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Promoting Creative and Critical Thinking in Mathematics: An Exploratory Study

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Abstract**—**The Japanese art of origami provides a rich context for designing exploratory mathematical activities for children and young people. By folding a simple sheet of paper, fascinating and surprising planar and spatial configurations emerge. Equally surprising is the unfolding process, which also produces striking patterns. The procedure of folding, unfolding, and folding again allows the exploration of interesting geometric patterns. When adequately and systematically done, we may deduce some of the mathematical rules ruling origami. As the child/youth folds the sheet of paper repeatedly, he can physically observe how the forms he obtains are transformed and how they relate to the pattern of the corresponding unfolding, creating space for the understanding/discovery of mathematical principles regulating the folding-unfolding process. As part of a 2023 Summer Academy organized by a Portuguese university, a session entitled "Folding, Thinking and Generalizing" took place. 23 students attended the session, all enrolled in the 2nd cycle of Portuguese Basic Education and aged between 10 and 12 years old. The main focus of this session was to foster the development of critical cognitive and socio-emotional skills among these young learners, using origami. These skills included creativity, critical analysis, mathematical reasoning, collaboration, and communication. Employing a qualitative, descriptive, and interpretative analysis of data, collected during the session through field notes and students' written productions, our findings reveal that structured origami-based activities not only promote student engagement with mathematical concepts in a playful and interactive but also facilitate the development of socio-emotional skills, which include collaboration and effective communication between participants. This research highlights the value of integrating origami into educational practices, highlighting its role in supporting comprehensive cognitive and emotional learning experiences.

Keywords**—**Active learning, hands-on activities, origami, creativity, critical thinking.

I. Introduction

LEARNING mathematics, whether in structured or informal environments, contributes significantly to solid knowledge acquisition and cognitive and socio-emotional skills development [1]. However, enthusiasm and appreciation for mathematics are most effectively awakened, nurtured, and promoted in informal contexts. In informal settings, learners typically have more control over their learning process. The self-regulating decision-making in selecting subjects, methodology, and timing of learning can considerably enhance motivation and engagement, [2], [3]. There is a growing consensus among educators, teachers, and researchers in educational sciences, that the involvement of children, young people, and even adults in practical experiences of making and tinkering contributes as a means of facilitating learning by fostering curiosity, creativity, and a deeper understanding of how things work [4]-[6]. As also reported by Davis et al. [7], the idea of "learning by doing" has gained significant credit in the field of education in general – and in particular, in the field of mathematics education, we would say – leading to the establishment of numerous dedicated makerspaces within educational institutions. This approach provides students with opportunities for practical and autonomous learning experiences, moving away from traditional teacher-student interaction.

In many official and recommended documents from different countries, including those from the Common Core State Standards and the National Council of Teachers of Mathematics (NCTM), it is worth highlighting the significant emphasis they place on the development of conceptual understanding in mathematics. Origami is a powerful context for this purpose as well as for developing critical and creative thinking, [8], [9].

The history of origami, the traditional art of paper folding, describing its evolution over time, is fascinating. It is not known exactly where it all started. As mentioned by Meloni et al. [10], some researchers associate its origin with the discovery of paper, others are convinced that it appeared in China, others take it for granted that it was in Japan and there are still those who claim that it is part of the European tradition. In short, its origin remains covered in mystery. However, what is known is that the use of the word origami to describe paper folding emerged, in the 20th century, in Japan.

Origami allows the transformation of abstract mathematical concepts into concrete visual representations. Through paper folding processes, various geometric shapes can be created, having in mind different learning objectives, aiming to understand specified mathematical principles such as symmetry, proportions, and spatial relationships, [11], [12].

According to Rachmawati & Kurniati, as cited by Wati, [13], the characteristics of creativity in children include a wide range of skills and attributes. These include, among others, the capacity to think critically, adapt flexibly to diversified situations, and generate innovative solutions. Creative children often focus on the minutest details, and demonstrate the ability to create new constructions or compositions by building upon existing knowledge or other available data.

