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Noise Source Identification on Urban Construction Sites Using Signal Time Delay Analysis

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***Abstract*—**The problem of identifying local noise sources on a construction site using a sensor system is considered. Mathematical modeling of detected signals on sensors was carried out, considering signal decay and signal delay time between the source and detector. Recordings of noises produced by construction tools were used as a dependence of noise on time. Synthetic sensor data was constructed based on these data, and a model of the propagation of acoustic waves from a point source in the three-dimensional space was applied. All sensors and sources are assumed to be located in the same plane. A source localization method is checked based on the signal time delay between two adjacent detectors and plotting the direction of the source. Based on the two direct lines' crossline, the noise source's position is determined. Cases of one dominant source and the case of two sources in the presence of several other sources of lower intensity are considered. The number of detectors varies from three to eight detectors. The intensity of the noise field in the assessed area is plotted. The signal of a two-second duration is considered. The source is located for subsequent parts of the signal with a duration above 0.04 sec; the final result is obtained by computing the average value.

***Keywords*—**Acoustic model, direction of arrival, inverse source problem, sound localization, urban noises.

I. Introduction

THE source identification problems form a branch of mathematical physics and engineering inverse problems with different statements and applications. For instance, the identification of pollution or heat sources is mainly described in terms of inverse source problems (ISP) for parabolic equations (see, for example, [1]-[3], the monography [4], and references therein); the inverse problems for elliptic equations have application in geophysics [4]; IPSs for hyperbolic equations are applied to identify the sources of electromagnetic, acoustic, or seismic waves [4]-[9].

Problems of determining several moving sources of electromagnetic waves are solved to identify the positions of flying machines [10], [11]; the issues of detecting multiple speech sources in the room are solved in similar ways [12]-[16].

To detect several sources acting simultaneously, there are different methods of localization, such as an estimate of time delay (DOA and TDOA) [12]-[17], maximum likelihood method (ML) [18]-[20], beamforming method (BM) [21]-[23], and Multiple Signal Classification (MUSIC) [1], [15], [16]. More detailed methods of identifying sound sources are discussed in the monography [24] and the review [25] and references therein. The review [25] is written with the aid of AI methods and gives not only the list of more relevant refences to that area, but additionally classifies sound localization methods presented recently.

The classical sound localization methods assume that the sources are localized at several points in space and are also detected by pointwise detectors. The corresponding mathematical model is represented by the MUSIC method [11] via the relation in (1):

$$x(t) = \sum_{i=1}^{M} A^T(\Phi_i)\, s_i(t) + n(t), \tag{1}$$

where the vector function x(t) of dimension N represents signals detected by N detectors; $s_i(t)$ is a wave amplitude generated by the i-th source, i = 1,…, M, where M is a number of sources; $A(\Phi_i)$ – is an N-dimensional vector that depends on i-th signal parameters, and $n(t)$ is a noise vector.

Representation in (1) allows one to write a matrix form of the model of detected signals as (2):

$$\begin{gathered} x(t) = As(t) + n(t), \\ A = [A(\Phi_1), A(\Phi_2), \dots, A(\Phi_M)], \\ s^T(t) = [s_1(t), s_2(t), \dots s_M(t)] \end{gathered} \tag{2}$$

From this representation, it follows that all detected signals belong to the span of the space of generated waves in noise-free cases. Then, the dimension of the linear span of detected signals cannot be greater than the dimension of the generated wave space. Hence, in [1], the fundamental fact was derived that the number of sources is determined by the rank of the matrix presented in (3):

$$S = x \times x^T \tag{3}$$

This fact helps to determine the number of sources in advance. However, this model has some restrictions related to the ratio noise/signal and assumptions of the incident waves, which we will discuss further.

We focus here on the identification of noises that are represented by acoustic sources in the construction sites. Compared to cases discussed in [11], our sources of interest are not speech, music, radio waves in fixed range frequencies, or

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other meaningful signals but noises. Then, the ratio noise/signal, an important parameter to estimate the detection quality, is not applicable in the cases considered here.

The paper is organized as follows: Section II consists of a statement on the basic mathematical model and formulates the solution to the direct problem in the case of 3D pointwise acoustic sources. Section III presents the statement of the inverse problem and analyses the applicability of the standard model (1) to the considered cases. Section IV describes the method to solve the inverse problem and the numerical modeling results. The paper is finalized by Conclusion.

