal traction. Electrically, the two end faces are electroded with a circuit between the electrodes, which can be switched on or off. Two cases of open and shorted electrodes (short circuit and open circuit) will be considered. Finally, a finite element model will be used to compare the results with an exact solution. The study uses ABAQUS (v.6.7) software to derive the finite element model of the ceramic rod.

***Keywords*—**Finite element method; Ceramic rod; Axial poling; Normal traction; Short circuit; Open circuit.

I. INTRODUCTION

PIEZOELECTRIC materials are used widely in transducers such as ultrasonic transmitters and receivers, sonar for underwater applications, and as actuators for precision positioning devices. Piezoelectric materials exhibit *Electromechanical Coupling*, which is useful for the design of devices for sensing and actuation. The coupling is exhibited in the fact that piezoelectric materials produce an electrical displacement when a mechanical stress is applied and can produce mechanical strain under the application of an electric field. Due to the fact that the mechanical-to-electrical coupling was discovered first, this property is termed the *direct piezoelectric effect*, while the electrical-to-mechanical coupling is termed the *converse piezoelectric effect* [1]. The physical basis for piezoelectricity in solids is widely studied by physicists and materials scientists. Most piezoelectric materials belong to a class of *crystalline* solids. Crystals are solids in which the atoms are arranged in a single pattern repeated throughout the body. Crystalline materials are highly ordered, and an understanding of the bulk properties of the material can begin by understanding the properties of the crystals repeated throughout the solid. The individual crystals in a solid can be thought of as building blocks for the material. Joining crystals together produces a three-dimensional arrangement of the crystals called a *unit cell*. One of the most important properties of a unit cell in relation to piezoelectricity is the *polarity* of the unit cell structure. Crystallographers have studied the structure of unit cells and classified them into a set of 32 crystal classes or *point groups*. Each point group is characterized by a particular arrangement of the constituent atoms. Of these 32 point groups, 10 have been shown to exhibit a polar *axis* in which there is a net separation between positive charges in the crystal and their associated negative charges. This separation of charge produces an *electric dipole*, which can give rise to piezoelectricity [1,2].

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Induced strain actuators like piezoelectric materials have been effectively used as integrated sensors and actuators for monitoring and further controlling the mechanical behavior of advanced structures [3,4]. Over the past decade, Finite Element Analysis (FEA) techniques have been employed to model the overall structural response involving the *electromechanical coupling effects* of the piezoelectric sensing/actuating elements [5]. Superior to analytical methods, the FEA technique provides greater geometric flexibility and allows use of more complex electrical and mechanical boundary conditions. Although much research effort has been devoted to finite element formulation for the electromechanical coupling effects of piezoelectric materials (Tzou and Tseng, 1990; Ha et al., 1991), fully electromechanical coupled piezoelectric elements have just recently become available in commercial FEA software [6].

Before the new piezoelectric capability was developed in commercial FEA codes, the induced strain actuation function of piezoelectric materials had been modeled using analogous thermal expansion/contraction characteristics of structural materials [7]. This method was helpful in the studies of the resulting stress distribution in actuators and *host substructures* and the overall deformation of integrated structures under static actuation. However, the intrinsic electromechanical coupling effects of piezoelectric materials cannot be modeled. Moreover, the dynamic actuation response of piezoelectric actuators on host substructures is difficult to implement by this method.

The new piezoelectric finite element capability in commercial FEA packages gives convenient access to perform both static and dynamic analysis for the fully coupled piezoelectric and structural response. In addition, since most commercialized FEA packages are generally equipped with well-developed pre and post-processors and user-friendly interactive graphics working environments, the time-consuming tasks of finite element model generation and solution extraction can be significantly reduced [7].

