

Performance Analysis of the First-Order Characteristics of Polling Systems Based on Parallel Limited ($k = 1$) Services Mode

Liu Yi, Bao Liyong

Abstract—Aiming at the problem of low efficiency of pipelined scheduling in periodic query-qualified service, this paper proposes a system service resource scheduling strategy with parallel optimized qualified service polling control. The paper constructs the polling queuing system and its mathematical model; firstly, the first-order and second-order characteristic parameter equations are obtained by partial derivation of the probability mother function of the system state variables, and the complete mathematical, analytical expressions of each system parameter are deduced after the joint solution. The simulation experimental results are consistent with the theoretical calculated values. The system performance analysis shows that the average captain and average period of the system have been greatly improved, which can better adapt to the service demand of delay-sensitive data in the dense data environment.

Keywords—Polling, parallel scheduling, mean queue length, average cycle time.

I. INTRODUCTION

POLLING is a dynamic control method, in which servers provide system services to customers sequentially from week to week. It has wide application and adoption in industries such as industrial control, public transport, communication networks, logistics, and economic forecasting due to the high reliability of its non-competitive scheduling process [1], [2].

Competition-based random scheduling of service resources is prone to congestion in high load service situations, whereas the periodic allocation and non-competitive access control mechanism provided by polling not only improves the stability of service resources in high load situations, but also has better delay characteristics. Therefore, the theoretical analyses of polling systems have been deepened and extended [3]-[5]. At present, various industries are in a period of unprecedented development, and the number of customers is growing dramatically, while the business applications also show a richer diversity [6], [7]. In particular, the application of information and communication technology has been expanded from the current human communication to the deep convergence communication between human, machine and object, and the system service not only responds to the changes of differentiated business diversity and real-time guarantee, but also emphasizes the system construction requirements for the super-large customer traffic, dense site access and ultra-high

reliability and low-latency service [8].

Limited ($k = 1$) service [9] means that when the server queries the current queue for customer arrivals, if the current queue has been reached by a customer, only one customer will be served or transferred if the current queue is still empty then the query is transferred to the next queue [10]-[12]. According to the control mechanism of the basic polling system, the overall service efficiency of the system is greatly reduced by the pipelined control process under light load and dense station. Therefore, how to improve the service efficiency of the polling system to meet the needs of the in-depth development of the system service is the focus of research by scholars at home and abroad [13]-[15]. At present, the research on how to further improve the service efficiency of the polling system is mainly divided into two directions. One is to obtain the reservation and site busy status information in advance during the polling process to reduce the number of sites queried by the server and narrow the query period to improve the system efficiency. The second is to improve the overall efficiency of the system by optimizing the traditional pipelined query service process in parallel [16]-[18]. The former is based on the premise that it is necessary to obtain the state information of each site in advance in order to prepare for the dynamic reorganization of sites, and the constant adjustment of the actual number of sites involved in the querying process will often consume more system resources. In the latter case, once the optimized system query service process has been defined, the system can complete the periodic system service stably and reliably [19]. The parallel limited ($k = 1$) polling system is clearly found to have better fairness and stability under high loads and super-multiple customers [20].

The mathematical model of the polling system is a complex stochastic system consisting of N-dimensional random variables, and the optimized control strategy makes the difficulty of solving the limited ($k = 1$) service polling system increase dramatically. In order to improve the service efficiency of the limited ($k = 1$) service polling system, this paper proposes a scheduling strategy for the parallel optimization of the query transfer and service process of the limited service polling system, using queuing theory and stochastic process theory to mathematically model the processes of customer arrival, server query and service, and transfer, and using the probability mother function of the N-dimensional random variables to

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construct a system state variable equation for the parallel optimization of the limited service polling system. The mathematical expressions for the average queue leader and the average query period are fully resolved. The statistical analysis results of the computer simulation experiments are verified with the theoretical calculations. Finally, the system performance of the polling system is evaluated.

II. SYSTEM MODEL

A. System Queuing Model Analysis

According to the queuing theory, the system can be viewed as an architecture consisting of N end stations and a single

server. The operation process of the traditional polling system is as follows: Server periodically queries each station in turn, and the service rule adopts the threshold service. The transition between the service and the query process, in t_{n-1} time server service $i-1$ station, after time β_{n-1} after the service of the customers in the site, and then after the transfer time and then in t_n moment query service After serving the customers in the site at time β_{n-1} , then after the transfer time, it inquires about serving site i at time t_n , serves the rest of the sites in the same way, and then returns to the first site, and then inquires about, transfers to, and qualifies the limited ($k = 1$) service in the same way.

