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Power Allocation in User-Centric Cell-Free Massive MIMO Systems with Limited Fronthaul Capacity

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Abstract—In this paper, we study two power allocation problems for an uplink user-centric (UC) cell-free massive multiple-input multiple-output (CF-mMIMO) system. Besides, we assume each access point (AP) is connected to a central processing unit (CPU) via fronthaul link with limited capacity. To efficiently use the fronthaul capacity, two strategies for transmitting signals from APs to the CPU are employed; namely: compress-forward-estimate (CFE), estimate-compress-forward (ECF). The capacity of the aforementioned strategies in user-centric CF-mMIMO are drived. Then, we solved the two power allocation problems with minimum Spectral Efficiency (SE) and sum-SE maximization objectives for ECF and CFE strategies.

Keywords—Cell-free massive MIMO, limited capacity fronthaul, spectral efficiency, power allocation problem.

I. Introduction

CELL-free massive MIMO is a paradigm where a large number of the APs serve, collaboratively, fewer number of users which it diminishes the cell edge interference by overcoming the traditional cell concept [1]. Compared to the traditional massive MIMO, the massive macro-diversity that CF-mMIMO brings with itself improves our system by overcoming shadow fading and path loss [2]. One of the main purposes of the cell-free massive MIMO is to provide fair performance across the users. User-centric (UC) cell-free mMIMO is an approach where each AP serves the limited number of users with better channel condition [3]. In [4] and [5], the UC approach is compared to the CF one where both APs and users have single antenna; it is shown that the UC approach can achieve better SE for the majority of users, and it can reduce the required backhaul capacity. In [5], authors propose power control strategies on the downlink aiming at maximizing the minimum users' SINR, and sum-rate of the system. Paper [6] compared CF-mMIMO with the UC counterpart with assuming that each AP and user is equipped with multiple antennas. Results show that UC approach requires less backhaul overhead and achieves better performance than CF-mMIMO. It also proposes power allocation strategies with sum-rate and minimum rate maximization objectives for both the uplink and downlink. Reference [7] explores cell-free and user-centric massive MIMO at millimeter wave frequencies. Same authors in [8] extend the paper [7] by investigating energy-efficient downlink power control.

Cell-free has a distributed nature in which each AP is connected to CPU via fronthaul link with limited capacity and shares its data with CPU. As fronthaul link with limited capacity limits the SE of the system, exploiting the capacity of the fronthaul link efficiently is a crucial factor in improving the performance of these systems. Several works have been done on CF-mMIMO with the limited capacity fronthaul. For instance, in [9], for sending signals from APs to CPU, APs multiply the received signal by the conjugate of the estimated channel; and then by quantizing the weighted signal, they send it to the CPU. This paper maximizes the minimum rate with the power and fronthaul capacity constraints. Reference [10] extends [9] and studies the energy efficiency in these systems. In [11], authors study the CF-mMIMO in the uplink; they use two strategies, estimate-and-quantize and quantize-and-estimate, for CSI acquisition at CPU. Paper [12] considers limited fronthaul capacity and hardware impairments in CF-mMIMO systems. In this paper, three strategies, ECF, CFE, and EMCF, for transmitting signals from APs to the CPU are used and their performance are compared with each other. Paper [13] considers user management in a limited fronthaul capacity CF-mMIMO systems, and performance of the cell-free massive MIMO when it is serving limited number of users and using three strategies, ECF, CFE, and EMCF, for transmitting signals from APs to the CPU has been investigated. This paper is extended version of [13], we consider two uplink power allocation problems in UC-cell free massive MIMO system, the first one is the sum-SE maximization and the second one is the minimum-SE maximization.

This paper is organized as follows. In Section II, the system model is proposed. In Section III, the numerical results for performance analysis of the solved problems are presented and finally a brief conclusion of the paper is given in Section IV.

