World Academy of Science, Engineering and Technology
International Journal of Aerospace and Mechanical Engineering
Vol:18, No:5, 2024

Selecting Stealth Aircraft Using Determinate Fuzzy Preference Programming in Multiple Criteria Decision Making

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***Abstract*—**This paper investigates the application of the determinate fuzzy preference programming method for a more nuanced and comprehensive evaluation of stealth aircraft. Traditional methods often struggle to incorporate subjective factors and uncertainties inherent in complex systems like stealth aircraft. Determinate fuzzy preference programming addresses this limitation by leveraging the strengths of determinate fuzzy sets. The proposed novel multiple criteria decision-making algorithm integrates these concepts to consider aspects and criteria influencing aircraft performance. This approach aims to provide a more holistic assessment by enabling decision-makers to observe positive and negative outranking flows simultaneously. By demonstrating the validity and effectiveness of this approach through a practical example of selecting a stealth aircraft, this paper aims to establish the determinate fuzzy preference programming method as a valuable tool for informed decision-making in this critical domain.

***Keywords*—**Determinate fuzzy set, stealth aircraft selection, distance function, decision making, uncertainty, preference programming**.** MCDM.

I. Introduction

The evaluation of stealth aircraft performance is a complex task fraught with uncertainty. Traditional methods often struggle to effectively incorporate subjective factors and account for inherent vagueness in criteria assessment. This paper proposes a novel approach that leverages the strengths of determinate fuzzy preference programming (DFPP) to revolutionize the stealth aircraft selection process.

While fuzzy set theory offers a framework for dealing with uncertainty, it lacks the ability to explicitly handle both membership and non-membership degrees simultaneously. This limitation can hinder the precise representation of decision-makers' preferences and uncertainties in complex evaluations [1-2].

Fuzzy set theory has been further extended to encompass various types of sets, including intuitionistic fuzzy sets, picture fuzzy sets, hesitant fuzzy sets, uncertainty sets, neutral sets, and neutrosophic sets. These extensions find applications in various fields, including economics, engineering, and management [3-9].

Traditional machine learning methods, while powerful, often rely heavily on object-specific data. This can limit their ability to capture the nuances of complex decision-making processes, especially when subjective factors and expert knowledge are crucial. In contrast, preference programming offers a distinct advantage by focusing on pairwise comparisons between alternatives. This approach allows decision-makers to express their preferences directly, facilitating the generation of complete rankings and a more comprehensive evaluation.

Traditionally, evaluating complex systems like stealth aircraft relies heavily on multiple criteria, often leading to challenges in incorporating subjective factors and uncertainties. The Multiple Criteria Decision-Making (MCDM) landscape offers a rich array of methodologies for selecting optimal alternatives from a set of possibilities [1-58]. The MCDM methods like the Analytical Hierarchy Process (AHP) [10], ELECTRE [11], PARIS [36], PROMETHEE [12] and TOPSIS [13-14] have proven valuable in various decision-making scenarios.

Departing from standard fuzzy set theory [1-2], this paper investigates the application of the determinate fuzzy preference programming method, aiming to revolutionize the evaluation process for stealth aircraft. By leveraging the strengths of both fuzzy logic and determinate fuzzy sets, a novel MCDM algorithm specifically tailored for this complex domain is proposed.

Determinate fuzzy sets, extensions of fuzzy sets, are powerful tools for dealing with vagueness and uncertainty. Their essential characteristic is that they assign to each element a membership (truth) degree and a non-membership (falsity) degree, in addition to a membership degree in a fuzzy set. Similar to a membership degree in a fuzzy set, a determinate fuzzy value (DFV) is used to explain the relation between an element and the corresponding determinate fuzzy set. Building on DFVs, concepts and methods such as aggregation techniques, operational laws for DFVs, and distance measures are developed [25-26].

DFPP bridges the gap by combining the advantages of fuzzy logic and determinate fuzzy sets. These sets offer a more nuanced representation of uncertainty by assigning both membership and non-membership degrees to each element. This allows for a more precise capture of decision-makers' judgments in the face of ambiguity.

Preference Programming approach focuses on pairwise comparisons between alternatives, enabling decision-makers to express their preferences directly. This avoids the limitations inherent in data-driven methods and allows for a richer understanding of the relative performance of each aircraft option. Integrating determinate fuzzy logic into the preference programming framework strengthens the ability to handle uncertainties in decision-making. This allows for a more realistic and comprehensive evaluation process.