II. LINEAR PIEZOELECTRICITY FOR INFINITESIMAL FIELDS

Nonlinear theory of Electroelasticity is used for *large deformations* and *strong electric fields.* In linear theory like Piezoelectricity, we can specialize the nonlinear equations to the case of *infinitesimal deformations* and *fields*, which results in the linear theory of piezoelectricity. For Linearization, we reduce the nonlinear electroelastic equations in the nonlinear theory to the linear theory of piezoelectricity for infinitesimal deformations and fields. We consider *small amplitude motions* of an electroelastic body around its reference state due to small mechanical and electrical loads [8]. It is assumed that the *displacement gradient* is infinitesimal in the following sense that :

$$\|u_{i,K}\| \ll 1 \quad (1)$$

Under some norm, e.g., $\|u_{i,K}\| = max|u_{i,K}|$. It is also assumed that the *electric potential gradient* $\phi_{,K}$ is infinitesimal.

$$\|\phi_{,K}\| \ll 1 \quad (2)$$

We neglect powers of $u_{i,K}$ and $\phi_{,K}$ higher than the first as well as their products in all expressions. The linear terms themselves are also dropped in comparison with any finite quantity such the Kronecker delta or 1. Under (1),

$$\frac{\partial u_i}{\partial X_K} = \frac{\partial u_i}{\partial y_k} y_{k,K} = \frac{\partial u_i}{\partial y_k}\left(\delta_{kK} + u_{k,K}\right) \cong \frac{\partial u_i}{\partial y_k}\delta_{kK} \quad (3)$$

$$\phi_{,K} = \phi_{,i} y_{i,K} \cong \phi_{,i}\delta_{iK}$$

Which implies that, to the first order of approximation, the displacement and potential gradients calculated from the material and spatial coordinates are numerically equal. Therefore, within the linear theory, there is no need to distinguish capital and lowercase indices. Only lowercase indices will be used in the linear theory. The material time derivative of an infinitesimal field variable $f(\boldsymbol{y}, t)$ is simply the partial derivative with respect to t :

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t}|_{X\ fixed} = \frac{\partial f}{\partial t}|_{y\ fixed} + \frac{\partial f}{\partial y_i}|_{t\ fixed}\frac{\partial y_i}{\partial t}|_{X\ fixed} = \frac{\partial f}{\partial t}|_{y\ fixed} + v_i\frac{\partial f}{\partial y_i} \cong \frac{\partial f}{\partial t}|_{y\ fixed} \quad (4)$$

For the finite strain tensor :

$$S_{KL} = \frac{1}{2}\left(u_{L,K} + u_{K,L} + u_{M,K}u_{M,L}\right) \cong \frac{1}{2}\left(u_{L,K} + u_{K,L}\right) \quad (5)$$

In the linear theory, the infinitesimal strain tensor will be denoted by :

$$S_{kl} = \frac{1}{2}\left(u_{l,k} + u_{k,l}\right) \quad (6)$$

The material electric field becomes :

$$E_K = E_i y_{i,K} \cong E_i\delta_{iK} \to E_k \quad (7)$$

Similarly,

$$\sigma_{ij}^E \cong 0\ , \sigma_{ij}^M \cong 0\ , \sigma_{ij} \cong \sigma_{ij}^S \cong \tau_{ij}$$
$$M_{Lj} \cong 0\ , K_{Lj} \cong F_{Lj} \cong \delta_{Ki}\sigma_{ij}\ , T_{KL}^S \cong \delta_{Ki}\delta_{Lj}\sigma_{ij} \quad (8)$$
$$\mathcal{P}_K \to P_k\ , \mathcal{D}_K \to D_k$$

Where :

σ_{ij}^E = Electrostatic strees tensor

σ_{ij}^M , M_{Lj} = Symmetric Maxwell stress tensor in spatial , two point

σ_{ij} = Cauchy stress tensor

σ_{ij}^S , F_{Lj} , T_{KL}^S = Symmetric stress tensor in spatial , two point , and material form