II. Direct Problem Solution

The sound propagation model inside a homogeneous medium is specified by the acoustic equations in three-dimensional space [26] as presented in (4):

$$\frac{\partial u}{\partial t}+\frac{1}{\rho}\nabla p=\sum_{j=1}^{M} f_j(t,x,y,z),\ \frac{\partial p}{\partial t}+\rho c^2\nabla\cdot u=0 \qquad (4)$$

Here, $(x,\ y,\ z)$ and t are Cartesian coordinates and time, c denotes the constant speed of sound, ρ is the density of the undisturbed medium, and p and u, respectively, are the changes in pressure and speed of medium particles at the vicinity of the steady-state values ρ_0 = const and u = 0. The functions $f_j(t,x,y,z)$ describe the acting disturbing sources. After eliminating the velocity vector from the equations, the direct problem for pressure, in which the intensities of external sources are assumed to be known, the model in (4) is reduced to the Cauchy problem for the following wave equation (5):

$$\begin{cases}\frac{\partial^2 p}{\partial t^2}=c^2\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}\right)+\sum_{j=1}^{M} S_j(t,x,y,z)\\ p(0,x,y,z)=0\end{cases} \qquad (5)$$

In the general case, the solution to the problem (5) is given by a convolution of the fundamental solution of the wave equation with the right-hand side. We consider the case when the characteristic sizes of the sensors and disturbing sources are at least three orders less in magnitude than the scale of the solution area. This makes it possible to model "hot spots" as concentrated pointwise sources. Accordingly, the source intensity function will be written in the form of a generalized δ-function, i.e., (6):

$$S_j(x,y,z,t)=\delta(r-\ r_j)H_j(t),\ j=\overline{1,M} \qquad (6)$$

The fundamental solution of the wave equation (5) has the following form [27] (7):

$$E_3(x,y,z,t)=\frac{1}{4\pi c}\frac{\delta(r-ct)}{r},\ r=|\mathrm{r}|=\sqrt{x^2+y^2+z^2} \qquad (7)$$

Then, computing the convolution of the function in (6) with the fundamental solution in (7), we obtain the expression for the solution of the Cauchy problem (5) for the source j as (8):

$$p_j(t,r)=\frac{1}{4\pi c}\int_0^{\infty} H_j(t-\tau)d\tau\iint_{|y|=c\tau}\frac{\delta(|y|-c\tau)\delta(|r-y|)}{|y|}dS_{c\tau}=\frac{1}{4\pi c}\frac{H_j(t-|r|/c)}{|r|} \qquad (8)$$

Due to linearity, for several simultaneously acting sources, the solution is written in the form shown in (9):

$$p(t,r)=\frac{1}{4\pi c}\sum_{j=1}^{M}\frac{H_j(t-|r-r_j|/c)}{|r-r_j|} \qquad (9)$$

The signal arrives from each source with a delay equal to the distance from the detector to the source r_{ij} divided by the wave speed c. When different sensors receive a signal, this difference in arrival time is usually applied to derive the system of equations for source localization [23]. On the other hand, the signal arrives with a decay of amplitude inversely proportional to the distance to the source. This relationship can also be considered when determining the position of the source.

Equation (9) made it possible to perform numerical modeling of signal propagation from various sources using MATLAB scripts and construct sound intensity distribution in the solution area. Based on this model, the propagation of a spherical sound wave was simulated, and theories for detecting signal sources are tested below.

III. Generation of Synthetic Data

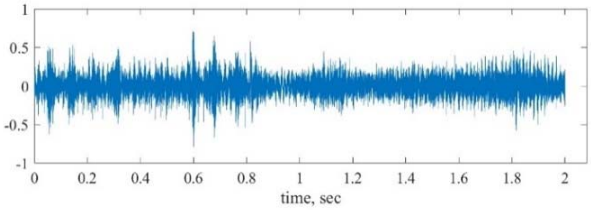
The model in (9) contains the functions $H_m(t)$ that describe disturbing signals at the source number m at the time moment t. To define those functions, we have used the real noises (like the sounds of some metal cutting tools and working construction machines) recorded on the construction site of residential complex "Athletic City" located in Astana, Kazakhstan, in July 2023. The sounds were recorded with a sample frequency of 48000 Hz by smartphone iPhone using an audio compression technology called Apple Lossless Audio Codec (ALAC). Length of recordings were ranging from 5 to 30 sec and a distance to the source was relatively close, around 5 m, due to the presence of extraneous noises. The analysis utilized MATLAB R2023b software to generate 2-second soundtracks as graphs from various noise sources (Fig. 1).

