

# A Model of a Non-expanding Universe Driven the Vacuum Space Properties

Yongbai Yin

**Abstract**—We propose a non-expanding model of the universe based on the non-changing fine-structure constant and the Einstein's space-time relativity theory by assuming that the vacuum space permittivity is a decaying function over the time span of the universe. This model consistently explains the Redshift, the "expanding" and the age of the universe as in the "Big Bang" model. It also offers an interpretation on the unexpected "accelerated expanding" universe and the origin of the mystery "Dark matters" which the "big Bang" model has failed to explain. This model predicts that the universe began with an "extremely cold" rather than "extremely hot" stage to explain the cosmic microwave background radiation and the age of the universe without introducing the problematic singularity and inflationary issues introduced in the "Big Bang" model. It predicts mathematically that galaxies could end into blackholes. Because blackholes should have the same space conditions as those of the vacuum space in the beginning of the universe in this model, this work paves the way to support the cyclic universes model without violating the first law of thermodynamics.

**Keywords**—Cosmic microwave background, dark energy, dark matters, expanding universe, evolution of the universe, blackholes.

## I. INTRODUCTION

CURRENT well accepted universe theory is the "Big Bang" model, supported or driven by the Redshift or "recession" observation. The "Big Bang" theory describes the universe started with an infinitely hot and dense single point, followed by a "Big Bang" to inflate and stretch from an unimaginably high speed. The understanding about the "Big Bang" comes from mathematical modelling and through a phenomenon known as the cosmic microwave background radiation. Besides the Big Bang theory, there are other much less popular or successful theories such as eternal inflation or an oscillating universe [1]-[4].

Fritz Zwicky proposed the "Dark matter" or invisible matter in 1934 to account for evidence of the "missing mass" in the orbital velocities of galaxies in clusters [5], [6]. Subsequently, other observations have indicated the presence of the mystery "Dark matter" [7], [8]. The "Dark matter" is not interacting with electromagnetic radiation or utterly transparent or nonbaryonic. The required "Dark matter" should consist over 85% of all matter in the universe and its existence indicates without introducing the "Dark matter" the behavior of stars, planets, galaxies, and the universe would be inexplicable.

The Einstein's relativity theory was based on that the speed of light is a constant to build his revolutionary relativity theory of space and time [10]. Attempts were continuously conducted

in searching evidence whether the speed of light is or is not a fundamental constant of nature, for example, measuring the fine-structure constant, though incredibly the fine-structure constant does not appear to have been much different from a long time ago [11]-[14].

## II. MODELLING

The speed of light in vacuum is a physics constant today defined as exactly equal to 299,792,458 meters per second, the second is defined by taking the unperturbed ground-state hyperfine transition frequency of the cesium-133 atom to be 9192631770, which is equal to  $s^{-1}$ , and the unit of meter is the light travelled length in  $1/299792458$  second in vacuum [9].

Considering light emissions from electrons-ions interactions in atoms, the photon energies are proportional to the square of the Coulomb constant,  $k_e^2$ , which suggests the frequency of light is also proportional to  $k_e^2$  by assuming the Planck constant is independent of the Coulomb constant. Further we take the fact that the fine-structure constant is independent of the age of the universe, therefore one can obtain that the speed of light in vacuum space is proportional to the  $k_e$ .

We chose a refence point of the universe (the current stage of our time and space, in this paper). In other stage of the universe (past or future), we express the "Coulomb constant" as  $k_p = k_e k_r$ , here we call  $k_r$  as a relative vacuum space property function. If the  $k_p$  is not a constant at other stages of the universe, then  $k_r$  is not equal to 1. The speed of light for other stages of the universe can be expressed as  $c_p = c k_r$ , where  $c$  is the speed of light in vacuum space determined currently at the current stage of the universe. The distance unit meter is thus inversely proportional to  $k_r$  (i.e.  $x_p = x_o k_r^{-1}$ , where  $x_o$  is the current value of the meter unit), and the time unit in second is  $s_p = s_o k_r^{-2}$ , where  $s_o$  is the current second unit. If  $k_r$  is not equal to 1 the values of  $s_p$  in second and the meter  $x_p$  will be different from those in today. The unitless quantity  $s_p c_p / x_p$  is an invariance parameter independent of  $k_r$  and does not violate the causality and the Einstein' relative space-time theory. A distance  $l_p$  measured using the meter definition  $x_p = x_o k_r^{-1}$  will be proportional to  $k_r$ . As  $c_p = c k_r$ , the time  $t_p$  required for a light to pass through the distance  $l_p$ , therefore, is also an invariance parameter independent of  $k_r$ . Thus, the invariance parameter  $s_p c_p / x_p$  can be extended to a form of  $t_p c_p / l_p$  for radiation traveling, regardless the value of the relative vacuum property function  $k_r$ .