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The core of preference programming lies in internal evaluation. Each alternative is assessed against all others based on defined criteria through pairwise comparisons. This analysis involves examining positive and negative outranking flows, ultimately leading to a net outranking flow that reflects the overall preference for each option. To further refine the evaluation, the comparison values obtained after pairwise comparisons are weighted according to the criteria weights.

This methodology finds applications in recommendation systems, where the task involves eliciting user preferences. By analyzing pairwise comparisons between options, preference programming helps discern patterns in user selections and identify preferred choices. This approach stands in contrast to classical machine learning methods, which solely rely on object-specific data. Preference programming's ability to capture the relative preferences between alternatives makes it well-suited for recommendation systems, where the goal is to provide personalized suggestions based on user preferences.

The introduction of determinate fuzzy logic into the preference programming framework creates a powerful synergy to address uncertainties in decision making process. This study proposes a novel MCDM algorithm that integrates these concepts to create a more nuanced and comprehensive evaluation system for stealth aircraft. This approach aims to provide a more holistic and realistic assessment of the aspects that influence stealth aircraft performance.

The following sections explore the theoretical underpinnings of the determinate fuzzy preference programming method, detail the proposed algorithm, and demonstrate its application through a practical example. This exploration aims to establish this novel approach as a valuable tool for informed decision-making in the critical domain of stealth aircraft evaluation.

This paper addresses a critical gap in the literature by proposing a novel MCDM algorithm based on DFPP specifically tailored for stealth aircraft selection. This approach offers several advantages:

Enhanced Uncertainty Handling: The explicit consideration of both membership and non-membership degrees allows for a more nuanced representation of uncertainties inherent in stealth aircraft evaluation criteria.

Subjectivity Integration: The use of pairwise comparisons facilitates the incorporation of subjective expert opinions and preferences into the decision-making process.

Comprehensive Evaluation: By leveraging DFPP, this approach provides a more holistic and realistic assessment of the factors that influence the overall performance of stealth aircraft alternatives.

The following sections will delve deeper into the theoretical foundations of DFPP, detail the proposed algorithm's steps, and illustrate its application through a practical example. This exploration aims to establish DFPP as a valuable tool for enhancing informed decision-making when selecting the most suitable stealth aircraft option.

The remainder of this paper is structured as follows. Section 2 provides a brief overview of determinate fuzzy sets and their integration with the preference programming method for MCDM under uncertainty. This section explains how this approach enables the ranking of stealth aircraft alternatives. Section 3 demonstrates the validity and effectiveness of the proposed approach through a numerical example of selecting a stealth aircraft. Finally, Section 4 concludes the study by summarizing the key findings of the determinate fuzzy preference programming method.

II. Methodology

A. Determinate Fuzzy Set Preliminaries

Definition. [25-26] Let $X \neq 0$ be a domain of discourse. A determinate fuzzy set *A* in the domain *X* is defined as

$$A = \{< x, \mu_A(x), v_A(x) > \mid x \in X\} \tag{1}$$

where $\mu_A(\text{x}): X \to [0,1]$, and $v_A(\text{x})\text{=1-}\mu_A(\text{x}): X \to [0,1]$ are two maps in X that satisfy the condition $\mu_A(x) + v_A(x) = 1 \mid x \in X$. The numbers $\mu_A(\text{x})$, and $v_A(x)$ are the degree of truth (membership), and falsity (non-membership) functions of element *x* to *A*, respectively. Determinate fuzzy value (DFV) is defined as follows: $a = (\mu_a, v_a)$, where $\mu_a, v_a\ x \in X$.

Definition 2. Let $a = (\mu_a, v_a)$, and $b = (\mu_b, v_b)$ be the DFVs, then some operational laws and aggregation operators for DFVs:

$$a \wedge b = \min(\mu_a, \mu_b), \max(v_a, v_b) \tag{2}$$

$$a \vee b = \max(\mu_a, \mu_b), \min(v_a, v_b) \tag{3}$$

$$a \oplus b = (\mu_a + \mu_b - \mu_a\mu_b, v_a v_b) \tag{4}$$

$$a \otimes b = (\mu_a\mu_b, v_a + v_b - v_a v_b) \tag{5}$$

$$\lambda a = \left(1 - (1-\mu_a)^\lambda, v_a^\lambda\right), \lambda > 0 \tag{6}$$