For further analysis, the typical graphs of these soundtracks for a time interval of 0.025 sec are taken (Fig. 2) as an example.

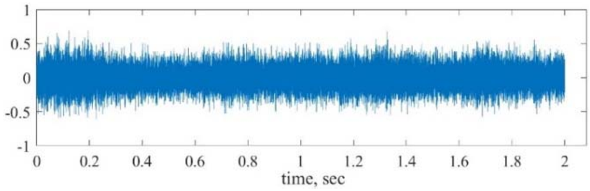
We need to choose the scales for the model's parameters to put numerical data in the admissible computing range. For modeling of sound propagation, the following characteristic scales were applied: length unit $[L]$=10 m, time unit $[T]$ = 0.01 sec, then the corresponding frequency of samples will be 480 samples/$[T]$, the speed scale will be measured in units of 100 m/sec, and the value of c in (4) is equal to 0.34 units. Furthermore, all computations below are made in the abovementioned scales.

By the (9), the n^{th} detector registers the following amplitude of the signal as (10):

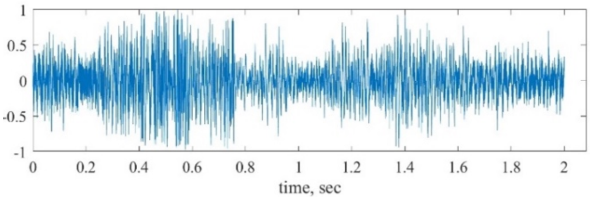
$$p_n(t,r_n)=\frac{1}{4\pi c}\sum_{m=1}^{M}\frac{H_m(t-|r_n-r_m|/c)}{|r_n-r_m|},\ n=\overline{1,N} \qquad (10)$$



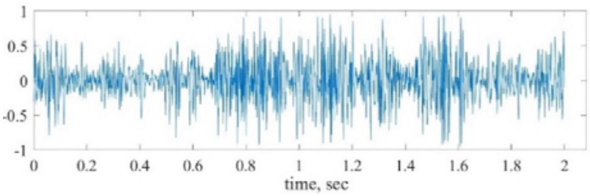
(a) Angle grinder switched on



(b) Grinding concrete



(c) Drilling concrete



(d) Hitting rebar with a hammer

Fig. 1 Soundtracks processed in MATLAB

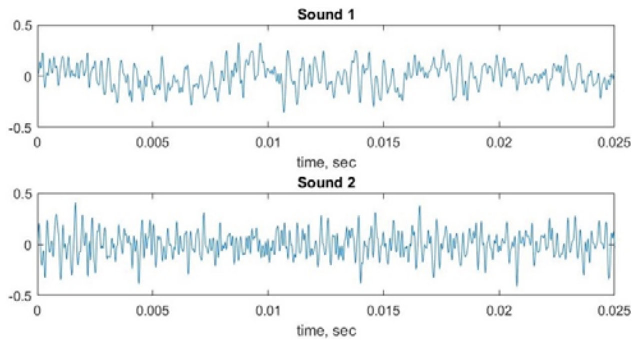


Fig. 2 Examples of sound signals from angle grinder in action (1200 samples are presented)

Here, M and N are the numbers of noise sources and sensors, respectively; r_n, r_m are radius vectors of sensors and sources positions, and $p(t, r_n)$ is a pressure deviation from the steady-state value at time t at the point r_n. The function $H_m(t)$ specifies the amplitude of the disturbance at the noise source number m at time moment t.

To simulate the measured data and sound propagation, a 1x1 square was considered, on the boundaries of which up to 8 sensors were located, and no more than four noise sources were randomly located inside of that square. Fig. 2 shows an example of source and sensors placement depicted together with the sound field amplitude distribution. Numerical amplitude values were obtained based on the sound propagation model (10).

The amplitude decay coefficients are set inversely proportional to the distance to the source, according to (10), and the signal delay associated with the finite speed of sound propagation is also considered.