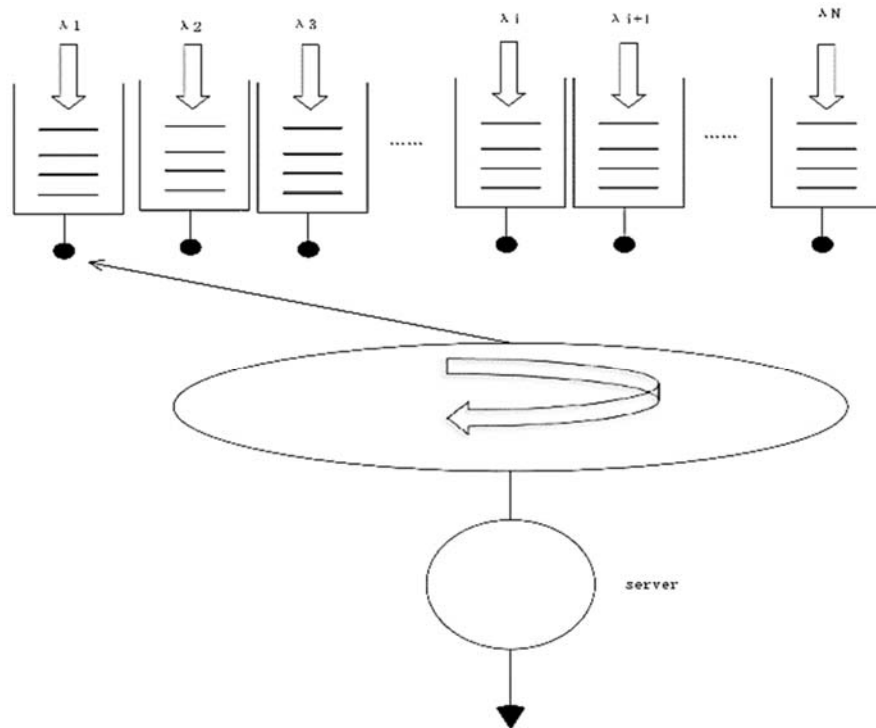


Fig. 1 Model diagram of the polling system

B. Variable Setting of System Random Process

The system operates in a discrete-time state, and the capacity of the site cache is sufficiently large and does not generate the phenomenon of customer loss. From the point of view of mathematical analysis, the arrival, transfer and service processes of the polling control system can be regarded as a composite stochastic system consisting of three N-dimensional stochastic processes.

Assume that the probability mother function of the arrival process in which the number of customers entering station i is given as $A_i(Z_i)$, it is the condition that satisfies the independent Poisson distribution whose mean $\lambda_i = A'_i(1)$, the variance is $\sigma_{\lambda_i}^2 = A''_i(1) + \lambda_i - \lambda_i^2$. After serving one site the system starts serving the next site $i+1$ through a transfer process, the mean and variance of this transfer process are

respectively $\gamma = R'(1)$, $\sigma_{\gamma_i}^2 = R''(1) + \gamma_i - \gamma_i^2$. Its probability mother function is set as $R_i(Z_i)$. The service process at the site is mean $\beta_i = B'_i(1)$, the variance is $\sigma_{\beta_i}^2 = B''_i(1) + \beta_i - \beta_i^2$, the independent of each other and obey the same probability distribution whose probability mother function is set as $B_i(Z_i)$.