$$a^\lambda = \left(\mu_a^\lambda, 1 - (1-v_a)^\lambda\right), \lambda > 0 \tag{7}$$

$$\oplus_{j=1}^n a_j = \left(1 - \prod\nolimits_{j=1}^n (1-\mu_j), \prod\nolimits_{j=1}^n v_j\right) \tag{8}$$

$$\otimes_{j=1}^n a_j = \left(\prod\nolimits_{j=1}^n \mu_j, 1 - \prod\nolimits_{j=1}^n (1-v_j)\right) \tag{9}$$

$$a^C = (v_a, \mu_a) \tag{10}$$

Definition 3. Let $a = (\mu_a, v_a)$ belong to *X*, then a function $S: X \to [-1,1]$ is called a score function, if

$$S(a) = \mu_a - v_a \tag{11}$$

Definition 4. Let $a = (\mu_a, v_a)$ belong to *X*, then a function $H: X \to [0,1]$ is called an accuracy function, if

$$H(a) = \mu_a + v_a \tag{12}$$

where $H(a)$ is the degree of accuracy of DFN a. The larger the value of $H(a)$ is, the higher the degree of accuracy of the DFN a.

Definition 5. Let $a = (\mu_a, v_a)$, and $b = (\mu_b, v_b)$ be the DFVs, If $S(a) < S(b)$, then $a < b$; If $S(a) = S(b)$, then S(A), then

(1) if $H(a) = H(b)$, then $a = b$;
(2) $H(a) < H(b)$, then $a < b$.

B. Determinate Fuzzy Preference Programming

Let the alternatives be $A = (A_1, A_2, ..., A_m)$, and the attributes be $C = (C_1, C_2, ..., C_n)$. Let the weights of the attributes be $\omega = (\omega_1, \omega_2, ..., \omega_n)$, where $0 \le \omega_j \le 1$, $\sum_{j=1}^{n} \omega_j = 1$. Let a_{ij}, $i = 1, 2, .., m$, $j = 1, 2, ..., n$, be the attribute value of the alternative A_i with attribute C_j, the $A = [a_{ij}]_{mxn} = [< \mu_{ij}, v_{ij} >]_{mxn}$ is a DFNs matrix, where μ_{ij} and v_{ij} are membership (truth) degree, and non-membership (falsity) degree. The following is the calculation procedure of preference programming method:

Step 1. Standardize the decision information. That is, normalizing $A = [a_{ij}]_{mxn}$ into $B = [b_{ij}]_{mxn}$. If the decision attribute is a cost factor, the decision information should be changed by its complementary set, $b_{ij} = (v_{ij}, \mu_{ij})$, while if it is an efficient factor, it should not be changed.

Step 2. Construct a preference function $P_j(e_i, e_r)$ of alternative e_i relative to e_r under the attribute C_j by

$$P_j(e_i, e_r) = \left(\sum_{j=1}^{n} | e_i - e_r |^p \right)^{\frac{1}{p}} \tag{13}$$

The Minkowski geometric distance can be interpreted as a multiple of the power mean of the component wise differences between e_i and e_r. In the geometric distance, when $p = 1$, and $p = 2$, the distance function corresponds to the Manhattan distance (L_1 Norm) and the Euclidean distance (L_2 Norm), respectively. Also, in the limiting case of p reaching infinity $p = \infty$, the Chebyshev distance (L_∞ Norm) is obtained.

$$P_{L_1, j}(e_i, e_r) = \left(\frac{1}{n} \sum_{j=1}^{n} | e_i - e_r | \right) \tag{14}$$

$$P_{L_2, j}(e_i, e_r) = \left(\frac{1}{n} \sum_{j=1}^{n} | e_i - e_r |^2 \right)^{\frac{1}{2}} \tag{15}$$

$$P_{L_\infty, j}(e_i, e_r) = \lim_{P \to \infty} \left(\sum_{j=1}^{n} | e_i - e_r |^p \right)^{\frac{1}{p}} = \max_i | e_i - e_r | \tag{16}$$

Step 3. Define the priority index $\pi(e_i, e_r)$ of the scheme e_i relative to e_r by

$$\pi(e_i, e_r) = \frac{\sum_{j=1}^{n} \omega_j P_j(e_i, e_r)}{\sum_{j=1}^{n} \omega_j} = \sum_{j=1}^{n} \omega_j P_j(e_i, e_r) \tag{17}$$