τ_{ij} , K_{Lj} = Total stress tensor in spatial , two point

$\mathcal{P}_K$ = Reference electric polarization vector

$\mathcal{D}_K$ = Reference electric displacement vector

Since the various stress tensors are either approximately zero (quadratic in the infinitesimal gradients) or about the same, we will use T_{ij} to denote the stress tensor that is linear in the *infinitesimal gradients*. This is according to the IEEE Standard on Piezoelectricity. The notation for the rest of the linear theory will also follow the IEEE Standard [9]. Then :

$$\sigma_{ij} \cong \sigma_{ij}^S \cong \tau_{ij} \to T_{ij}$$
$$K_{Lj} \cong F_{Lj} \cong \delta_{Li}\sigma_{ij} \to T_{ij} \quad (9)$$
$$T_{KL}^S \cong \delta_{Ki}\delta_{Lj}\sigma_{ij} \to T_{kl}$$

For small fields the *total free energy* can be approximated by :

$$\rho_0\hat{\psi}(S_{KL}, E_K) = \rho_0\psi(S_{KL}, E_K) - \frac{1}{2}\varepsilon_0 J E_K E_K \cong \frac{1}{2}c_{2\ ABCD}S_{AB}S_{CD} - e_{ABC}E_A S_{BC} - \frac{1}{2}\chi_{2\ AB}E_A E_B - \frac{1}{2}\varepsilon_0 J E_K E_K \to \frac{1}{2}c_{ijkl}^E S_{ij}S_{kl} - e_{ijk}E_i S_{jk} - \frac{1}{2}\varepsilon_{ij}^S E_i E_j = H(S_{kl}, E_k) \quad (10)$$

Where :

$$\varepsilon_{ij}^S = \chi_{2\ ij} + \varepsilon_0\delta_{ij} \quad (11)$$

The superscript E in c_{ijkl}^E indicates that the independent electric constitutive variable is the electric field $\boldsymbol{E}$. The superscript S in ε_{ij}^S indicates that the mechanical constitutive variable is the strain tensor $\boldsymbol{S}$. We have also denoted the total free energy of the linear theory by H which is usually called the *electric enthalpy.* The electrical enthalpy (H) in a piezoelectric body is an energy quantity similar to strain energy in an elastic structure. The constitutive relations generated by H are :

$$T_{ij} = \frac{\partial H}{\partial S_{ij}} = c_{ijkl}^E S_{kl} - e_{kij}E_k$$
$$D_i = -\frac{\partial H}{\partial E_i} = e_{ikl}S_{kl} + \varepsilon_{ik}^S E_k \quad (12)$$

Where :

c_{ijkl}^E = Elastic stiffness constants

e_{kij} = Piezoelectric stress constants

ε_{ik} = Dielectric constants

Hence T , D and P are also infinitesimal. The material constants in Equation (12) have the following symmetries :

$$c_{ijkl}^E = c_{jikl}^E = c_{klij}^E$$
$$e_{kij} = e_{kji} \quad (13)$$
$$\varepsilon_{ij}^S = \varepsilon_{ji}^S$$

We also assume that the *elastic* and *dielectric material tensors* are positive definite in the following sense :

$$c_{ijkl}^E S_{ij}S_{kl} \geq 0\ for\ any\ S_{ij} = S_{ji}$$
$$and\ c_{ijkl}^E S_{ij}S_{kl} = 0 \to S_{ij} = 0$$
$$\varepsilon_{ij}^S E_i E_j \geq 0\ for\ any\ E_i \quad (14)$$
$$and\ \varepsilon_{ij}^S E_i E_j = 0 \to E_i = 0$$

III. Compact Matrix Notation

We now introduce a compact matrix notation. This notation consists of replacing pairs of indices ij or kl by single indices p or q where i, j, k and l take the values of 1, 2, and 3, and p and q take the values 1, 2, 3, 4, 5, and 6 according to [8] :

ij or kl :	11	22	33	23 or 32	31 or 13	12 or 21
p or q :	1	2	3	4	5	6

(15)