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Origami, with its folding rules and techniques to model complex 3D shapes, provides a dynamic platform for enhancing children's creative thinking skills; it challenges children to analyze patterns, follow instructions, and make strategic decisions during the folding process. This engagement together with well-structured problem-solving tasks fosters critical thinking skills as they may have to transform intentionally their construction according to the nature of the problem they have to cope with. Origami often requires adaptability, especially when a folding model does not go as planned or when trying to create alternative forms, starting from a known folding model. This flexibility encourages children to explore multiple approaches to a problem and adapt their techniques accordingly, promoting flexible thinking. It also encourages children to explore new possibilities. The experiments with different folding lines, combinations, and modifications of existing models, foster their capacity for innovative thinking. The precision required in origami models demands careful attention to every construction step. Children practicing origami learn to focus their attention on the properties of the folding process, refining their observational skills and patience. Origami often begins with simple basic folds that can be transformed into quite complex creations. It serves as an engaging and hands-on educational tool that not only develops creativity but also does so in a fun and interactive manner. It stands as a promising resource to promote and develop the attributes of creativity in young minds, equipping them with helpful problem-solving and creative thinking skills for the future.

II. Method

This work was developed within the scope of the 2023 Summer Academy, hosted by a Portuguese university, to provide students in the 2nd and 3rd cycles of the Portuguese Basic Education with an opportunity to immerse themselves in a university-life atmosphere and participate in several scientific, cultural, athletic and recreational activities. The first phase of our work consisted of designing a workshop specifically targeted at children between 10 and 12 years old, who would be attending, in two months, the 5th and 6th year of schooling, in line with the general objectives of the Summer Academy, where mathematizable concepts, introduced in an enjoyable, relaxed, and non-explicit way, would be explored. Following this principle, we conceived a task set, presented in a two-column format, enriched with figures and tables.

The proposed workshop, entitled, *Folding, Thinking, and Generalizing*, which we will detail later, was implemented in a non-formal context, and evaluated, aiming to understand its contribution to the triggering/development of cognitive and socio-emotional skills, in line with the following research guiding questions (RQ):

RQ1. What methods or approaches were used in the "Folding, Thinking, and Generalizing" session to increase creativity, critical thinking, mathematical logic, teamwork, and communication among the students involved?

RQ2. How did the origami session contribute to the development of critical cognitive and socio-emotional skills of the participating students?

The design and implementation of the workshop proposal were guided by the lesson study's phases [14]: (1) *study*; (2) *plan*; (3) *implementation* (teaching); (4) *reflection*.

In the initial phase, themes were established to be considered in the workshop proposal to involve mathematical thinking in a creative and entertaining context, in which socio-emotional skills were also stimulated. In the planning phase, learning objectives were established, resources were created and strategies and methodologies were defined to address the selected themes, implement, and discuss the tasks. During the implementation phase, the impact of educators' themes, resources, methodology, and conversations on student performance, as well as collaboration and communication between students, was observed and analyzed. In the last phase, we reflected on the contribution of the workshop to the triggering/development of cognitive and socio-emotional skills of the participating students.

20 students participated in the workshop, 16 from the 6th year of schooling and four from the 5th year. The workshop was facilitated by two researchers and supported by a team of five university students, three of them in initial teacher training, specifically studying for a degree in Basic Education, and the other two holding a starting research grant of the GEOMETRIX thematic line. During its implementation, data were collected through direct observation by the support team and researchers, through photographic records and student productions.

III. Results

In this section, we will present the results relating to the different phases that cover the conception, implementation, and subsequent reflection on the proposed workshop.