We consider a model of the universe: in the beginning of the

Yongbai Yin is a physicist at Sydney, NSW, Australia (e-mail: yongbaiyin@gmail.com).

universe, the value of the relative vacuum property parameter  $k_r$  of the vacuum space is very small and then the  $k_r$  increases over the life span of the universe, i.e. the vacuum space is decaying over the life span of the universe. Note that in our model the photon emission frequency  $f_p = f_e k_r^2$  and its wavelength  $\lambda_p = \lambda_e k_r^{-1}$ , where  $f_e$  and  $\lambda_e$  are our current frequency and wavelength values respectively.

We start at a naturally decaying reciprocal vacuum property function (proportional to the vacuum space permittivity)  $k^{-1}(t) = Ae^{-bt}$  ( $A$  and  $b$  are positive and limited constants), or a relative vacuum property function  $k_r(t) = e^{-b(T-t)}$ , normalized by our current  $k_r$  at the defined age  $T$  of the universe. In this study we call the accumulated time  $t$  for a light passing through the space as “the universe time” thus the  $T$  is the age of the universe in the universe time. We assume in this model that all current fundamental physics laws are firmly hold including Einstein’s space-time theory and quantum physics.

### III. RESULTS

We consider a first light emitted at universe time close to zero and finally reached us by travelling an accumulated or proper distance  $D(0)$  at universe age time  $T$  in a “static” universe scenario, we have:

$$D(0) = \int_0^T c e^{-b(T-t)} dt = \frac{c}{b} (1 - e^{-bT}). \quad (1)$$

When  $e^{-bT} \approx 1$ ,  $D(0) \approx cT$ ; and when  $e^{-bT}$  approaches zero the proper distance  $D(0)$  approaches the constant  $c/b$ . For a general case of a light emitted at time  $t$ , the proper distance  $D(t)$  can be expressed as:

$$D(t) = \int_t^T c e^{-b(T-t)} dt = \frac{c}{b} (1 - e^{-b(T-t)}). \quad (2)$$

Equation (2) shows that the “expansion” rate of the distance  $\frac{dD(t)}{d(T-t)} = c e^{-bT+bt}$  is an increasing function with the universe time  $t$  and of course with our current time defined in the current universe age because both time functions are monotonically increasing. The second derivative of  $D(t)$  is also a non-zero positive and increasing function. Thus, this model gives a straightforward explanation on the “recession” and on the observed Redshift result [15] that the universe was acceleratingly “expanding”, which is originated from the decaying vacuum space property according to this model.

For nearby stars, the Hubble constant  $H$  in the Hubble’s law:  $v = HD = cz$ , was recently determined as 72.0 or 69.8 km/s/Mpc in well cited two separate determinations and its reciprocal value was used to define the age the universe [16], [17], where  $v$  is recessional velocity,  $D$  is the proper distance, and  $z$  is the Redshift equal to a normalized wavelength shift. Using the vacuum property model, wavelength  $\lambda(t) = \lambda_e k_r^{-1}(t)$ , wavelength shift  $\Delta\lambda(t) = \lambda_e k_r^{-1}(t) - \lambda_e$ , or  $\lambda_e k_r^{-2}(t) \Delta k_r(t)$ , for the nearby proper distance, we have:

$$HD = cH(T - t) = cz = c \frac{\Delta\lambda}{\lambda_e} \approx cb(T - t). \quad (3)$$

Equation (3) shows that the constant  $b$  in the relative vacuum property function is equal to the Hubble constant in the near universe age approximation. When  $T - t$  is not small, using  $\Delta\lambda(t) = \lambda_e k_r^{-1}(t) - \lambda_e$  and (3), the relationship between  $H$  and  $b$  is:

$$H \frac{c}{b} (1 - k_r(t)) = c \frac{1 - k_r(t)}{k_r(t)}, \text{ or } H(t) = b k_r^{-1}(t). \quad (4)$$