II. System Model

Our system model consists of M single antenna APs serving K single antenna users which are randomly distributed in an area. The mth AP where $m \in \{1, 2, \dots, M\}$ is connected to the CPU through fronthaul link with limited capacity $C_m[bits/s/Hz]$. It is considered that the coherence interval of wireless channels between APs and UEs modeled as block fading is T samples. The channel between user kth where $k \in \{1, 2, \dots, K\}$ and mth AP is modeled as follows,

$$g_{mk} = \sqrt{\beta_{mk}} h_{mk}, \text{ for } m = 1, \dots, M, k = 1, \dots, K, \quad (1)$$

where β_{mk} is the large-scale fading, and h_{mk} is small-scale fading coefficients with independent and identically

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distribution, i.e. $i.i.d$, zero-mean and unit variance complex Gaussian random variables, which is shown by $h_{mk} \sim CN(0,1)$. We analyse the uplink performance of the system. The ϕ_k is the pilot vector with length τ and power ρ_p. We consider pilot sequences are orthogonal to each other which means τ must be equal or greater than K. Here, we assume τ is equal to K. The pilot received by mth AP is $y_{p,m}$ which is as follows

$$y_{p,m} = \sum_{k=1}^{K} g_{mk}\sqrt{\tau\rho_p}\phi_k + n_m. \tag{2}$$

s_k is the signal with power ρ_u which is sent by kth user. The factor η_k is the power coefficient which is assigned to user k. The received signal is as follows

$$y_m = \sum_{k=1}^{K} g_{mk}\sqrt{\eta_k\rho_u}s_k + n_m. \tag{3}$$

For assigning users to the APs, we go for an heuristic approach. we consider that each AP serves a few number of users with better channel gains and there is no user without connection. To define whether kth user is connected to mth AP, we use the following parameter,

$$x_{mk} = \begin{cases} 1 & \text{if } k\text{th user is connected to } m\text{th AP}, \\ 0 & \text{else.} \end{cases} \tag{4}$$

In the following, we write down our constraints about the number of users connected to each AP which is denoted by NCUE and every user having at least one connection with one of the APs.

$$\sum_{k=1}^{K} x_{mk} = \text{NCUE},\ x_{mk} \in \{0,1\},\ \sum_{m=1}^{M} x_{mk} \geq 1, \\ \forall m \in \{1,2,\dots,M\},\ \forall k \in \{1,2,\dots,K\} \tag{5}$$

III. Performance Analysis

In this section, we study the performance of the two strategies used for sending signals from APs to CPU with considering user allocation. In CFE strategy, both the received data and channel state information are quantized at the AP and sent to the CPU through the fronthaul link with limited capacity; the quantized data are combined at CPU based on x_{mk}. In ECF strategy, quantized signals of the users which are not connected to the AP are not sent to the CPU, but similar to CFE, the quantized signals are combined at CPU based on x_{mk}. In what follows, we explicate the performance of these systems.

A. Compress-Forward-Estimate (CFE) Strategy

In CFE strategy, first, each AP quantizes the received signal, then sends it to CPU via fronthaul link. $\hat{y}_{p,m}$ is the quantized signal received which is as following

$$\hat{y}_{p,m} = y_{p,m} + q_{p,m} \tag{6}$$

with using MMSE as estimator at CPU, the estimated channel is

$$\tilde{g}_{mk} = \frac{E\{\bar{y}_{p,mk} g^*_{mk}\}}{E\{|\bar{y}_{p,mk}|^2\}}\bar{y}_{p,mk} =: \lambda_{mk}\bar{y}_{p,mk}, \tag{7}$$

whence,

$$\bar{y}_{p,mk} = \phi_k^H \hat{y}_{p,m} = \sqrt{\tau\rho_p} g_{mk} + \phi_k^H n_m + \phi_k^H q_{p,m}, \tag{8}$$

and

$$\lambda_{mk} = \frac{\sqrt{\tau\rho_p}\beta_{mk}}{\tau\rho_p\beta_{mk} + N + \frac{1}{\tau}\sum_{k'}^{K} Q_{p,mk'}}. \tag{9}$$

The variance of the estimated channel is $\gamma_{mk} := E\{|\tilde{g}_{mk}|^2\} = \sqrt{\tau\rho_p}\beta_{mk}\lambda_{mk}$.