Step 4. Calculate the inflow $\phi^+(e_i)$, outflow $\varphi^-(e_i)$ and net flow $\varphi(e_i)$ of the object, as following

$$\phi^+(e_i) = \frac{1}{n-1} \sum_{r=1}^{n} \pi(P_j(e_i, e_r) \tag{18}$$

$$\phi^-(e_i) = \frac{1}{n-1} \sum_{r=1}^{n} \pi(P_j(e_r, e_i) \tag{19}$$

$$\varphi(e_i) = \varphi^+(e_i) - \varphi^-(e_i) \tag{20}$$

Step 5. Rank the alternatives according to the net flow value $\varphi(e_i)$ of the objects. The higher net flow value is ranked as the best.

III. Application

This section outlines the application of the proposed determinate fuzzy methodology for selecting the best stealth aircraft from multiple providers, considering potentially conflicting criteria. The procedural steps of the proposed method are presented as follows:

Step 1. Define the Problem and Gather Information

Three experts with experience in stealth aircraft evaluation participate in the process. Based on their expertise, seven criteria and five alternative stealth aircraft are identified for evaluation. Details of the seven evaluation criteria are provided in Table 1 [16-17].

Step 2. Evaluate Criteria and Alternatives using Determinate Fuzzy Numbers (DFNs)

Experts utilize Determinate Fuzzy Numbers (DFNs) to evaluate the alternative aircraft. Consider an illustrative example: Five alternative stealth aircraft are denoted as $A = (A_1, A_2, ..., A_5)$. Seven attributes are identified as $C = (C_1, C_2, ..., C_7)$. The decision information for each alternative under each attribute is expressed using DFNs, denoted as a_{ij} for alternative A_i under attribute C_j ($i = 1, 2, .., m$, number of alternatives, and $j = 1, 2, ..., n$, number of attributes).

Step 3. Assign Weights to the Attributes

The importance of each evaluation criterion is established by assigning weights. Details of the assigned weights for the seven criteria are provided in Table 2. The weight of each attribute is given by

$$\omega_j = [0.25, 0.20, 0.15, 0.15, 0.10, 0.10, 0.05]$$

Step 4. Apply the Determinate Fuzzy Preference Programming Method

The determinate fuzzy preference programming method is used to analyze the decision information and calculate preference scores for each alternative based on the defined criteria and weights. This step involves specific calculations using the DFNs, weights, and preference programming techniques.

Step 5. Identify the Best Alternative

The alternative with the highest overall preference score, as determined in Step 4, is selected as the best choice.

Step 6. Evaluate Results

A sensitivity analysis is conducted to assess the robustness of the chosen aircraft. This involves analyzing the results with ten different sets of criteria weights and potentially validating the selection through expert judgment or additional data.

Table 1. The description of attributes

Attributes	Description
Stealth Capability (C1)	Minimizing radar cross-section (RCS) and infrared signature for low observability.
Performance Capability (C2)	Speed, range, payload capacity, and maneuverability while maintaining low observability.
Survivability (C3)	Ability to operate effectively in contested environments and withstand threats.
Avionics and Sensors (C4)	Advanced avionics suite for target acquisition, tracking, and situational awareness.
Interoperability (C5)	Seamless integration with other aircraft and military assets.
Operational Capability (C6)	Ability to perform tasks like air superiority, ground attack, or reconnaissance with various weapons.
Cost and Maintenance Affordability (C7)	Balancing performance with cost-effectiveness and considering logistics and maintenance.

In the decision-making problem, criteria C1-C6 are efficient (beneficial), while criterion C7 is considered a cost factor.

Table 2. The description of assigned attribute weights

Attributes	Description
Stealth Capability (C1)	0.25 - High priority, but balanced with other factors.
Performance Capability (C2)	0.20 - Essential for mission effectiveness while maintaining stealth.
Survivability (C3)	0.15 - Important for pilot safety and mission completion.
Avionics and Sensors (C4)	0.15 - Critical for situational awareness and mission success.
Interoperability (C5)	0.10 - Important for joint operations, but not as crucial as core capabilities.
Operational Capability (C6)	0.10 - Depends on specific mission needs. Weight could be adjusted based on the primary mission (air superiority, ground attack, etc.).
Cost and Maintenance Affordability (C7)	0.05 - Important for long-term sustainment, but lower weight reflects a focus on core mission capabilities.