A. Study

The Summer Academy is an event aiming to provide students with a variety of experiences integrating science, culture, and sport, among others, in a captivating and interesting environment. Bearing this premise in mind and the objective of conducting an engaging workshop centered on Mathematics, which aligns with our training, for a group of children, from which it was only known, the age range and the schooling years they attended, our initial focus, in this phase, was the careful selection of topics/themes. In this context, origami has emerged as an exceptionally promising choice. Through paper folding, it is possible not only to create replicas of various elements surrounding us but also to develop abstract thoughts, and creative and fun generalizations. In turn, the unfolding process can also reveal surprising patterns. In fact, in addition to origami being a pleasurable activity, from which it is possible to explore mathematical ideas, it is an activity that arouses children's interest and promotes socio-emotional skills, namely encouraging the sharing of ideas, promoting communication, and helps to alleviate stress and anxiety [15], [16].

Origami, as an educational tool, offers a multifaceted approach to the development of crucial cognitive skills, particularly spatial thinking, [15]. This cognitive ability plays a crucial role in exploring a wide range of geometric elements and concepts, including straight lines, rays, angles, areas, polygons, polyhedra, and reflection and rotational symmetries. Furthermore, it serves as a valuable tool for modeling complex shapes, ranging from solid geometric figures to complex fractals [17]. According to the Mathematics Curricular Guidelines of the Portuguese education system [18], students in the 2nd cycle of Basic Education (5th and 6th years) cover, in the Geometry domain, the geometric concepts and elements mentioned above, and in the domain of Algebra, the identification of regularities and patterns. Therefore, when designing the tasks, we took these mathematical contents into account, in addition to the principles, areas of competence, and values outlined in the student profile at the end of compulsory schooling [19], and also in characteristics of teaching in non-formal contexts, [2].

B. Plan

The workshop was carefully structured to include essential components, starting with introductory and contextualization segments on relevant topics and tasks. Specific periods were also assigned for students to actively engage in problem-solving activities. In addition, extensive group discussions were scheduled to deepen and explore the ideas that emerged from the proposed tasks. The workshop plan assumed the structure in Table I, which will be described in more detail later.

TABLE I
WORKSHOP'S PLAN

Moments	Description
Introduction/contextualization	Introduction to origami characteristics and types of folds (researcher)
*TASK 1- **Simple folds – What I observe!***	Simple folds and pattern identification (students)
Large group discussion	Large group discussion of Task 1 emerging ideas (researchers and students)
*TASK 2 – **From a flexahedron to a parallelepiped***	Construction of polyhedra and analysis of their characteristics (students)
Large group discussion	Large group discussion of Task 2 resulting ideas (researchers and students)
*Task 3 - **Finger traps***	Playful moment - Finger traps construction (students)

Throughout the workshop, students are organized into five groups, each group with four children. To increase efficiency and speed up the monitoring of students in carrying out the tasks assigned to them, a support team is created made up of five members, each responsible for monitoring a group of students.

During the workshop, researchers take on the responsibility of contextualizing topics/themes and tasks, promoting discussions in large and small groups, clarifying doubts, and closely observing students' progress. Next, we present and characterize the different moments outlined in the workshop plan (as described in Table I), specifying the resources, methodology, and learning objectives associated with each one of them.

C. Introduction and Contextualization

Initially, our main objective is to give students an understanding of origami, present the tasks they will be working on, and clarify the dynamics of the session. To achieve this, we employ a structured PowerPoint presentation to serve as an informative tool to elucidate students on the basics of origami and folding techniques. This moment is triggered by one of the researchers, in conversation with the large group.

Task 1- Simple Folds: What I Observe!

The initial task is designed to engage students in the art of paper folding while fostering their ability to recognize emerging patterns upon unfolding. Here are the steps for this task:

1. Begin by folding an A4 sheet of paper in half along its length, and then promptly unfold it. What do you observe?
2. With a pencil, pen, or marker, trace the visible marks that become apparent as the paper is unfolded.
3. Perform several consecutive folds on a separate sheet of paper, and then unfold it. Take note of the distinctive pattern that materializes upon the paper's unfolding. What aspects of the pattern capture your attention? Write a short text about what you observed.
4. Imagine that you continued to make additional folds. What do you predict would occur when you eventually unfold the paper? Share your thoughts on the anticipated outcome.