IV. Detection of Sources' Location

A. Case of One Source

The case of one source with $M = 1$ is quite simple, and the location and time dependency can be recovered by the TDOA (Time Delay of Arrival) method. For that aim, three detectors are sufficient. To compute the time delay for two sensors with numbers k and l, we calculate the correlation function in (11):

$$B_{kl}(\tau) = \sum_{i=1}^{K} S_k(t_i) S_l(t_i - \tau) \tag{11}$$

The time delay is defined by the value $\tau_{\max}$, where the function in (11) obtains its maximum as (12):

$$\Delta_{kl} = \arg\max B_{kl}(\tau) \tag{12}$$

To compute that function, we use the MATLAB software's function *alignsignals*(·,·). The maximum of the correlation function in (11) gives the time delay between two signals Δ_{kl}. Suppose that the amplitude of the signals at detectors are measured and equal to A_k and A_l , and r_k and r_l are distances from sensors to the source. If the amplitude of the signal at the start is A, then $A_k = A/ r_k$, $A_l = A/ r_l$, and the following system of equations to define r_k and r_l can be formulated as (13):

$$r_k - r_l = \Delta_{kl}, A_l r_l - A_k r_k = 0 \tag{13}$$

The only exception is the case of $A_k = A_l$, when the system in (13) has infinite possible solutions, is $r_k = r_l$. For that case, we need to use data on additional sensor where the time delay is not zero. However, the solution for one source can be applied for the case of one dominant source. Knowing the distances between the source and two sensors helps to find the coordinates of sources geometrically. The accuracy of the source position strongly depends on the accuracy of the time delay definition. Computations show that the most important parameter to define the time delay is the length K of analyzed signal in (10). For instance, the values $K = 200$, 400, 2000 were checked and the last value $K = 2000$ gave the most accurate results in computing time delay and solving the system in (13). The larger values of K did not affect the results. Let us notice that the in Fig. 3, the exact position of the source (denoted by the red star) is imposed to the recovered location and intensity of the sound field is shown by gradation of color. The recovered location coincides with the exact one.

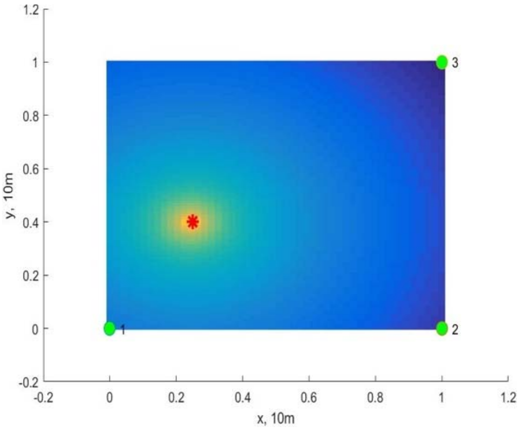


Fig. 3 The exact field intensity distribution and the recovered position of noise (the recovery coordinates are (0.2501, 0.3999) against actual values (0.25, 0.4))

The advantage of that model is its simplicity and that it works with a few numbers of detectors, even with two sensors. It is possible to apply that model in the case of one dominant noise source. In Fig. 4, we represent the result of source location in the case of three sources where one of them is dominant. To make one of sources dominate, we divide the signals from other sources by 10. Then, the corresponding norms of the signals equal to (37.5, 4.62, 3.82) for dominant source #1, (3.75, 46.3, 3.82) for source #2, and (3.7, 4.6, 38.2) for source #3. The number of samples $K = 2000$.

The position of signal source in Fig. 4 is obtained for the sound duration above 0.041 sec, but the duration of real noises is 2 sec.

Further, we consider the possibility to locate the source by using subsequent parts of the signal, for instance with duration of 2000 samples. In Fig. 4, the results of subsequent computations of source position are depicted for signal of 96000 samples, i.e., 48 parts of signal have the total duration equal to 2 sec. The final position of the source is computed as an average value of those 48 positions. Let us notice that if the source is moving, then this method helps to follow the trajectory of the source.

It is seen from Fig. 5 that in the presence of additional sources not all locations based on parts of the signal are computed correctly (green asterisks), however, the average value is obtained with satisfactory accuracy. Table I compares the exact coordinates of the dominant source and the recovered ones.