Thus

$$c_{ijkl} \to c_{pq}\ , e_{ikl} \to e_{ip}\ , T_{ij} \to T_p \quad (16)$$

For the strain tensor, we introduce S_p such that :

$$S_1 = S_{11}\ , S_2 = S_{22}\ , S_3 = S_{33}$$
$$S_4 = 2S_{23}\ , S_5 = 2S_{31}\ , S_6 = 2S_{12} \quad (17)$$

The constitutive relations in Equation (12) can then be written as :

$$T_p = c_{pq}^E S_q - e_{kp} E_k$$
$$D_i = e_{iq} S_q + \varepsilon_{ik}^S E_k \quad (18)$$

In matrix form, Equation (18) becomes :

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} c_{11}^E & c_{12}^E & c_{13}^E & c_{14}^E & c_{15}^E & c_{16}^E \\ c_{21}^E & c_{22}^E & c_{23}^E & c_{24}^E & c_{25}^E & c_{26}^E \\ c_{31}^E & c_{32}^E & c_{33}^E & c_{34}^E & c_{35}^E & c_{36}^E \\ c_{41}^E & c_{42}^E & c_{43}^E & c_{44}^E & c_{45}^E & c_{46}^E \\ c_{51}^E & c_{52}^E & c_{53}^E & c_{54}^E & c_{55}^E & c_{56}^E \\ c_{61}^E & c_{62}^E & c_{63}^E & c_{64}^E & c_{65}^E & c_{66}^E \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{pmatrix} - \begin{pmatrix} e_{11} & e_{21} & e_{31} \\ e_{12} & e_{22} & e_{32} \\ e_{13} & e_{23} & e_{33} \\ e_{14} & e_{24} & e_{34} \\ e_{15} & e_{25} & e_{35} \\ e_{16} & e_{26} & e_{36} \end{pmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \quad (19)$$

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{pmatrix} e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\ e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} \\ e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36} \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{pmatrix} + \begin{pmatrix} \varepsilon_{11}^S & \varepsilon_{12}^S & \varepsilon_{13}^S \\ \varepsilon_{21}^S & \varepsilon_{22}^S & \varepsilon_{23}^S \\ \varepsilon_{31}^S & \varepsilon_{32}^S & \varepsilon_{33}^S \end{pmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}$$

IV. Displacement – Potential Formulation

In summary, the linear theory of piezoelectricity consists of the *equations* of *motion* and *charge* [8] :

$$T_{ji,j} + \rho f_i = \rho u_i \quad , \quad D_{i,i} = \rho_e \quad (20)$$

Constitutive relations :

$$T_{ij} = c_{ijkl} S_{kl} - e_{kij} E_k \quad , \quad D_i = e_{ijk} S_{jk} + \varepsilon_{ij} E_j \quad (21)$$

And the strain-displacement and electric field-potential relations :

$$S_{ij} = (u_{i,j} + u_{j,i})/2 \quad , \quad E_i = -\phi_{,i} \quad (22)$$

Where u is the mechanical displacement vector, T is the stress tensor, S is the strain tensor, E is the electric field, D is the electric displacement (electric flux density), ϕ is the electric potential, ρ is the known reference mass density, ρ_e is the body free charge density, and f is the body force per unit mass. We have neglected the superscripts in the material constants. With successive substitutions from Equations (21) and (22), Equation (20) can be written as four equations for u and :

$$c_{ijkl} u_{k,lj} + e_{kij} \phi_{,kj} + \rho f_i = \rho \ddot{u}_i$$
$$e_{ikl} u_{k,li} - \varepsilon_{ij} \phi_{,ij} = \rho_e \quad (23)$$

V. Extension of a Ceramic Rod

Consider a cylindrical rod of length L made from polarized ceramics with axial poling. The cross-section of the rod can be arbitrary. The lateral surface of the rod is traction-free and is unelectroded. The two end faces are under a uniform normal traction p, but there is no tangential traction. Electrically, the two end faces are electroded with a circuit between the electrodes, which can be switched on or off. Two cases of open and shorted electrodes (short circuit and open circuit) will be considered. This problem is an electrostatic case which is very formal in the piezoelectric problems. Figure 1 shows an axially poled ceramic rod.