Dissecting the model on the time axis, at moment t_n the parallel polling system points to site i to start serving it with a limited ($k = 1$) strategy, at which point the number of customers waiting to be served in site i is set to be $\zeta_i(n)$, $\zeta_i(n)$ is the number of customers waiting for service at site i ($i = 1, 2, 3, \dots, N$) at time t_n site, at which time the whole system status is:

$$\{\zeta_1(n), \zeta_2(n), \zeta_3(n), \dots, \zeta_N(n)\}$$

Subsequently, after the parallel query for the number of customers at site $i+1$, at t_{n+1} the system starts to qualify limited ($k = 1$) service for site $i+1$, at which time the state of the system is:

$$\{\zeta_1(n+1), \zeta_2(n+1), \zeta_3(n+1), \dots, \zeta_N(n+1)\}$$

C. System State Equation and Probability Generating Function

According to the above process, the number of customers waiting in each site with the server periodic query service change rule, the thesis gives the following system of state transfer equation. The server starts to serve site $i+1$ at time t_{n+1} , the number of customers in site j and site i at time t_{n+1} .

$$\begin{cases} \xi_j(n+1) = \xi_j(n) + \eta_j(v_i), \\ \xi_i(n+1) = \xi_i(n) + \eta_j(v_i) - 1, & \xi_i(n) \neq 0 \\ \xi_j(n+1) = \xi_j(n) + u_j(u_i), \\ \xi_i(n+1) = u_i(u_i), & \xi_i(n) = 0 \end{cases} \quad (1)$$

where $u_i(n)$ is defined as the query transition time for the server to move from site i to the next site; $v_i(n)$ is the time for the server to serve site i . $\mu_j(u_i)$ is the number of customers entering site j at time $u_i(n)$; $\eta_j(v_i)$ is the number of customers entering site j at time $v_i(n)$; $\zeta_j(n)$ is the number of customers entering site j at time t_{n+1} ; $\zeta_j(n+1)$ is the number of customers at time t_{n+1} at time t_{n+1} at time t_{n+1} .

When the system is in the steady state condition, the state of the system is a chi-square, non-approximable, non-periodic Markov stochastic process and has a unique stable distribution. Therefore, the server starts serving site $i+1$ at time t_{n+1} and the probability mother function of the system state variable in steady state is:

$$\begin{aligned} G_{i+1}(z_1, z_2, \dots, z_i, \dots, z_N) &= \lim_{n \rightarrow \infty} E \left[\prod_{i=1}^N z_i^{\xi_j(n+1)} \right] \\ &= \lim_{n \rightarrow \infty} \sum_{k=1}^{\infty} E \left[\prod_{\substack{j=1 \\ j \neq i}}^N z_j^{\xi_j(n) + \eta_j(v_i)} \cdot z_i^{\xi_i(n) + \eta_j(v_i) - 1} \right] + \\ &R_i \left[\prod_{j=1}^N A_j(z_j) \right] G_i(z_1, z_2, \dots, z_{i-1}, 0, z_{i+1}, \dots, z_N) \\ &= B_i \left[\prod_{j=1}^N A_j(z_j) \right] \frac{1}{z_i} [G_i(z_1, z_2, \dots, z_i, \dots, z_N) \\ &- G_i(z_1, z_2, \dots, z_{i-1}, 0, z_{i+1}, \dots, z_N)] \\ &+ R_i \left[\prod_{j=1}^N A_j(z_j) \right] [G_i(z_1, z_2, \dots, z_{i-1}, 0, z_{i+1}, \dots, z_N)] \\ &+ G_i(z_1, z_2, \dots, z_{i-1}, 0, z_{i+1}, \dots, z_N) \\ &i = 1, 2, \dots, N \end{aligned} \quad (2)$$

III. THE FIRST-ORDER CHARACTERISTIC ANALYSIS PROCESS OF THE SYSTEM

A. Define

Average queue length of customers in the queue in the polling system:

$$g_i(j) = \frac{\partial G_i(z_1, z_2, \dots, z_i, \dots, z_N)}{\partial z_j}, i=1, 2, \dots, N; j=1, 2, \dots, N \quad (3)$$

$$g_{i0}(j) = \lim_{z_1, z_2, \dots, z_j, \dots, z_N \rightarrow 1} \frac{\partial G_i(z_1, z_2, \dots, z_{i-1}, 0, z_{i+1}, \dots, z_N)}{\partial z_j}, \quad (4)$$

$$i = 1, 2, \dots, N; j = 1, 2, \dots, i-1, i+1, N$$

To find the second order partial derivatives, it is obtained by iterating through the obtained equation:

$$g_i(j, k) = \lim_{z_1, z_2, \dots, z_j, \dots, z_k, \dots, z_N \rightarrow 1} \frac{\partial^2 G_i(z_1, z_2, \dots, z_j, \dots, z_k, \dots, z_N)}{\partial z_j \partial z_k}, \quad (5)$$