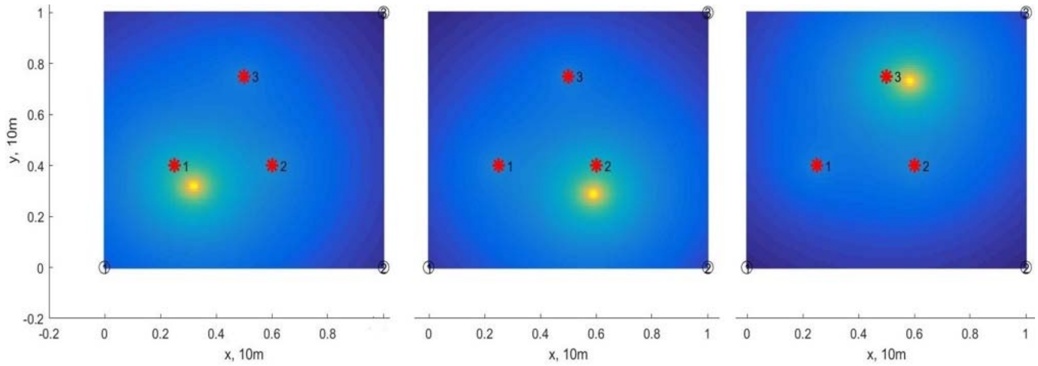


Fig. 4 Detection of one dominant source in the case of two additional pointwise noise

TABLE I
COMPARISON OF EXACT AND RECOVERED POSITIONS OF DOMINANT SOURCE

Source #	1		2		3	
	exact	recovered	exact	recovered	exact	recovered
(x, y)	(0.25, 0.4)	(0.24, 0.399)	(0.6, 0.4)	(0.595, 0.4)	(0.5, 0.75)	(0.57, 0.64)

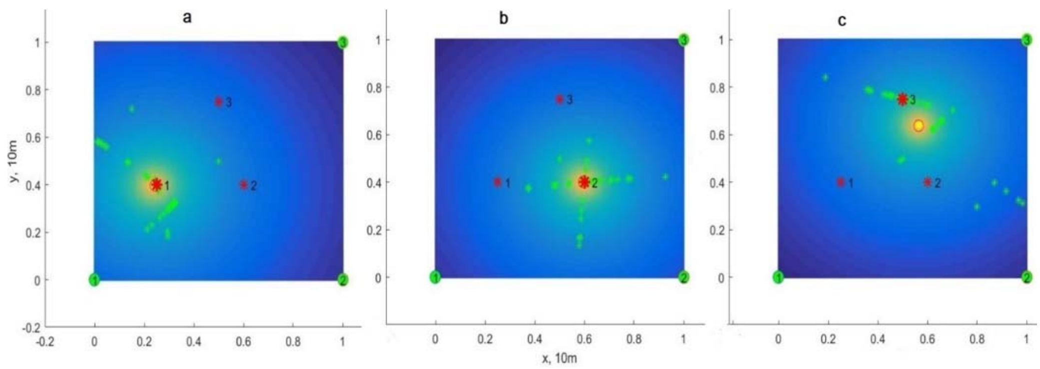


Fig. 5 Location of dominant sources: a – source #1, b – source #2, c – source #3 (red stars denote positions of pointwise signals, red circles denote recovered position of dominant source; green stars are positions computed using parts of the signal)

B. Detection of Two Sources

Suppose we have two simultaneously acting noise sources and at least four pairs of detectors. Fig. 6 shows the position of sources and detectors.

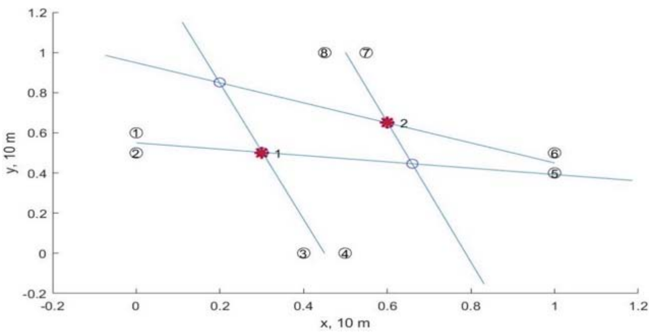


Fig. 6 Sources detection by four pairs of sensors, numbered from 1 to 8 (red stars represent exact positions of sources; intersections of lines are possible candidates for location of sources)

The case of two sources turns out to be more complicated compared the case of one dominant source. At the first step, we have checked the idea of detecting the number of sources by computing the range of the matrix defined by the detected signal via (3). It turns out that the range of the matrix *S* in (10) is not equal to the number of sources. We have checked that rule for the number of sources up to 7. Let us notice that the equality is obtained only in the cases when the delay time is neglected, i.e., is set to zero. That means that in our mathematical model, we cannot define the number of acting sources based on the theory described in [11], because the delay time is not negligible.