This task aims to stimulate critical and mathematical thinking through pattern recognition, namely in understanding that the number of straight-line segments (unfolding lines) concurrent at the same point is even, and trigger a deeper appreciation of the complexities of paper folding.

From a mathematical point of view, this task serves multiple learning objectives: it encourages students to perceive fundamental concepts related to straight lines and their relative position; it fosters the capacity to identify and discriminate recurring patterns and regularities, and it aims to develop students' ability to formulate informed conjectures based on the observed regularities. During the execution of this task, the support team's intervention is intentionally kept to a minimum to avoid undue influence on the students' responses. Researchers monitoring the students' progress, whenever necessary, will thoughtfully ask questions, ensuring that such inquiries do not affect their work.

Task 2- From a Flexahedron to a Parallelepiped

In this task, we introduce the construction of a flexahedron - a non-convex polyhedron with the remarkable ability to move through itself "infinitely" (as illustrated in Fig. 1). To facilitate the construction, each group receives guidance from a support team member, who gradually presents a tutorial video accessible on the YouTube platform. The video, hosted by Jeremy Shafer, serves as a precious resource for mastering the flexahedron's intricate folds, and it can be accessed via [20].

When needed, support team members were available to assist students with the origami construction of the flexahedron. It is important to note that a flexahedron can be manipulated so that it can be transformed into a parallelepiped, and this transformation can be repeated "infinitely" by specific movements.

Fig. 1 A flexahedron

In this task, the following subtasks and questions were presented to the students:

1. Begin by constructing the flexahedron.
2. When you manipulate the flexahedron, what do you observe? Take note of your observations.
3. When manipulating the flexahedron, do the resulting geometric solids resemble any you are familiar with? Are there any noticeable differences? If so, please describe these differences.

Subsequently, students are encouraged to transform the flexahedron into a parallelepiped and answer the following question:

4. Suppose you were asked to decorate both the flexahedron and the parallelepiped with colored sticker paper, covering the entire visible surface. Would the amount of sticker paper needed to cover each of these solids be the same? Please describe your reasoning.

Regarding question 2, the main objective is to encourage students to verify that, when manipulating the flexahedron, they consistently obtain the same solids. Given the emphasis on convex polyhedra in the educational curriculum at these levels, in question 3 we look for evidence of students' ability to identify differences between the flexahedron and the well-known polyhedral shapes they have encountered, along with their ability to identify any other attributes to distinguish them.

Question 4, focused on the visible surfaces of the flexahedron during manipulation, intends to find out whether students conclude that the amount of adhesive paper needed to cover completely the flexahedron and the parallelepiped is the same.

As the flexahedron and the parallelepiped represent distinct three-dimensional configurations, our objective is to evaluate whether students formulate their answers based simply on visual assessments or if they apply strategies to determine, quantitatively, the surface area of each geometric solid. This task has the following learning objectives: develop spatial thinking; distinguish convex polyhedra from non-convex polyhedra; compare and relate areas of polygons and measure areas using non-standard units of measurement. During the task, the member of the group support team, in addition to showing the tutorial video on the construction of the flexahedron, assists students in their origami constructions and clarifies any doubts that may arise related to the interpretation of the questions asked. The two researchers will circulate through the different groups, observing the students' performance, clarifying possible doubts, or asking additional questions to sharpen their sense of observation.

Task 3- Finger Traps

In the final phase of the session, we introduce the construction of finger traps, as illustrated in Fig. 2, to conclude the workshop with a playful and engaging moment. During this finger trap construction activity, we employ an instructional video accessible via the YouTube platform [21].

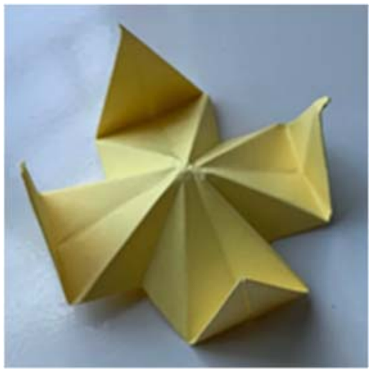

Fig. 2 Finger traps

As in Task 2, the video is presented to each group by a member of the support team, who will assist students during construction whenever necessary.