$$i = 1, 2, \dots, N; j = 1, 2, \dots, N; k = 1, 2, \dots, N$$

B. Average Queue Leader

According to the definition of queue leader, the average queue leader of the system can be obtained by taking the first order derivative of the probability mother function of the system state variable. t_n the moment the server is serving station i , the average queue leader of station i at this moment:

$$\begin{aligned} g_i(i) &= \frac{N\lambda^2\gamma}{2(1-N\rho)(1-N\rho+N\lambda\gamma)} \left\{ (1-N\rho) \frac{R''(1)}{\gamma} \right. \\ &+ [1 - (N-1)\lambda(\beta-\gamma)] \frac{A''(1)}{\lambda^2} + N\lambda B''(1) \\ &\left. + (N-1)\gamma + (N-1)\lambda(\beta-\gamma) + \frac{2}{\lambda}(1-N\rho) \right\} \end{aligned} \quad (6)$$

C. Average Cycle Time

The query cycle of the system is the time that the server serves all site queries once, and its average value reflects the response speed of the system, by the mathematical relationship between the average queue leader and the average cycle time can be obtained as the average query period.

$$\begin{aligned} E(\theta) &= \sum_{i=1}^N [\gamma + g_i(i)\beta + g_i(i)\rho\beta + g_i(i)\rho^2\beta + \dots + g_i(i)\rho^n\beta] \\ &= \frac{N\gamma}{1-N\lambda\beta+N\lambda\gamma} \end{aligned} \quad (7)$$

IV. SIMULATION EXPERIMENT AND PERFORMANCE ANALYSIS

In order to verify the above theoretical results and evaluate the system performance, a computer simulation experiment

platform is built based on MATLAB R 2022b. The theoretical model and the simulation environment are set to have the same conditions, i.e., the customer arrivals at each station obey the Poisson process with parameter λ , and the system satisfies the stability condition of $N(\rho+\lambda\gamma) < 1$. Finally, the theoretically calculated values of the key performance indicators of the system are compared with the statistical values of the simulation experiments to illustrate the correctness of the system analysis. Comparison of the theoretical computed values of the parallel optimization limited polling service system and the statistical analysis values of the simulation experiment. The analysis results are shown below:

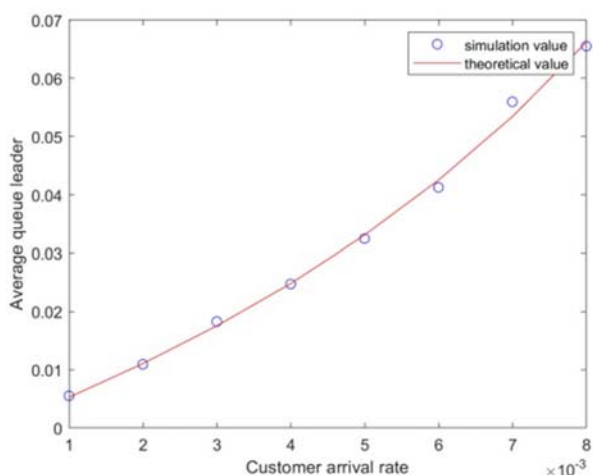


Fig. 2 Variation curve of average queue length with customer arrival rate ($N = 5, \beta = 10, \gamma = 1$)

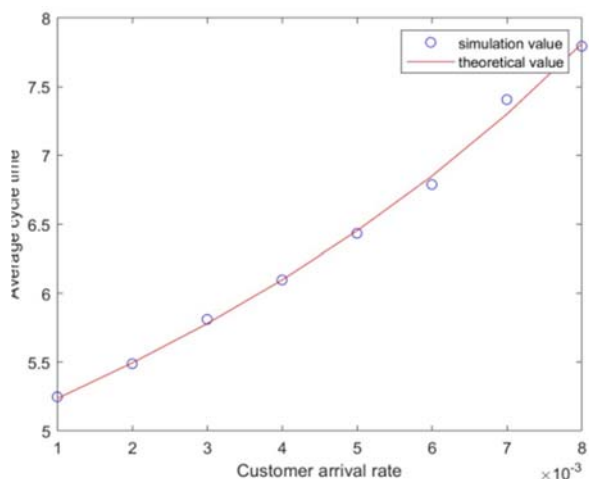


Fig. 3 Variation curve of average cycle time degree with customer arrival rate ($N = 5, \beta = 10, \gamma = 1$)

Figs. 2 and 3 both reflect the reasonable change trend of the first-order system characteristic indexes increasing with the increasing customer arrival rate. Especially in the whole curve, the curve calculated by the theoretical formula and the statistical analysis value of the computer simulation experiment basically coincide with each other, which shows that the theoretical analysis of the key indicators of the system and the

results of the simulation experiment have a good rationality and consistency.