From another side, the examples above show the applicability of methods based on computing TDOA to locate the signals. That is the reason why we apply the method of detecting sources by computing TDOA between sensors.

We follow the idea similar to [12] of detecting the direction of the source by using a group of sensors placed in close vicinity each other. In the 2D case, we use pairs of sensors. The time delay of arrival δt is computed for each pair of sensors via the covariance (11). The angle θ between the line connecting sensors and the direction to the source is calculated by the following approximate relation in (14):

$$\cos\theta = \frac{\delta t \cdot c}{d} \tag{14}$$

Here, d is the distance between sensors in the pair. Let us explain the formula above. Suppose a source is placed at the point (x, y) on the plane, and two detectors are placed at points $(0, 0)$ and $(d, 0)$. Let (r, θ) be the polar coordinates of the source. Then, the difference in distances from the source to the sensors is equal to (15):

$$\Delta = \sqrt{x^2+y^2} - \sqrt{(x-d)^2+y^2} = \frac{-d^2+2xd}{\sqrt{(x-d)^2+y^2}+\sqrt{x^2+y^2}} \approx \frac{-d^2}{2r} + d\cos\theta \tag{15}$$

If $d << r$ then $\Delta \approx d\cos\theta$, and $\cos\theta \approx \frac{\Delta}{d} = \delta t \cdot c/d$.

The angle θ gives the direction to the source from the pair of sensors. The location of source is defined by the crosspoint of two direct lines corresponding to adjacent pairs of sensors. That imposes the restriction on the number of possible sources to be detected; namely, it is less or equal than half of the number of pairs.

In Figs. 6 and 7, the directions for each pair of detectors are built. The intersections of direct lines define the possible positions of sources. As is seen from Fig. 6, for instance, the adjacent pair of sensors labeled by (5, 6) and (7, 8) gives one possible intersection at point *A*, as well as the pairs (3, 4) and (5, 6) define another point *B*. To choose the correct position, compare the sum of distances from points A and B to the pairs of sensors as in (16):

$$Sum1 = |AC| + |AD|, \text{ and } Sum2 = |CB| + |BE| \tag{16}$$

The position with the lower value of that sum is considered to be a solution. For this example, the correct position is the point *A*. The Fig. 7 shows the recovered and the actual positions of sources. It is seen that they are located correctly.

Let us notice that the most important parameter to locate the noise source is the time delay. Because of highly irregular and noisy signal in the construction area, that delay can be computed with mistakes. It depends on number of samples to compute the correlation function in (11). The most admissible value of that parameters for considered examples is equal to 2000. In practice, it corresponds to the duration of signal equal to 2000/48000 ≈ 0.041 sec.