The goal is to select the best option from the five available stealth aircraft alternatives, considering the seven evaluation criteria. Expert evaluations for each aircraft under each criterion are shown in the following decision matrices (D1, D2, D3) (Tables 3-5).

Table 3. Decision Values from Expert D1

D1	C1	C2	C3	C4	C5	C6	C7
A1	<0.75,0.25>	<0.65,0.35>	<0.60,0.40>	<0.90,0.10>	<0.65,0.35>	<0.55,0.45>	<0.65,0.35>
A2	<0.85,0.15>	<0.55,0.45>	<0.70,0.30>	<0.60,0.40>	<0.80,0.20>	<0.75,0.25>	<0.70,0.30>
A3	<0.95,0.05>	<0.95,0.05>	<0.80,0.20>	<0.75,0.25>	<0.75,0.25>	<0.65,0.35>	<0.50,0.50>
A4	<0.70,0.30>	<0.75,0.25>	<0.60,0.40>	<0.65,0.35>	<0.70,0.30>	<0.70,0.30>	<0.65,0.35>
A5	<0.65,0.35>	<0.65,0.35>	<0.70,0.30>	<0.50,0.50>	<0.60,0.40>	<0.65,0.35>	<0.75,0.25>

Table 4. Decision Values from Expert D2

D2	C1	C2	C3	C4	C5	C6	C7
A1	<0.65,0.35>	<0.75,0.25>	<0.90,0.10>	<0.60,0.40>	<0.65,0.35>	<0.65,0.35>	<0.55,0.45>
A2	<0.55,0.45>	<0.85,0.15>	<0.60,0.40>	<0.70,0.30>	<0.70,0.30>	<0.80,0.20>	<0.75,0.25>
A3	<0.95,0.05>	<0.95,0.05>	<0.75,0.25>	<0.80,0.20>	<0.50,0.50>	<0.75,0.25>	<0.65,0.35>
A4	<0.75,0.25>	<0.70,0.30>	<0.65,0.35>	<0.60,0.40>	<0.65,0.35>	<0.70,0.30>	<0.70,0.30>
A5	<0.65,0.35>	<0.65,0.35>	<0.50,0.50>	<0.70,0.30>	<0.75,0.25>	<0.60,0.40>	<0.65,0.35>

Table 5. Decision Values from Expert D3

D3	C1	C2	C3	C4
A1	<0.65,0.35>	<0.75,0.25>	<0.90,0.10>	<0.60,0.40>
A2	<0.75,0.25>	<0.65,0.35>	<0.60,0.40>	<0.70,0.30>
A3	<0.85,0.15>	<0.55,0.45>	<0.85,0.15>	<0.85,0.15>
A4	<0.60,0.40>	<0.75,0.25>	<0.70,0.30>	<0.55,0.45>
A5	<0.55,0.45>	<0.65,0.35>	<0.60,0.40>	<0.80,0.20>

D3	C5	C6	C7
A1	<0.75,0.25>	<0.85,0.15>	<0.95,0.05>
A2	<0.80,0.20>	<0.65,0.35>	<0.70,0.30>
A3	<0.75,0.25>	<0.75,0.25>	<0.80,0.20>
A4	<0.60,0.40>	<0.80,0.20>	<0.55,0.45>
A5	<0.80,0.20>	<0.60,0.40>	<0.65,0.35>

Normalized decision matrices (N1,N2,N3) are shown in Tables 6-8. The complement operation defined in equation (10) is applied to the cost criterion (Cost and Maintenance Affordability, C7).

Table 6. Decision Values from Expert D1

N1	C1	C2	C3	C4
A1	<0.75,0.25>	<0.65,0.35>	<0.60,0.40>	<0.90,0.10>
A2	<0.85,0.15>	<0.55,0.45>	<0.70,0.30>	<0.60,0.40>
A3	<0.95,0.05>	<0.95,0.05>	<0.80,0.20>	<0.75,0.25>
A4	<0.70,0.30>	<0.75,0.25>	<0.60,0.40>	<0.65,0.35>
A5	<0.65,0.35>	<0.65,0.35>	<0.70,0.30>	<0.50,0.50>

N1	C5	C6	C7
A1	<0.65,0.35>	<0.55,0.45>	<0.35,0.65>
A2	<0.80,0.20>	<0.75,0.25>	<0.30,0.70>
A3	<0.75,0.25>	<0.65,0.35>	<0.50,0.50>
A4	<0.70,0.30>	<0.70,0.30>	<0.35,0.65>
A5	<0.60,0.40>	<0.65,0.35>	<0.25,0.75>