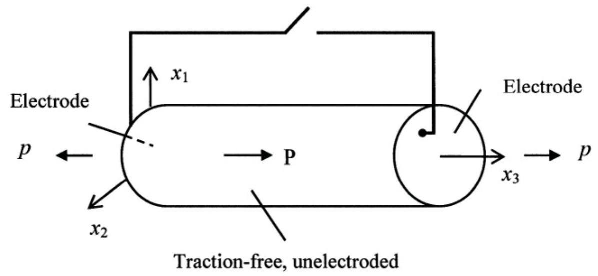

Fig. 1 An axially poled ceramic rod [8]

A. *Boundary Value Problem*

The boundary value problem is :

$$T_{ji,j} = 0 \quad , \quad D_{i,i} = 0 \quad \text{in} \quad V$$
$$S_{ij} = s_{ijkl}^E T_{kl} + d_{kij} E_k \quad , \quad D_i = d_{ikl} T_{kl} + \varepsilon_{ik}^T E_k \quad in \quad V$$
$$\varepsilon_{ijk} \varepsilon_{lmn} S_{il,jm} = 0 \quad , \quad \varepsilon_{ijk} E_{k,j} = 0 \quad \text{in} \quad V$$
$$T_{ji} n_j = 0 \quad , \quad D_i n_i = 0 \quad \text{on the lateral surface}$$
$$T_{31} = 0 \quad , \quad T_{32} = 0 \quad , \quad T_{33} = p \quad , \quad E_1 = E_2 = 0 \quad , \quad x_3 = 0, L \quad (24)$$
$$\phi(x_3 = 0) = \phi(x_3 = L) \quad , \quad \text{if the end faces are shorted}$$
$$\int D_3 \, dA = 0 \quad , \quad x_3 = 0, L \quad , \quad \text{if the end faces are open}$$

Where we have chosen the stress components and the electric displacement components as the primary unknowns. Many of these components are known on the lateral surface, and it is easy to guess what they are like inside the cylinder. Since many components of *T* will vanish, it is convenient to use constitutive relations with *T* as the independent constitutive variable. In this formulation the compatibility conditions on strains and the curl-free condition on the electric field have to be satisfied. As suggested by the boundary conditions on the lateral surface we consider the following *T* and *D* fields

$$T_{33} = p \quad , \text{all other} \quad T_{ij} = 0$$
$$D_3 = \text{constant} \quad , \quad D_1 = D_2 = 0 \quad (25)$$

Which satisfy the equation of motion and the charge equation. Since the T and D fields are constants, the constitutive relations imply that the S and E fields are also constants. Therefore, the compatibility conditions on S and the curl-free condition on E are satisfied. (25) also satisfies the boundary conditions on the lateral surface and the mechanical boundary conditions on the end faces. From the constitutive relations

$$S_{23} = S_{31} = S_{12} = 0$$
$$S_{33} = s_{33}^E p + d_{33} E_3 \quad , \quad S_{11} = S_{22} = s_{13}^E p + d_{31} E_3 \quad (26)$$
$$E_1 = E_2 = 0 \quad , \quad D_3 = d_{33} p + \varepsilon_{33}^T E_3$$

Hence the electrical boundary conditions of $E_1 = E_2 = 0$ (constant electric potential on an electrode) on the end electrodes are also satisfied. We consider two cases as follows.