1) Achievable Rate: CPU detects the effective symbol of kth user by using MRC as combiner and use-and-then-forget (UatF) technique as following

$$\begin{aligned} r_k = \sum_{m=1}^{M} x_{mk}\hat{y}_m\tilde{g}^*_{mk} = & \underbrace{E\{\sum_{m=1}^{M} x_{mk}\sqrt{\rho_u\eta_k} g_{mk}\tilde{g}^*_{mk}\}\, s_k}_{DS_k} \\ & + \underbrace{\sqrt{\rho_u\eta_k}\{\sum_{m=1}^{M} x_{mk}[g_{mk}\tilde{g}^*_{mk} - E\{g_{mk}\tilde{g}^*_{mk}\}]\}\, s_k}_{BU_k} \\ & + \sum_{k' \neq k}^{K} \underbrace{\sqrt{\rho_u\eta_{k'}}\{\sum_{m=1}^{M} x_{mk} g_{mk'}\tilde{g}^*_{mk}\}\, s_{k'}}_{IUI_{kk'}} \\ & + \underbrace{\sum_{m=1}^{M} x_{mk} n_m\tilde{g}^*_{mk}}_{RN_k} + \underbrace{\sum_{m=1}^{M} x_{mk} q_{d,m}\tilde{g}^*_{mk}}_{QN_k} \end{aligned} \tag{10}$$

- DS is the desired part of the received signal.
- BU comes from Beamforming Uncertainty.
- IUI stands for inter user interference.
- QN and RN are quantization noise and channel noise, respectively.

Theorem 1: The achievable rate for kth user is

$$R_k = \frac{T-\tau}{T}\log_2(1 + \frac{\rho_u\eta_k(\sum_{m=1}^{M} x_{mk}\gamma_{mk})^2}{\rho_u\sum_{k'=1}^{K}\eta_{k'}[\sum_{m=1}^{M} x_{mk}\gamma_{mk}\beta_{mk'}] + \sum_{m=1}^{M} x_{mk}(N+Q_{d,m})\gamma_{mk}}) \tag{11}$$

Note: for the sake of space limitation, the proof of the theorem is not presented here.

2) Fronthaul Capacity: We can obtain the quantization noise based on C_m.

- For computing $C_{p,m}$, we have:

$$C_{p,m} = \frac{1}{T} I(y_{p,m};\hat{y}_{p,m}) \leq \frac{K}{T}\log[1 + \frac{\rho_p\sum_{k=1}^{K}\beta_{mk} + N}{Q_{p,m}}], \tag{12}$$

  where $Q_{p,m} = Q_{p,m1} = \dots = Q_{p,mK}$.
- For computing $C_{d,m}$, we have:

$$C_{d,m} = \frac{T-\tau}{T} I(y_m;\hat{y}_m) \leq \frac{T-\tau}{T}\log[1 + \frac{\rho_u\sum_{k=1}^{K}\eta_k\beta_{mk} + N}{Q_{d,m}}]. \tag{13}$$

So, by having $C_{p,m} + C_{d,m} = C_m$, we can calculate the quantization noises, $Q_{p,m}$ and $Q_{d,m}$.

B. Estimate-Compress-Forward (ECF) Strategy

In ECF strategy, first, APs estimate the channels, then after quantizing the channel gains, they are sent to CPU through fronthaul link. The estimated channel is as following

$$\tilde{g}_{mk} = \frac{E\{\bar{y}_{p,mk} g^*_{mk}\}}{E\{|\bar{y}_{p,mk}|^2\}} \bar{y}_{p,mk} =: \lambda_{mk} \bar{y}_{p,mk}, \tag{14}$$

where

$$\bar{y}_{p,mk} = \phi_k^H y_{p,m} = \sqrt{\tau \rho_p} g_{mk} + \phi_k^H n_m, \tag{15}$$

and

$$\lambda_{mk} = \frac{\sqrt{\tau \rho_p} \beta_{mk}}{\tau \rho_p \beta_{mk} + N}. \tag{16}$$