Large Discussion Group

Large group discussions follow after Tasks 1 and 2, respectively, and serve as opportunities for sharing ideas related to the questions posed. These discussions are facilitated by one of the researchers to actively listen to students' arguments, stimulating the emergence of ideas, and challenge students with thought-provoking questions.

Cognitive, Social, and Emotional Skills

Throughout the workshop presentation, learning objectives related to the domain of mathematics were presented. However, it is essential to highlight that this workshop aims to develop skills in students, not only in the cognitive domain but also in the socio-emotional domain. In pursuit of these objectives, the proposals, resources, and methodology that support this action plan were selected, to prioritize the development of key attributes, including responsibility and integrity, emphasizing self-respect and respect for others' well-being; curiosity, reflection, and innovation through reflective, critical and creative thinking; aesthetic and artistic sensitivity, [19].

D.Implementation

In this section, we describe the implementation of the workshop, aiming to give an overall picture of the moments and opportunities that allowed us to see how topics, resources, teaching methods, and interactions among students, and between students and the support team, including the two researchers, influenced the development of students' critical cognitive and socio-emotional skills.

The workshop took place in a room with round tables. As soon as the students arrived in the room, they were divided into small groups of four members, with each group seated at one of the tables. A member of the support team was assigned to oversee and facilitate the group's work at each table. Interestingly, despite the students coming from different classes or schools, the process of group formation proved to be straightforward and self-directed, as the students themselves formed the groups without any intervention from the researchers or support team.

In the initial phase (Fig. 3) of the workshop, where we introduced the workshop theme and explained the different types and basic folding techniques, we had the opportunity to assess the familiarity of each student with the art of origami and check their enthusiasm for the upcoming activities. It became clear that all students had prior experience with origami creations, and their interactions with the researcher passed on an amazing eagerness and enthusiasm to engage in the folding tasks.

Fig. 3 Workshop’s introduction

Task 1, although not directly associated with construction but focused on exploring folds and patterns, captivated the students' attention, leading them to engage actively in responding to the posed questions (Fig. 4). During the subsequent discussion of this task in the large group, the students revealed a high level of participation and enthusiasm in sharing their thoughts. Among the various drawn conclusions, their observation of the regularity pattern in the parity of the number of segment straight lines (unfolding marks) intersecting at the same point (vertex) was an impressive detection.

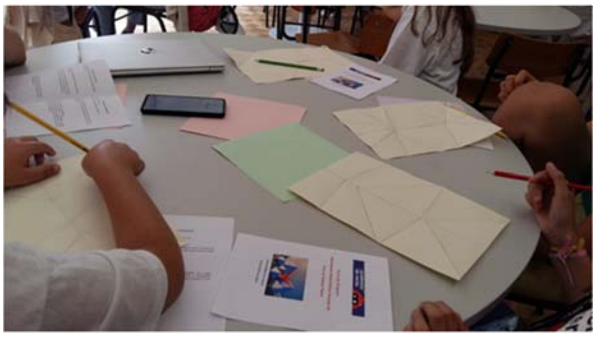

Fig. 4 Students solving Task 1

Task 2, involving the construction of the flexahedron, proved to be demanding due to its elaborate execution (Fig. 5). In addition to following the provided tutorial, some students required guidance from the respective support group team member.

Fig. 5 Students solving Task 2

During the construction phase, a point worth highlighting was the mutual assistance observed within the (small) groups, where students who completed their constructions willingly helped those facing difficulties (Fig. 6).

Fig. 6 Peer support

The difficulties experienced by some students during the flexahedron’s construction were mainly due to issues related to visual-motor coordination, manual dexterity, and some lack of patience and perseverance.