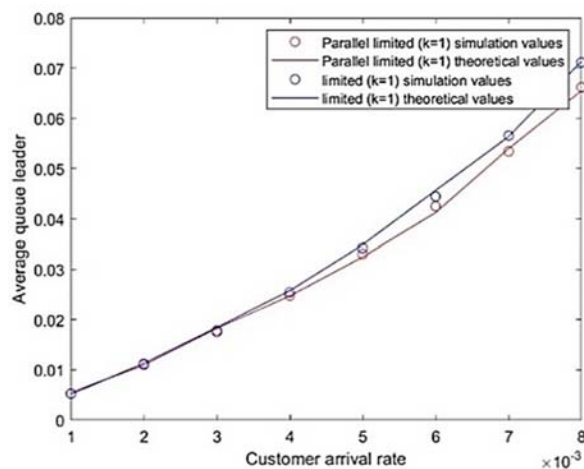


Fig. 4 Variation curve of average queue length with customer arrival rate ($N = 5, \beta = 10, \gamma = 1$)

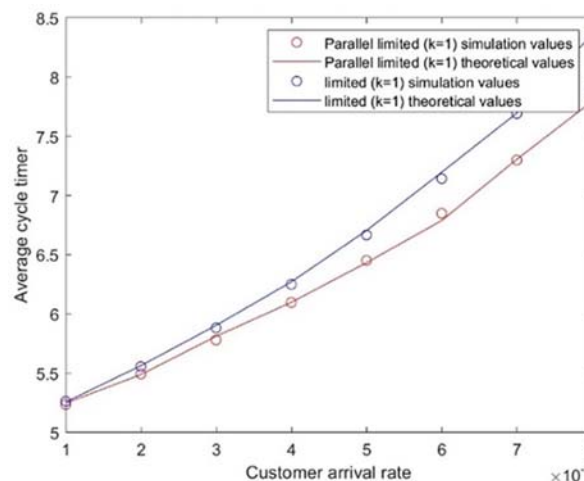


Fig. 5 Variation curve of average cycle time degree with customer arrival rate ($N = 5, \beta = 10, \gamma = 1$)

As shown in Figs. 4 and 5, with the increase of customer's arrival rate, the number of waiting people and customer's waiting time in the terminal station of the system are increasing, and the average queue leader and average cycle time of the limited ($k = 1$) and parallel optimization limited ($k = 1$) service modes are distributed hierarchically up and down without crossing each other, which is in line with the system characteristics of the polling system of conflict-free scheduling. Parallel limited ($k = 1$) has better average queue length and average cycle period, which indicates that the polling system has improved the queue length and cycle period of customers through the structural adjustment of parallel optimization, which significantly improves the service efficiency of the polling system.

V. CONCLUSION

At present, the theory of polling systems is an important

theoretical basis for the allocation of service resources and the management of shared access rights. With the broader cross-fertilization and ubiquitous development of information technology and all walks of life, it will have a greater impact on the service field of the whole society. In particular, the coming era of the Internet of Things (IoT), the system service demands of large customer traffic, dense site access, and ultra-high reliability and low-latency services put forward higher requirements for the service efficiency of the basic polling system and the expansion of the polling application space. Therefore, based on the in-depth analysis of the control mechanism of the periodic query-qualified service system, the paper carries out the parallel optimization of the pipelined serial scheduling of server transfer query and service processing. The system performance analysis shows that the parallel optimization of the query-qualified service polling system reduces the queue leader and waiting delay in the service process, improves the system service response speed, has better system stability, and makes the control technique of the polling system more efficient. The mathematical analysis process of the thesis expands more space for the evolution research and application of the polling service system.

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