Variance of the estimated channel is $\gamma_{mk} := E\{|\tilde{g}_{mk}|^2\} = \sqrt{\tau \rho_p} \beta_{mk} \lambda_{mk}$. The vector of quantized channel gains received at CPU is $\hat{g}_m$ which is as following:

$$\hat{g}_m = [x_{m1}\hat{g}_{m1}, x_{m2}\hat{g}_{m2}, \ldots, x_{mK}\hat{g}_{mK}] \tag{17}$$

and

$$\tilde{g}_{mk} = \hat{g}_{mk} + q_{p,mk}, \forall\{m,k\} \tag{18}$$

where $q_{p,mk}$ is the quantization noise, and $\hat{\gamma}_{mk} := E\{|\hat{g}_{mk}|^2\} = E\{\hat{g}^*_{mk} g_{mk}\} = E\{|\tilde{g}_{mk}|^2\} - Q_{p,mk}$ is variance of the quantized channel gain.

1) Achievable Rate: CPU by using MRC as combiner and UatF technique, detects the effective symbol of kth user as following

$$\begin{aligned} r_k = & \sum_{m=1}^{M} x_{mk} \hat{y}_m \hat{g}^*_{mk} = \underbrace{E\{\sum_{m=1}^{M} x_{mk} \sqrt{\rho_u \eta_k} g_{mk} \hat{g}^*_{mk}\} s_k}_{DS_k} \\ & + \underbrace{\sqrt{\rho_u \eta_k} \{\sum_{m=1}^{M} x_{mk} [g_{mk}\hat{g}^*_{mk} - E\{g_{mk}\hat{g}^*_{mk}\}]\} s_k}_{BU_k} \\ & + \sum_{k' \neq k}^{K} \underbrace{\sqrt{\rho_u \eta_{k'}} \{\sum_{m=1}^{M} x_{mk} g_{mk'} \hat{g}^*_{mk}\} s_{k'}}_{IUI_{kk'}} \\ & + \underbrace{\sum_{m=1}^{M} x_{mk} n_m \hat{g}^*_{mk}}_{RN_k} + \underbrace{\sum_{m=1}^{M} x_{mk} q_{d,m} \hat{g}^*_{mk}}_{QN_k}, \end{aligned} \tag{19}$$

where BU, DS, IUI, RN, and QN are as defined in the previous section.

Theorem 2: The achievable rate for kth user is

$$R_k = \frac{T-\tau}{T} \log_2\Big(1 + \frac{\rho_u \eta_k (\sum_{m=1}^{M} x_{mk}\hat{\gamma}_{mk})^2}{\rho_u \sum_{k'=1}^{K} \eta_{k'} [\sum_{m=1}^{M} x_{mk}\hat{\gamma}_{mk}\beta_{mk'}] + \sum_{m=1}^{M} x_{mk}(N + Q_{d,m})\hat{\gamma}_{mk}}\Big) \tag{20}$$

Note: for the sake of space limitation, the proof of the theorem is not presented here.

2) Fronthaul Capacity: For computing $C_{p,m}$ capacity, we have

$$\begin{aligned} C_{p,m} = & \frac{1}{T} I(\tilde{g}_m; \hat{g}_m) = \frac{1}{T}[h(\tilde{g}_m) - h(\tilde{g}_m|\hat{g}_m)] \leq \\ & \frac{1}{T} \log[\frac{|E\{\tilde{g}_m \tilde{g}_m^H|\}}{|E\{\tilde{q}_{p,m}\tilde{q}_{p,m}^H\}|}] = \frac{1}{T} \sum_{k=1}^{K} x_{mk} \log[\frac{\gamma_{mk}}{Q_{p,mk}}], \end{aligned} \tag{21}$$

where, $\hat{g}_m := [x_{m1}\hat{g}_{m1}, x_{m2}\hat{g}_{m2}, \ldots, x_{mK}\hat{g}_{mK}]$ and $\tilde{g}_m := [x_{m1}\tilde{g}_{m1}, x_{m2}\tilde{g}_{m2}, \ldots, x_{mK}\tilde{g}_{mK}]$. As shown in [12], selecting $Q_{p,mk}$ as following can greatly improve the performance of the system.