Equation (4) shows that the  $H(t)$  is inversely proportional to the relative vacuum space property function. Taking  $b = H(T)$  and  $(1 - k_r(t))/k_r(t) = z(t)$ , we can express the “Hubble constant” as a time dependent function  $H(t)$  on  $z(t)$  for a radiation signal emitted at time  $t$  close to zero as:

$$H(t) = H(T) e^{H(T)(T-t)} = H(T) (1 + z(t)). \quad (5)$$

Using the largest observed value of  $z(t)$  and assuming the value is for a light emission close to  $t = 0$ , we can express the age of the universe in the universe time scale as:

$$T \geq \frac{\ln(1+z(0))}{H(T)}. \quad (6)$$

Applying the large  $z$  Redshift value 4.658 observed recently [18] and  $H^{-1}(T) = 13.8$  billion years to (6), we obtained the  $T$  about 23.92 billion universe years; and using the highest  $z$  Redshift 10.957 reported earlier [19], we obtained the  $T$  about 34 billion universe years, which can be simply scaled to our current time using the scaling equation:

$$T_c(t) = \int_t^T e^{-b(T-t)} dt = \frac{1}{b} (1 - e^{-b(T-t)}). \quad (7)$$

This scaling equation is capped at the value of  $T_c = b^{-1}$ , showing the consistency with the “Big Bang” model. Using (7) we obtained the  $T_c$  time for the  $z$  Redshift of 4.658 and 10.957 light was from 11.4 and 12.7 billion current years respectively. By taking  $t = 0$  and when  $bT \gg 1$ ,  $T_c(0)$  approaches the current defined or accepted universe age =  $b^{-1} = 13.8$  billion current years.

We argue that when the vacuum space property is time dependent then the vacuum space must contain energy and mass. We apply Einstein’s mass-energy equation  $E = mc^2$  to this model, the mass-energy equation is modified as  $E = mc^2 k_r^2(t)$ . When the  $k_r(t)$  is small at the beginning of the universe, i.e., at  $t = 0$ ,  $E = mc^2 k_r^2(0) = mc^2 e^{-2bT}$ , here  $c^2 e^{-2bT}$  is a non-zero invariance constant independent of  $T$ , indicating an “extremely cold” rather than “extremely hot” early universe. As the universe becomes older and older, the energy level of an ordinary mass relative to the energy level of vacuum space becomes larger and larger.

We propose that the energy level of the vacuum space reduces while the energy of mass matters increases by assuming that an enclosed universe obeys the energy. Using the current observable proper distance 13.8 billion light current years and “Dark matter” = 5.4 times of ordinary matter [20] and  $k_r = 1$ , we estimate that the energy density level of the vacuum space has dropped by  $9 \times 10^{-9} J/m^3$  since the beginning of the universe.

Accept the fact that the vacuum space does not travel in the speed of light, it must have a rest mass property otherwise it violates the Einstein's time-space relativity theory. This implies that we must assume that the vacuum space has a non-condensable super-fluid property and does not interact directly with ordinary matters or radiations other than gravitational forces or is a form of matter which has not been included in the Standard Model. When an ordinary matter is in a uniform and isotropic vacuum space, the integrated net gravitational force from the isotropic uniform vacuum space environment will be cancelled as if the ordinary matter were floating in a super fluid space with a nil net interaction.

We argue that the vacuum space property cannot be homogeneous locally. If the vacuum space has a property of rest mass to interact gravitationally with localized stars/blackholes and galaxies, it will lead to an inhomogeneous vacuum space around stars/blackholes or within or around galaxies to appear as a mystery "Dark matter" around or within galaxies. We use a relative mass density ring function  $\rho_m(r)$  to simulate the mass effect of the vacuum space reference to the mass level of the remote vacuum space for a case of a galaxy that its stars orbit the galaxy with a constant orbiting period  $T_p$ , where the relative mass density ring function is an integrated effect of an approximated ring (or a cylindrical shell) of vacuum space centered at the center of the galaxy with radius  $r$ . For a star at distance  $R$  from the center of the galaxy, any point at distance  $r < R$  from the center of the galaxy, the contribution of the relative mass density ring function  $\rho_m(r)$  of the vacuum space to the star at  $R$  is  $\rho_m(r) - \rho_m(R)$ ; if  $r > R$ , the contribution of the mass ring function of the vacuum space is zero. We use a linear function truncated at distance equal to the maximum possible radius of the galaxy:  $\rho_m(r) = \rho_{mo} - \sigma r$ , where  $\rho_{mo}$  and  $\sigma$  are constants satisfying the equation  $\rho_{mo} - \sigma R_{max} = 0$ , and the  $R_{max}$  is the maximum orbiting radius of stars at the edge of the galaxy. Using the "Dark matter" model for a given galaxy with a constant rotational period and assuming the thickness  $h(r)$  of the galaxy satisfies a truncated parabola function:  $h(r) = \beta(R_m^2 - r^2) \geq 0$ , where  $\beta$  is a constant and  $R_m$  is the radius of the galaxy, we have:

$$\frac{2\pi G \int_0^R \beta(R_m^2 - r^2)(\rho_m(r) - \rho_m(R))r dr}{R^2} \approx \frac{1}{3} \sigma \beta \pi G R_m^2 R = \frac{4\pi^2 R}{T_p^2}. \quad (8)$$

It satisfies the requirement of a constant period  $T_p$ , which is equal to  $(12\pi/\sigma Gh(0))^{1/2}$ . Note that if one takes a uniform thickness  $(\beta R_m^2)$  of a galaxy instead of the truncated parabola function, (8) will be exactly equal. Using the Milky Way galaxy as an example,  $T_p = 225 \times 10^6$  years and  $h(0) = 1000$  light years, we obtained  $\sigma = 1.2 \times 10^{-39}$ . The mass effect of the vacuum property should not affect significantly the motions of local stars or planets even at the center of galaxies. We assume that the solar system is in the Milky Way galaxy center without interactions from others ordinary matters, the ratio of the interaction force on the Earth from the mass of the vacuum space to the interaction force from the Sun is about  $10^{-15}$ , indeed negligible. The  $\sigma$  value cannot be more than a certain value for

a galaxy, otherwise the entire galaxy could fall into a blackhole. Using the Schwarzschild radius [21], the requirement to prevent the blackhole formation of the Milky Way galaxy is:

$$r_s > \frac{2GM_m}{c^2} = \frac{2\pi Gh(0)R_m^3\sigma}{3c^2}. \quad (9)$$

where  $M_m$  is the supposed total mass of the "Dark matter" in the Milky Way galaxy, and  $r_s$  is the Schwarzschild radius. We found that the  $\sigma$  value cannot be more than  $2.7 \times 10^{-34}$ , which is 5 order of magnitude higher than the current value. At future time  $t > T$ , we can have the condition to prevent a blackhole to form as:

$$c^2 > \frac{2}{3} \pi Gh(0)R_m^2\sigma(t)k_r(t). \quad (10)$$

Equation (10) indicates that as long as the  $\sigma(t)$  is an increasing parameter over time, eventually the galaxy will fall into a blackhole as the  $k_r(t)$  increases continuously in this model.

The cosmic microwave background radiation [22] can be interpreted consistently as from the early "super cold" universe without introducing the gravitational singularity and the inflationary period in the "Big Bang" model. We use the hydrogen atom formation as the first radiation signals in the beginning of the universe. At the beginning of the extremely cold universe, the decaying vacuum property brought electrons and protons to appear without or with minimum thermal energy. The combination of electrons and protons to form hydrogen atoms would emit photons with  $13.6e^{-2bT}$  eV energy. The emitted photons would be absorbed by hydrogens or protons or electrons in the space. When the thermal equilibrium was reached and hydrogen atoms formation was largely completed the average thermal energy of each hydrogen atom  $= 3kT_b/2 = 13.6e^{-2bT}$  eV (where  $k$  is the Boltzman constant and the  $T_b$  = the microwave background radiation temperature 2.73K), we can obtain the  $T = 72.8$  billion universe years. Using the scaling equation (7) we have  $T_c = 13.73$  billion current years, which occurred 0.07 billion current years from the start of the universe.

As the universe becomes older and older and the speed of light in free vacuum space becomes larger and larger, the effect of the resulted "Dark matter" will more and more dominantly control the motions of a galaxy. This model suggests that eventually the overall mass of each galaxy will become large enough to crash the galaxy into a single blackhole. It is interesting to ask if all galaxy-blackholes be attracted together to form a grand super blackhole in the universe. Taking  $r_s(T) = 10$  times of the Milky Way galaxy radius  $R_m$  to cover a distance to reach nearby galaxies/blackholes, we can have that  $\sigma$  should be about  $2.7 \times 10^{-33}$ , 6 order of magnitude higher than the current value. When it happens, the mass of "Dark matter" will increase with  $r_s^3$ , an accelerated expanding of the Schwarzschild radius will occur, leading to a "contraction" universe, i.e. the formation of a single super blackhole in the observable universe is mathematically possible.