B. Shorted Electrodes

In this case, there is no potential difference between the end electrodes. Since E_3 is constant alon the rod, we must have

$$E_3 = 0 \tag{27}$$

Which implies that

$$D_3 = d_{33}p \quad , \quad S_{33} = s_{33}^E p \tag{28}$$

The mechanical work done to the rod per unit volume during the static extensional process is

$$W_1 = \frac{1}{2}T_{33}S_{33} = \frac{1}{2}s_{33}^E p^2 \tag{29}$$

C. Open Electrodes

In this case, there is no net charge on he end electrodes. Since D_3 is constant over a cross-section, we must have

$$D_3 = 0 \tag{30}$$

Which implies that

$$E_3 = -\frac{d_{33}}{\varepsilon_{33}^T}p$$

$$S_{33} = s_{33}^E p - d_{33}\frac{d_{33}}{\varepsilon_{33}^T}p = s_{33}^E\left(1 - \frac{d_{33}^2}{\varepsilon_{33}^T s_{33}^E}\right)p \tag{31}$$

The mechanical work done to the rod per nit volume is

$$W_2 = \frac{1}{2}T_{33}S_{33} = \frac{1}{2}s_{33}^E\left(1 - \frac{d_{33}^2}{\varepsilon_{33}^T s_{33}^E}\right)p^2 \tag{32}$$

D. lectromechanical Coupling Fa tor

Since

$$\frac{d_{33}^2}{\varepsilon_{33}^T s_{33}^E} > 0 \tag{33}$$

We have

$$W_1 > W_2 \tag{34}$$

Therefore, the rod appears to be stiffer hen the electrodes are open and an axial electric field is produced. This is called the piezoelectric stiffening effect. he following ratio is called the longitudinal electromechani al coupling factor for the extension of a ceramic rod with xial poling, and is denoted by

$$(k_{33}^{'})^2 = \frac{W_1 - W_2}{W_1} = \frac{d_{33}^2}{\varepsilon_{33}^T s_{33}^E} \tag{35}$$

For PZT-5H, a common ceramic, fr m the following material constants [8] :

$s_{11} = 16.5 \quad , \quad s_{33} = 20.7 \quad , \quad s_{44} = 43.5$

$s_{12} = -4.78 \quad , \quad s_{13} = -8.45 \times 10^{-12}\, m^2/N$

$d_{31} = -274 \quad , \quad d_{15} = 741 \quad , \quad d_{33} = 593 \times 10^{-12}\, C/N$

$\varepsilon_{11} = 3130\varepsilon_0 \quad , \quad \varepsilon_{33} = 3400\varepsilon_0$

$\varepsilon_0 = 8.854 \times 10^{-12}\, F/m$

We have

$$(k_{33}^{'})^2 = \frac{(593 \times 10^{-12})^2}{(3400 \times 8.85 \times 10^{-12})(20.7 \times 10^{-12})} = 0.56$$

$$k_{33}^{'} = 0.75$$

Which is typical for ceramics. Graphic lly, W_1 , W_2 and their difference are represented by area in the following figure. This figure confirms that a st ffer rod has less mechanical work, in other words, in sho t circuit case, the mechanical work done to the rod is mor than open circuit case ($W_1 > W_2$).

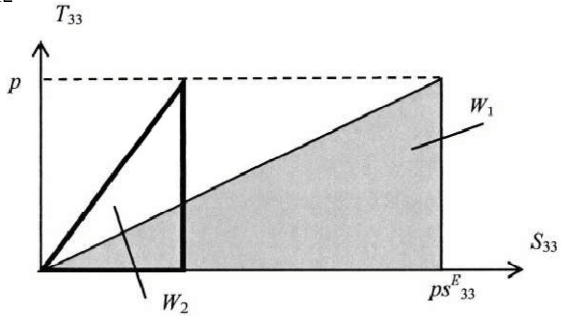

Fig. 2 Work done to the cera ic rod per unit volume along different aths [8]

VI. Finite Ele ent Method

In this section, a finite elem nt model of the ceramic rod will be studied. The geom trical configuration of the piezoceramic rod is shown in f gure 3.