$$\begin{aligned} & x_{mk} \log[\frac{\gamma_{mk}}{Q_{mk}}] = \frac{\gamma_{mk}}{\sum_{k=1}^{K} x_{mk}\gamma_{mk}} T C_{p,m} \rightarrow \\ & Q_{p,mk} = x_{mk}\gamma_{mk} 2^{-\frac{\gamma_{mk}}{\sum_{k=1}^{K} x_{mk}\gamma_{mk}} T C_{p,m}}. \end{aligned} \tag{22}$$

Optimal $C_{p,m}$ and $C_{d,m}$ can be obtained by an one dimension exhaustive search on $C_{p,m} + C_{d,m} = C_m$.

IV. Power Allocation for CFE and ECF Strategies

In this section, we consider two optimization problems, sum-rate and minimum rate maximization of the system; here, we have three optimization parameters, power, users, fronthaul capacity for data and pilot. Considering the complexity of the problem, we cannot optimize them jointly. Therefore, we solve the user allocation problem with the heuristic approach as described in previous sections, and fronthaul capacity optimization for sending data and pilot, C_d and C_p, with the exhaustive approach. Power allocation algorithms are described as following:

A. Sum-SE Maximization of the System

We focus on sum-rate maximization problem by controlling the transmitting power of the users. Basically, this approach improves the system performance with reducing the inter user interference. The optimization problem is as following,

$$\mathcal{P}_1 : \begin{cases} \underset{\{\eta_k \geq 0\}}{\text{maximize}} & \frac{T-\tau}{T} \sum_{k=1}^{K} \log(1 + \text{SINR}_k) \\ \text{subject to} & \\ \eta_k \leq 1, & k = 1,2,...,K, \\ \sum_{k=1}^{K} x_{mk} = \text{NCUE} & \forall m \in \{1,2,\ldots,M\}, k=1,2,...,K, \\ \sum_{m=1}^{M} x_{mk} \geq 1 & \forall k \in \{1,2,\ldots,K\}, m=1,2,...,M. \end{cases} \tag{23}$$

where SINR_k is the signal to noise ratio of user$k$th for each ECF and CFE approach. We can approximate the problem $\mathcal{P}_1$ as problem $\mathcal{P}_2$,

Theorem 3: $\mathcal{P}_1$ can be rewritten as linear programming problem as following

$$\mathcal{P}_2:\begin{cases}\underset{\{\eta_k\geq 0,\ t_k\geq 0\}}{\text{maximize}} & \prod_{k=1}^{K} t_k\\ \text{subject to} & \\ \frac{t_k}{A_k\eta_k}\sum_{k'=1}^{K}\eta_{k'}B_{kk'}+\frac{L_k}{A_k\eta_k}\leq 1, & k=1,2,...,K,\\ \eta_k\leq 1, & k=1,2,...,K,\\ \sum_{k=1}^{K}x_{mk}=\text{NCUE} & \forall m\in\{1,2,\ldots,M\},k=1,2,...,K,\\ \sum_{m=1}^{M}x_{mk}\geq 1 & \forall k\in\{1,2,\ldots,K\},m=1,2,...,M.\end{cases} \tag{24}$$

where,

$$\begin{aligned}&\Gamma_{kk'}=(\sum_{m=1}^{M}\gamma_{mk}\frac{\beta_{mk}}{\beta_{mk'}}),\quad \Omega_{kk'}=\sum_{m=1}^{M}\gamma_{mk}\beta_{mk'},\\ &A_k=\rho_k\Gamma_{kk},\quad \mathcal{S}_{kk'}=\rho_u\sum_{m=1}^{M}\Big[2^{\frac{T}{T-\tau}C_{d,m}}-1\Big]^{-1}\gamma_{mk}\beta_{mk'},\\ &B_{kk'}=\rho_u\Omega_{kk'}+\mathcal{S}_{kk'},\\ &L_k=\sum_{m=1}^{M}\bigg(1+\Big[2^{\frac{T}{T-\tau}C_{d,m}}-1\Big]^{-1}\bigg)N\gamma_{mk}.\end{aligned} \tag{25}$$