This model requires a mass reverse mechanism of blackholes

to bring the relative vacuum space function close to zero or the energy level of the vacuum space close to that of ordinary matter (if any exists), so that the decay of the vacuum space can start again for the next universe, otherwise our universe would be unique. The invariance parameter  $c^2e^{-2bT}$  tells us that the energy level of a mass cannot be zero, i.e., the  $k_r(t)$  function inside the Schwarzschild radius cannot be zero, or the speed of light cannot be zero and must be greater or equal to  $ce^{-bT}$ . Taking the case of the speed of light inside the Schwarzschild radius equal to  $ce^{-bT}$  and the value  $k_r$  inside the blackhole equal to  $e^{-bT}$ , we can fortunately see that it is the same conditions of the beginning of the universe in this model, which offers a strong indication that the reverse mechanism is occurring within blackholes whenever the blackholes expand. Therefore, this model indicates that our universe was originated from a super blackhole.

#### IV. DISCUSSION

A few conclusions resulted directly from the proposed non-expanding universe model are supported by the observed well-known results, including the “Redshift” of the galaxies, the “recession” speed of the galaxies, the observable universe age (13.8 billion current years). This model is the first time to explain mathematically the puzzling observation of the “accelerated expanding” universe, and first to model completely the origin of the mystery “Dark matters” well agreed with the constant orbiting speeds of stars in galaxies. The “extremely cold” temperature of the early universe resulted from this model is well supported by the cosmic microwave background radiation at 2.73K. The lens effect of the galaxies and in particular the blackholes support the conclusion that the vacuum space properties in their vicinity of the objects are different from those in the free vacuum space. Our model shows that the yearly change rate of the vacuum space property function or the Coulomb constant (about  $9 \times 10^9$ ) is inversely proportional to the universe age, much smaller than the uncertainty of the Coulomb constant. Particularly, the Coulomb constant is commonly measured using the speed of light, which is correlated with the vacuum space property function. In particular, the product of the speed of light and the vacuum space property function is an invariance parameter, therefore, the results of the Coulomb constant measurement also support our model. In a word, this model is supported by all cosmic observations of the universe. A few feasible experiments are suggested to give further confirmation or otherwise on the validity of this model, for example:

1. The rotation speed of a galaxy was becoming faster and faster over the life span of the universe, or the remote galaxies appear to rotate slower, and the sizes of remote galaxies/clusters were larger and became smaller and smaller over the life span of the universe.
2. The quantity of the “Dark matter” in galaxies was growing, thus their lens effects of galaxies were enhanced over the life span of the universe.
3. By assuming the  $k_r$  is a continuous function outside the Schwarzschild radius of a blackhole, the  $k_r$  value near the

Schwarzschild radius must be extremely small.

#### V. SUMMARY

A non-expanding universe model is proposed by introducing a decaying vacuum space property function. This model can explain the Redshift and the “recession” of the universe, the beginning and the age of the universe without introducing the singularity and inflationary issues that introduced in the “Big Bang” model. Furthermore, it can explain the surprising and puzzling “accelerated expanding” of the universe and the “unexpected” constant orbiting period of stars in galaxies i.e., the origin of the mystery “Dark matter”, which the “Big Bang” model has failed to explain. The cosmic microwave background radiation observation supports this model as evidence of an “extremely cold” early universe, rather than an unimaginable “extremely hot and small point” process in “Big Band” model. It shows mathematically that galaxies could be eventually crashed into a large blackhole. We argue that the initial vacuum space conditions of the universe should be the same to that within the Schwarzschild radius of blackholes therefore a process should occur within blackholes to bring their vacuum space back to their initial values whenever the blackballs expand, opening a possibility of cycling universes without violating the first law of thermodynamics.