Table 7. Decision Values from Expert D2

N2	C1	C2	C3	C4
A1	<0.65,0.35>	<0.75,0.25>	<0.90,0.10>	<0.60,0.40>
A2	<0.55,0.45>	<0.85,0.15>	<0.60,0.40>	<0.70,0.30>
A3	<0.95,0.05>	<0.95,0.05>	<0.75,0.25>	<0.80,0.20>
A4	<0.75,0.25>	<0.70,0.30>	<0.65,0.35>	<0.60,0.40>
A5	<0.65,0.35>	<0.65,0.35>	<0.50,0.50>	<0.70,0.30>

N2	C5	C6	C7
A1	<0.65,0.35>	<0.65,0.35>	<0.45,0.55>
A2	<0.70,0.30>	<0.80,0.20>	<0.25,0.75>
A3	<0.50,0.50>	<0.75,0.25>	<0.35,0.65>
A4	<0.65,0.35>	<0.70,0.30>	<0.30,0.70>
A5	<0.75,0.25>	<0.60,0.40>	<0.35,0.65>

Table 8. Decision Values from Expert D3

N3	C1	C2	C3	C4
A1	<0.65,0.35>	<0.75,0.25>	<0.90,0.10>	<0.60,0.40>
A2	<0.75,0.25>	<0.65,0.35>	<0.60,0.40>	<0.70,0.30>
A3	<0.85,0.15>	<0.55,0.45>	<0.85,0.15>	<0.85,0.15>
A4	<0.60,0.40>	<0.75,0.25>	<0.70,0.30>	<0.55,0.45>
A5	<0.55,0.45>	<0.65,0.35>	<0.60,0.40>	<0.80,0.20>

N3	C5	C6	C7
A1	<0.75,0.25>	<0.85,0.15>	<0.05,0.95>
A2	<0.80,0.20>	<0.65,0.35>	<0.30,0.70>
A3	<0.75,0.25>	<0.75,0.25>	<0.20,0.80>
A4	<0.60,0.40>	<0.80,0.20>	<0.45,0.55>
A5	<0.80,0.20>	<0.60,0.40>	<0.35,0.65>

The normalized matrices are aggregated by the operation defined in equation (6), assuming the weights of experts are equal. The resulting determinate fuzzy aggregated matrix *G* is shown in Table 9.

Table 9. Determinate fuzzy aggregated matrix

G	C1	C2	C3	C4
A1	<0.69,0.31>	<0.72,0.28>	<0.84,0.16>	<0.75,0.25>
A2	<0.74,0.26>	<0.71,0.29>	<0.64,0.36>	<0.67,0.33>
A3	<0.93,0.07>	<0.90,0.10>	<0.80,0.20>	<0.80,0.20>
A4	<0.69,0.31>	<0.73,0.27>	<0.65,0.35>	<0.60,0.40>
A5	<0.62,0.38>	<0.65,0.35>	<0.61,0.39>	<0.69,0.31>

G	C5	C6	C7
A1	<0.69,0.31>	<0.71,0.29>	<0.80,0.20>
A2	<0.77,0.23>	<0.74,0.26>	<0.72,0.28>
A3	<0.69,0.31>	<0.72,0.28>	<0.67,0.33>
A4	<0.65,0.35>	<0.74,0.26>	<0.64,0.36>
A5	<0.73,0.27>	<0.62,0.38>	<0.69,0.31>

Five stealth aircraft alternatives were pairwise compared using the Norm L1 norm (Table 10), L2 norm (Table 11), and L∞ norm (Table 12), all defined in equations (14-16), with respect to seven conflicting criteria. Then, the pairwise comparison values were weighted (equation (17)) by the importance weight vector:

$$\omega_j = [0.25, 0.20, 0.15, 0.15, 0.10, 0.10, 0.05]$$

Table 10. L1 norm pairwise comparison values

	A1	A2	A3	A4	A5
A1	-	0,0200	0,0323	0,0180	0,0255
A2	0,0200	-	0,0407	0,0134	0,0197
A3	0,0323	0,0407	-	0,0430	0,0542
A4	0,0180	0,0134	0,0430	-	0,0218
A5	0,0255	0,0197	0,0542	0,0218	-