Proof: By high signal to noise ratio approximation, we can write the lower band of the objective function of $\mathcal{P}_1$ as $\frac{T-\tau}{T}\log\Big[\prod_{k=1}^{K}\text{SINR}_k\Big]$ since $\log(x)$ is an increasing function of x and $\frac{T-\tau}{T}$ is a constant parameter. So, the objective function can be rewritten as $\prod_{k=1}^{K}\text{SINR}_k$, by using t_k as an auxiliary parameter where $\text{SINR}_k\geq t_k$, we can obtain the approximate of the $\mathcal{P}_1$ as $\mathcal{P}_3$,

$$\mathcal{P}_3:\begin{cases}\underset{\{\eta_k\geq 0,\ t_k\geq 0\}}{\text{maximize}} & \prod_{k=1}^{K} t_k\\ \text{subject to} & \\ t_k\leq \text{SINR}_k, & k=1,2,...,K,\\ \eta_k\leq 1, & k=1,2,...,K,\\ \sum_{k=1}^{K}x_{mk}=\text{NCUE} & \forall m\in\{1,2,\ldots,M\},k=1,2,...,K,\\ \sum_{m=1}^{M}x_{mk}\geq 1 & \forall k\in\{1,2,\ldots,K\},m=1,2,...,M.\end{cases} \tag{26}$$

where $SINR_k$ for CFE is as following

$$\text{SINR}_k^{CFE}=\frac{A_k\eta_k}{\sum_{k'=1}^{K}\eta_{k'}B_{kk'}+L_k}, \tag{27}$$

where A_k, $B_{kk'}$, L_k are given at (25). After some mathematical computation, we can achieve problem $\mathcal{P}_2$ which is a linear programming problem and can be solved by tools such ad MOSEK and CVX. It is worth noting that we can employ similar problem for CFE. ■

B. Minimum-SE Maximization of the System

We focus on minimum-rate maximization problem by controlling the transmitting power of the users. This approach improves the fairness of the system performance. The optimization problem is as following,

$$\mathcal{P}_1:\begin{cases}\underset{\{\eta_k\geq 0\}}{\text{maximize}} & \underset{k}{\text{minimum}}\ \frac{T-\tau}{T}\log(1+\text{SINR}_k)\\ \text{subject to} & \\ \eta_k\leq 1, & k=1,2,...,K,\\ \sum_{k=1}^{K}x_{mk}=\text{NCUE} & \forall m\in\{1,2,\ldots,M\},k=1,2,...,K,\\ \sum_{m=1}^{M}x_{mk}\geq 1 & \forall k\in\{1,2,\ldots,K\},m=1,2,...,M.\end{cases} \tag{28}$$

where SINR_k is the signal to noise ratio of user kth for each ECF and CFE approach. For solving this problem, we turn it into a convex problem. We can approximate the problem$\mathcal{P}_1$ as problem $\mathcal{P}_2$, as following

Theorem 4: $\mathcal{P}_1$ can be rewritten as linear programming problem as following

$$\mathcal{P}_2:\begin{cases}\underset{\{\eta_k\geq 0,\ t\geq 0\}}{\text{maximize}} & t\\ \text{subject to} & \\ \frac{t}{A_k\eta_k}\sum_{k'=1}^{K}\eta_{k'}B_{kk'}+\frac{L_k}{A_k\eta_k}\leq 1,\ k=1,2,...,K, & \\ \eta_k\leq 1, & k=1,2,...,K,\\ \sum_{k=1}^{K}x_{mk}=\text{NCUE} & \forall m\in\{1,2,\ldots,M\},k=1,2,\ldots,K,\\ \sum_{m=1}^{M}x_{mk}\geq 1 & \forall k\in\{1,2,\ldots,K\},m=1,2,...,M.\end{cases} \tag{29}$$