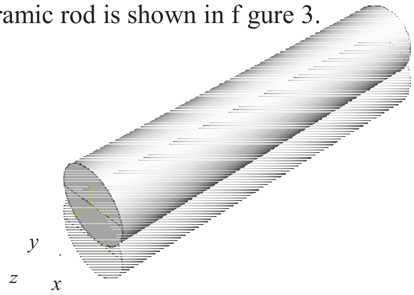

Fig. 3 The geometrical confi uration of the ceramic ro

The loaded configuration of the piezoceramic rod is shown in figure 4.

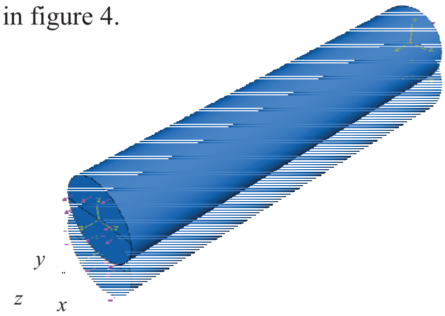

Fig. 4 The loaded configu ation of the ceramic rod

The length of the ceramic ro is assumed to be 10 cm. the radius of the rod is 1.0 cm. T e uniform normal traction is assumed to be 1 N/m^2. A typic l finite element model of the ceramic rod is shown in figu e 5. It should be noted that ceramic rod consists of e gh -node 3D linear brick iezoelectric elements (C3D8). The finite element mesh consists of 1872 elements for piezoceramic rod [10].

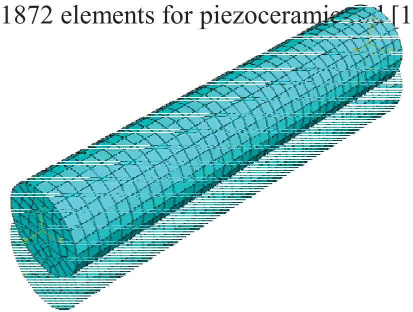

Fig. 5 Typical finite elemen model of the ceramic rod

Figure 6 shows the longitudinal displacement of the iezoceramic rod obtained by inite element analysis.

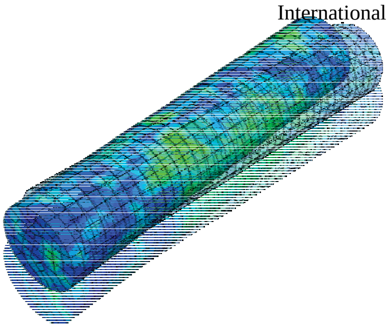

Fig. 6 Longitudinal displacement of the cer mic rod by finite element analysis

Two cases of *short circuit* (S.C) and *pen circuit* (O.C) as the electrical boundary conditions was investigated and an exact solution was presented for eac case. It is shown that the results obtained by the finite element analysis matches very well with the exact s lutions for each boundary condition. The results obtai ed by the finite element analysis and exact solution ar presented in the following table I

TABLE I
THE RESULTS OBTAINED BY THE FINITE ELEMENT A ALYSIS AND EXACT SOLUTION FOR PZT-5H CERAMIC OD

Case	D_3^{Exact}	S_{33}^{Exact}	E_3^{Exact}	D_3^{FEM}	S_{33}^{FEM}	E_3^{FEM}
S.C	593×10^{-12}	20.7×10^{-12}	0	594×10^{-12}	20.7×10^{-12}	0
O.C	0	9.11×10^{-12}	-1.97×10^{-2}	0	9.16×10^{-12}	-1.95×10^{-2}

VII. CONCLUSION

The piezoelectric finite element c pability recently made available in commercial FEA pac ages allows both static and dynamic analysis of fully co pled piezoelectric and structural responses. This paper reviewed the capability of the piezoelectric element provided y commercialized FEA codes, and discussed a simple case of static finite element analysis involving piezoelect ic and structural coupling.

Two cases of short circuit and o en circuit as the electrical boundary conditions was in estigated and an exact solution was presented for each c se. It was shown that the results obtained by the finite element analysis matches very well with the exact s lutions for each boundary condition.

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