#### REFERENCES

- [1] Belenkiy, A.; (2012). “Alexander Friedmann and the origins of modern cosmology”. *Physics Today*. 65(10): 38–43.
- [2] Nemiroff, R.J.; Patla, B.; (2008). “Adventures in Friedmann cosmology: A detailed expansion of the cosmological Friedmann equations”. *American Journal of Physics*. 76(3): 265–276.
- [3] Carroll, S.M.; Kaplinghat, M.; (2002). “Testing the Friedmann equation: The expansion of the universe during big-bang nucleosynthesis”. *Physical Review D*. 65(6): 063507.
- [4] Mörtzell, E.; (2016). “Cosmological histories from the Friedmann equation: The Universe as a particle”. *European Journal of Physics*. 37(5): 055603.
- [5] Zwicky, F.N.; (1933). “The red shift of extragalactic nebulae”. *Helvetica Physica Acta*. 6:110–127.
- [6] Zwick, F.N.; (1937). “On the Masses of Nebulae and of Clusters of Nebulae”. *The Astrophysical Journal*. 86: 217–246.
- [7] Taylor, A. N.; Gravitational lens magnification and the mass of Abell 1689. *The Astrophysical Journal*. 501(2), 539–553, (1998).
- [8] Refregier, A.; (2003). “Weak gravitational lensing by large-scale structure”. *Annual Review of Astronomy and Astrophysics*. 41(1): 645–668.
- [9] The International System of Units (SI), 9th ed.; Bureau International des Poids et Mesures: Sèvres, France, 2019. <https://www.bipm.org/documents/20126/41483022/SI-Brochure-9-EN.pdf/2d2b50bf-f2b4-9661-f402-5f9d66e4b507>
- [10] Einstein, A.; (1916). “The Foundations of the General Theory of Relativity”, *Ann. Phys.* 49.
- [11] Murphy, M.T.; Webb, J.K.; Flambaum, V.V.; Dzuba, V.A.; Churchill, C.W.; Prochaska, J.X.; et al. (2001). “Possible evidence for a variable fine-structure constant from QSO absorption lines: motivations, analysis and results”. *Monthly Notices of the Royal Astronomical Society*. 327(4): 1208–1222.
- [12] Webb, J.K.; Murphy, M.T.; Flambaum, V.V.; Dzuba, V.A.; Barrow, J.D.; Churchill, C.W.; et al. (2001). “Further evidence for cosmological evolution of the fine structure constant”. *Physical Review Letters*. 87(9): 091301.
- [13] Murphy, M.T.; Webb, J.K.; Flambaum, V.V.; (2003). “Further evidence for a variable fine-structure constant from Keck/HIRES QSO absorption spectra”, *Monthly Notices of the Royal Astronomical Society*. 345(2): 609–638.
- [14] Chand, H.; Srianand, R.; Petitjean, P.; Aracil, B.; (2004). “Probing the

- cosmological variation of the fine-structure constant: Results based on VLT-UVES sample". *Astronomy & Astrophysics*. 417(3): 853–871.
- [15] Frieman, J.A.; Turner, M.S.; Huterer, D.; (2008). "Dark Energy and the Accelerating Universe". *Annual Review of Astronomy and Astrophysics*. 46 (1): 385–432.
- [16] Keel, W.C.; (2007). "The Road to Galaxy Formation (2nd ed.). Springer. pp. 7–8. ISBN 978-3-540-72534-3.
- [17] Freedman, W. L.; (2001). "Final Results from the Hubble Space Telescope Key Project to measure the Hubble constant". *The Astrophysical Journal*. 553 (1): 47–72.
- [18] Carnall, A.C.; McLure, R.J.; Dunlop, J.S.; McLeod, D.J.; Wild, V.; Cullen, F.; et al. (2023). "A massive quiescent galaxy at redshift 4.658" *Nature*, 619, 716–719.
- [19] Jiang, L.; Kashikawa N.; Wang, S.; Walth, G.; Ho, L.C.; Cai, Z.; et al. (2021). "Evidence for GN-z11 as a luminous galaxy at redshift 11". *Nature Astronomy*. 5: 256-261.
- [20] Jarosik, N.; Bennett, C.L.; Dunkley, J.; Gold, D.; Greason, M.R.; Halpern, M.; et al. (2011). "Seven-year Wilson microwave anisotropy probe (WMAP) observations: Sky maps, systematic errors, and basic results". *Astrophysical Journal Supplement*. 192(2): 14.
- [21] Kutner, Marc Leslie. (2003). "Astronomy: a physical perspective" (2nd ed.). Cambridge, U.K.; New York: Cambridge University Press. p. 148. ISBN 978-0-521-82196-4.
- [22] Komatsu, Eiichiro. (2022). "New physics from the polarized light of the cosmic microwave background". *Nature Reviews Physics*. 4(7): 452-469.