where,

$$\begin{aligned}&\Gamma_{kk'}=(\sum_{m=1}^{M}\gamma_{mk}\frac{\beta_{mk}}{\beta_{mk'}}),\quad \Omega_{kk'}=\sum_{m=1}^{M}\gamma_{mk}\beta_{mk'},\\ &A_k=\rho_k\Gamma_{kk},\quad \mathcal{S}_{kk'}=\rho_u\sum_{m=1}^{M}\Big[2^{\frac{T}{T-\tau}C_{d,m}}-1\Big]^{-1}\gamma_{mk}\beta_{mk'},\\ &B_{kk'}=\rho_u\Omega_{kk'}+\mathcal{S}_{kk'},\\ &L_k=\sum_{m=1}^{M}\bigg(1+\Big[2^{\frac{T}{T-\tau}C_{d,m}}-1\Big]^{-1}\bigg)N\gamma_{mk}.\end{aligned} \tag{30}$$

Proof: By considering lower band of SINR_k and using t as an auxiliary parameter where $\text{SINR}_k\geq t$, we can obtain the approximate of the $\mathcal{P}_1$ by the epigraph equivalent $\mathcal{P}_3$,

$$\mathcal{P}_3:\begin{cases}\underset{\{\eta_k\geq 0,\ t\geq 0\}}{\text{maximize}} & t\\ \text{subject to} & \\ t\leq \text{SINR}_k, & k=1,2,...,K,\\ \eta_k\leq 1, & k=1,2,...,K,\\ \sum_{k=1}^{K} x_{mk}=\text{NCUE} & \forall m\in\{1,2,\dots,M\}, k=1,2,...,K,\\ \sum_{m=1}^{M} x_{mk}\geq 1 & \forall k\in\{1,2,\dots,K\}, m=1,2,...,M.\end{cases} \tag{31}$$

where,

$$\text{SINR}_k^{CFE}=\frac{A_k\eta_k}{\sum_{k'=1}^{K}\eta_{k'}B_{kk'}+L_k}, \tag{32}$$

where A_k, $B_{kk'}$, L_k are given at (30). After some mathematical computation, we can achieve problem $\mathcal{P}_2$ which is a linear programming problem and can be solved by tools such ad MOSEK and CVX. It is worth noting that we can employ similar problem for CFE. ■

V. Numerical Result

We consider a square area with the length of each side is 1 km where M number of APs are randomly and uniformly distributed in the area, and K number of users are distributed based on Poisson Point Process. In order to lessen the effect of the boundary, we used wrap-around technique. Regarding to this technique, 8 copies of the area are considered around the first area. We define the pass-loss between kth user and mth AP as PL_{mk} which is as following

$$\text{PL}_{mk}=\begin{cases}-L\text{–}35\log_{10}(d_{mk}) & if\ d_{mk}\geq d_0,\\ -L\text{–}15\log_{10}(d_1)\text{–}20\log_{10}(d_{mk}) & if\ d_0\geq d_{mk}\geq d_1,\\ -L\text{–}15\log_{10}(d_1)\text{–}20\log_{10}(d_0) & if\ d_{mk}\geq d_1,\end{cases} \tag{33}$$

where,

$$L\triangleq 46.3+33.9\log_{10}(f)\text{–}13.82\log_{10}(h_{AP})\text{–}(1.1\log_{10}(f)\text{–}0.7)h_u + (1.56\log_{10}(f)-0.8),$$

In the above equation, $d_0=10m, d_1=50m$, and f is the frequency in MHz. We assume it is 1.9 MHz. h_{AP} and h_u are the heights of the access point and users in meter which are 1.5 and 1.65, respectively. Large scale fading according to path-loss, shadow fading is modeled as following

$$\beta_{mk}=10^{-\frac{\text{PL}_{mk}}{10}}\times 10^{\frac{\sigma_{as}z_{mk}}{10}}, \tag{34}$$

In the above equation, PL_{mk} is the path-loss and $10^{\frac{\sigma_{sh}z_{mk}}{10}}$ is the shadowing effect where σ_{sh} is standard deviation and $z_{mk}\sim N(0,1)$. N is the noise power as following

$$N=B\times k_B\times T_0\times NF.$$

NF is the noise figure and equals to 9dB, and B is bandwidth equals to 20MHz. T_0 is temperate of the noise equals to 290 Kelvin degree, and k_B is the Boltzmann constant. We assume that coherence interval is 200 samples corresponding to coherence bandwidth of $20KHz$ and coherence time of $1ms$. Data and pilot power, ρ_u and ρ_p, are $10mw$. We consider pilot sequence equal or greater than number of users. The total number of the users and APs are 40 and 200, respectively.