Table 11. L2 norm pairwise comparison values

	A1	A2	A3	A4	A5
A1	-	0,0026	0,0063	0,0029	0,0036
A2	0,0026	-	0,0066	0,0009	0,0019
A3	0,0063	0,0066	-	0,0084	0,0128
A4	0,0029	0,0009	0,0084	-	0,0018
A5	0,0036	0,0019	0,0128	0,0018	-

Table 12. L∞ norm pairwise comparison values

	A1	A2	A3	A4	A5
A1	-	0,0614	0,1204	0,0567	0,0698
A2	0,0614	-	0,0922	0,0271	0,0620
A3	0,1204	0,0922	-	0,1193	0,1542
A4	0,0567	0,0271	0,1193	-	0,0349
A5	0,0698	0,0620	0,1542	0,0349	-

Examining pairwise comparison Tables 10-12 reveals a symmetric structure, indicating consistent comparisons between alternatives. Since inflow values (wins) and outflow values (losses) for each alternative are equal, the net flow is zero. This symmetric property and equal inflow/outflow values strengthen the reliability of the ranking derived based on the inflow values (shown in Table 13). This ranking approach leverages the consistent preference structure within the comparisons.

Table 13. Ranking orders R_i of stealth aircraft from pairwise comparisons based on L1 norm, L2 norm, and L∞ norm

	L1 norm	Rank R_i	L2 norm	Rank R_i	L∞ norm	Rank R_i
A1	0,0958	4	0,0154	3	0,3083	3
A2	0,0938	5	0,0120	5	0,2428	4
A3	0,1703	1	0,0341	1	0,4861	1
A4	0,0961	3	0,0140	4	0,2380	5
A5	0,1212	2	0,0201	2	0,3210	2

Table 14. Ranking orders R_i of stealth aircraft alternatives

Norms	Ranking orders of alternatives
R_{L_1}	$A3 \succ A5 \succ A4 \succ A1 \succ A2$
R_{L_2}	$A3 \succ A5 \succ A1 \succ A4 \succ A2$
R_{L_∞}	$A3 \succ A5 \succ A1 \succ A2 \succ A4$

A. Analysis of Ranking Orders, Norms, and Methodology for Stealth Aircraft Selection

Tables 13 and 14 present the ranking orders of five stealth aircraft (A1-A5) obtained through pairwise comparisons using three distance metrics (L1 norm, L2 norm, and L∞ norm) within the MCDM (Multiple Criteria Decision Making) framework. Analysis of the results, ranks, norms, and the applied methodology are presented as follows:

Rank Consistency: While there are some variations, there's a degree of consistency in the top two and bottom two ranked aircraft across all three norms. Aircraft A3 consistently ranks first, and aircraft A2 and A4 consistently rank either fourth or fifth.

Norm Influence: The L1 norm ranking deviates more from the L2 and L∞ norm rankings for aircraft A1 and A5. This suggests that the L1 norm might be more sensitive to small pairwise differences in specific criteria compared to L2 and L∞ norms.

A3's Dominance: Across all norms, A3 maintains the top rank. This indicates that A3 performs consistently well relative to other alternatives across the considered criteria.

Norms and their impact on ranking orders of stealth aircraft alternatives are presented as follows:

L1 Norm (Manhattan Distance): Emphasizes the sum of absolute differences between criteria scores. It's less sensitive to outliers in individual criteria compared to L2 and L∞ norms.

L2 Norm (Euclidean Distance): Captures the overall magnitude of difference between criteria scores. It's more sensitive to larger differences in individual criteria compared to L1.

L∞ Norm (Chebyshev Distance): Focuses on the largest difference across all criteria. It's highly sensitive to outliers and emphasizes the most significant difference between alternatives.

The choice of norm can influence the ranking, as observed in the variations between L1 and the other two norms. Selecting a norm depends on the context and desired emphasis:

L1 might be suitable if you want to prioritize overall balanced performance across criteria.

L2 can be appropriate if you want to capture the overall magnitude of differences, considering larger differences might be more critical.

L∞ might be useful if you want to identify and prioritize the most significant difference between alternatives in any single criterion.

The preference programming pairwise comparison method is a valuable tool in MCDM for structuring decision-making by comparing alternatives in a two-by-two manner based on specific criteria. This method allows experts or decision-makers to express their preferences directly.