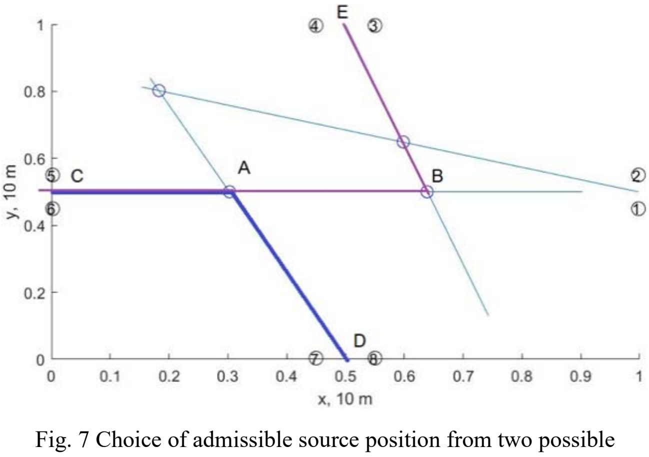


Fig. 7 Choice of admissible source position from two possible options A and B

But the real sound is much longer, for instance, for the case of 96000 samples, the duration is 2 second. Then that data is available for analysis too. For this case we set two identical noises at positions *A* and *B* and applied the method described above for subsequent parts of the detected sounds. The total number of parts is equal to 48, but not all parts are suitable to compute the TDOA. For instance, if the value Δ is greater than d, the direction cannot be defined. These parts were omitted, finally, 41 parts of total 48 were used in numerical computations. The final position is computed by average value of the obtained subsequent data. Fig. 8 shows the final positions of recovered noise locations. The computations were repeated for different kind of identical sounds placed at two different positions. It turned out that for identical sounds their positions are located with high accuracy. Then two different sounds were placed at two different positions.

In acoustics the energy of the wave is proportional to the square of the amplitude. Then, to compare the level of sounds we computed the squared Euclidian norm of the signal.

In this case the sound with higher energy is located correctly, but the sound with lower energy is located worse. In Fig. 9a, the norms of the sounds are 37.55 and 46.3, respectively, for sources 1 and 2. In Fig. 9b those sounds are in the exchanged positions. In both cases the position of the louder source is located better.

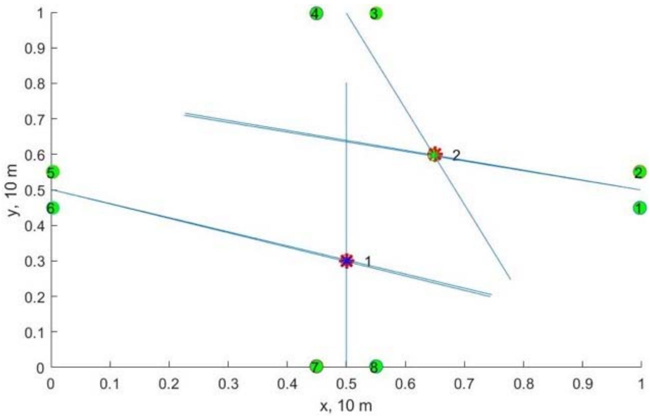


Fig. 8 Recovered locations of sources (red asterisks are exact positions (0.5, 0.3) and (0.65, 0.6); blue and green positions are recovered locations (0.5, 0.302) and (0.648, 0.598))

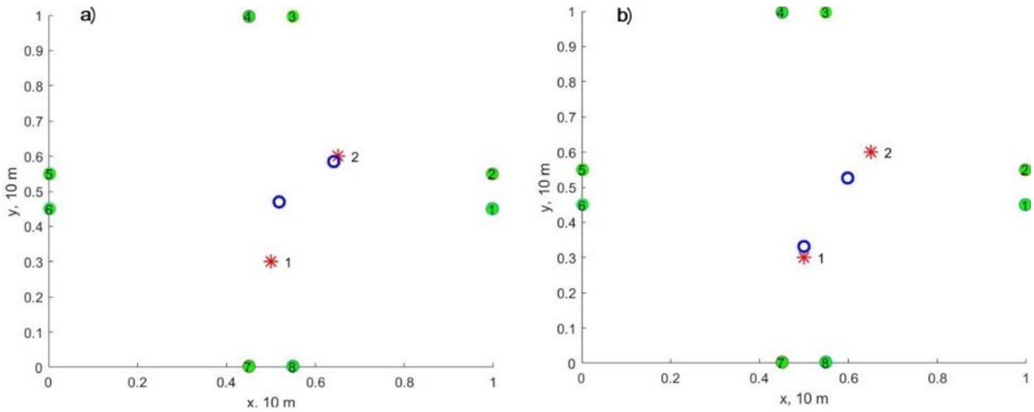


Fig. 9 Detection of two sources with different energy, where the source with higher energy is placed at: a – point 2 (res asterisk); b – point 1

V. Conclusion

Here, we made the mathematical modeling of the sound propagation and detection via equations of acoustic and by using several real noises written in urban areas. The computations show that model for one and two dominant signals that can be detected via relative low number of sensors and at short time interval. The case of higher number of sources needs additional study. However, the mathematical modeling helps to study the methods of modeling and detection of sounds sources before deployment of an experimental installation. We suppose that this approach has practical interest in monitoring the urban living area and checking disturbing noises.

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