We analyse the performance of the proposed algorithm for the sum-rate maximization and minimum rate maximization. For this purpose, we assume that $C=1[bits/s/Hz]$, and number of users and APs are 20 and 200, respectively. We bring the results for two different cases when the number of the connected users to each AP is 2 and 6 for each algorithm. It is shown in Figs. 1 and 2 that for the sum-SE maximization, the performance of the proposed algorithm improves for both ECF and CFE cases. In Figs. 3 and 4, we can see that for the minimum-SE maximization, the performance of the proposed algorithm improves for both ECF, but for the CFE case, the improvement is not significant. It can be interpreted by the fact that CFE uses the fronthaul capacity for forwarding the whole signals. Therefore, as what we have in rate distortion theorem, power allocation does not have great impact on the capacity of the fronthaul, and so on the performance of the system.

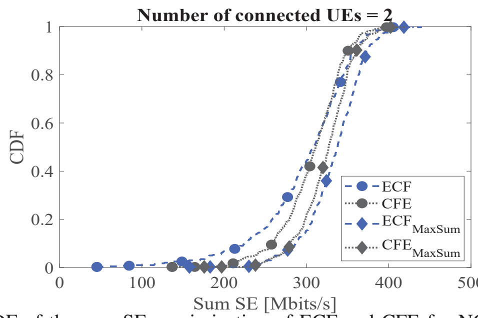

Fig. 1 CDF of the sum-SE maximization of ECF and CFE for NCUE=2

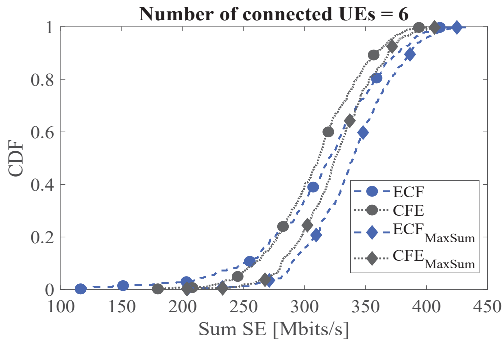

Fig. 2 CDF of the sum-SE maximization of ECF and CFE for NCUE=6

VI. Conclusion

We studied two power allocation problems in user-centric cell-free massive MIMO system with limited backhaul capacity which is close to the practical scenario. The objectives of these problems were sum-SE and minimum SE

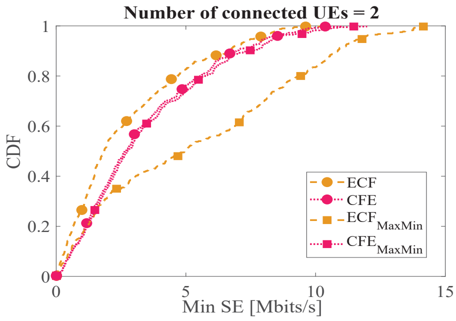

Fig. 3 CDF of the Minimum SE maximization of ECF and CFE for NCUE=2

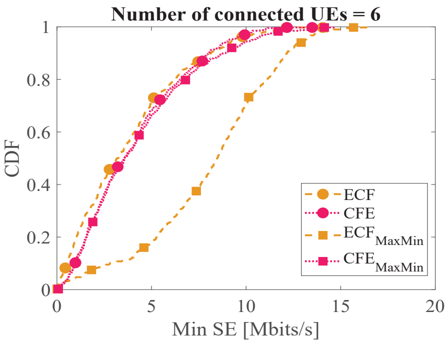

Fig. 4 CDF of the Minimum SE maximization of ECF and CFE for NCUE=6

maximization objectives for CFE and ECF strategies. We solved these problems by changing them to convex problem and specifically linear programming problem. It is shown that the proposed algorithm works and improves the performance of the system.

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