The solution to question 4 involved transforming the flexahedron into a parallelepiped, followed by an examination of the polygons marked on its surfaces and the establishment of relationships between their respective areas (Fig. 7).

Fig. 7 Students comparing the surface areas of the solids

After completing this task, some students took the opportunity to show their creativity through, for instance, compositions of the group constructions, as illustrated in Fig.8.

Fig. 8 An aesthetic composition created by a student

During the discussion of Task 2, within the large group, the concepts of convex and non-convex solids came to the forefront. As the conversation turned towards comparing the surface areas of the flexahedron and the parallelepiped, diverse perspectives among the students emerged. Some students, relying on their visual observations and the striking dissimilarity between the two solids, concluded that the surfaces must indeed be different. On the other hand, some argued that since the parallelepiped was a transformation of the flexahedron, they should share the same surface area. Interestingly, another group of students went beyond mere intuition; they managed to establish connections between the areas of the polygons marked on the respective surfaces, offering evidence to support their argument that the two solids possessed equivalent surface areas.

As we reached this stage of the workshop, it was apparent that some students were struggling with fatigue and agitation, which may have hindered their willingness or ability to put forth the effort required to fully justify their answers regarding the comparison of the surface areas of the discussed solids.

In the final task, involving the construction of finger traps, students showed great enthusiasm and mutual support, transcending any signs of fatigue they had previously shown. They eagerly engaged with one another, sparking spontaneous games and activities that incorporated their newly created constructions.

IV. Reflection

Using origami to improve mathematical reasoning and social-emotional skills is aligned with the best contemporary pedagogical practices [22]. The plan, method, and implementation outlined for the "Folding, Thinking and Generalizing" workshop, described in the previous sections, effectively fulfill the objectives of combining practical skills with theoretical knowledge, illustrating a well-thought-out and effective educational strategy for the use of origami, and where the balance between students' pre-existing knowledge and the promotion of new learning was taken into consideration. The learning objectives to be achieved in each of the workshop tasks and the types of data to be collected for analysis were strategically outlined. The team proactively engaged in task experimentation to anticipate students' thinking, ensuring a well-informed teaching approach based on an understanding of students' perspectives and learning needs, and improving the overall effectiveness of this non-formal educational experience.

A simulation of this experience should have also been carried out in a classroom environment. This would allow the research and support team to identify possible challenges, such as more specific questions aimed at the outlined learning objectives and time management in execution for the different moments of the work session.

The development of the workshop plan substantially enriched the understanding of team members, as a whole, not only on the topic involved but, above all, on dynamic teaching methodologies in a non-formal context. This experience provided valuable insights into ways to increase student engagement and innovative integration of mathematical concepts with origami.

The collaborative planning effort deepened the understanding of learning processes through hands-on activities. For planning future workshops in the spirit of summer academies, incorporating broader, interdisciplinary perspectives and collaborative brainstorming sessions could increase opportunities for new and innovative teaching methodologies.

V. Conclusions

The research work developed in the "Folding, Thinking and Generalizing" workshop aimed to explore the impact of an origami learning session, in a non-formal context, on cognitive and socio-emotional skills in the participant students, youths aged between 10 and 12 years old.

Concerning the guiding research questions RQ1 (In the "Fold, Think and Generalize" workshop, what methods or approaches were used to increase creativity, critical thinking, mathematical logic, teamwork, and communication among participating students?) and RQ2 (How did the origami session contribute to the development of the cognitive and socio-emotional skills of the students involved?), we direct the reader to Sections III and IV of this article. In these sections, we describe how the proposed tasks and applied methodologies were used as tools to promote mathematical, critical, and creative thinking and improve socio-emotional skills in work groups. We present evidence indicative of how the practical and creative aspect of origami, combined with structured tasks and group discussions, not only reinforced existing mathematical knowledge but also created space for the acquisition of new knowledge. Moreover, this approach supported the development of socio-emotional skills, promoting qualities such as collaboration, communication, and emotional well-being among the participating students.

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