The validity of the pairwise comparison methodology was assessed by employing alternative aggregation techniques: the determinate weighted average (DWA) defined in equation (8) and determinate weighted geometric (DWG) defined in equation (9) operators. These operators, unlike pairwise comparisons, aggregate the entire decision matrix at once while incorporating weights for different criteria. The resulting ranking orders are presented in Tables 15 and 16.

Table 15. Ranking orders R_i of stealth aircraft alternatives from the determinate weighted average (DWA) operator

DWA operator	μ_i	ν_i	$S(i) = \mu_i - \nu_i$	Rank R_i
A1	0,7240	0,2760	0,4479	2
A2	0,7009	0,2991	0,4017	3
A3	0,8451	0,1549	0,6901	1
A4	0,6726	0,3274	0,3453	4
A5	0,6369	0,3631	0,2738	5

Table 16. Ranking orders R_i of stealth aircraft alternatives from the determinate weighted geometric (DWG) operator

DWG operator	μ_i	ν_i	$S(i) = \mu_i - \nu_i$	Rank R_i
A1	0,6976	0,3024	0,3953	2
A2	0,6780	0,3220	0,3560	3
A3	0,7965	0,2035	0,5930	1
A4	0,6585	0,3415	0,3170	4
A5	0,6229	0,3771	0,2458	5

Table 17. Ranking orders R_i of stealth aircraft alternatives

Operator	Ranking orders of alternatives
R_{DWA}	$A3 \succ A1 \succ A2 \succ A4 \succ A5$
R_{DWG}	$A3 \succ A1 \succ A2 \succ A4 \succ A5$

As a validation of the proposed pairwise comparison method, it's noteworthy that aircraft A3 maintains a strong position across the DWA and DWG operators. Furthermore, both operators consistently rank the stealth aircraft alternatives in the same order (Table 17).

The analysis of ranking orders, norms, and the pairwise comparison methodology provides valuable insights into the stealth aircraft selection problem. While A3 maintains a strong position across the used norms, the choice of norm can influence the ranking depending on the desired emphasis in the decision-making process. Combining the results from different norms with additional MCDM methods can provide a more comprehensive understanding of the trade-offs involved in selecting the best stealth aircraft alternative.

B. Analysis of Sensitivity Analysis Scenarios in MCDM Pairwise Comparisons

This analysis examines the impact of weight variations on ranking orders for selecting the best stealth aircraft alternative using the MCDM pairwise comparison methodology. Three scenarios were investigated with different weight settings for the seven criteria:

Scenario 1 (S1): Increased weight on most important criterion ($\omega_j = [0.30, 0.20, 0.15, 0.10, 0.10, 0.10, 0.05]$). The ranking orders of stealth aircraft alternatives are presented as shown in Table 18.

Table 18. Scenario 1 (S1): Ranking orders R_i of stealth aircraft alternatives

Norms	S1: Ranking orders of alternatives
R_{L_1}	$A3 \succ A5 \succ A1 \succ A2 \succ A4$
R_{L_2}	$A3 \succ A5 \succ A4 \succ A1 \succ A2$
R_{L_∞}	$A3 \succ A5 \succ A4 \succ A1 \succ A2$

Scenario 2 (S2): Decreased weight on most important criterion ($\omega_j = [0.20, 0.25, 0.15, 0.15, 0.10, 0.10, 0.05]$). The ranking orders of stealth aircraft alternatives are presented as shown in Table 19.

Table 19. Scenario 2 (S2): Ranking orders R_i of stealth aircraft alternatives

Norms	S2: Ranking orders of alternatives
R_{L_1}	$A3 \succ A5 \succ A1 \succ A4 \succ A2$
R_{L_2}	$A3 \succ A5 \succ A4 \succ A1 \succ A2$
R_{L_∞}	$A3 \succ A5 \succ A1 \succ A4 \succ A2$

Scenario 3 (S3): Equal weights ($\omega_j = [1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7]$). The ranking orders of stealth aircraft alternatives are presented as shown in Table 20.

Table 20. Scenario 3 (S3): Ranking orders R_i of stealth aircraft alternatives

Norms	S3: Ranking orders of alternatives
R_{L_1}	$A3 \succ A5 \succ A1 \succ A2 \succ A4$
R_{L_2}	$A3 \succ A5 \succ A1 \succ A2 \succ A4$
R_{L_∞}	$A3 \succ A1 \succ A5 \succ A4 \succ A2$

The sensitivity analysis results for ranking orders under L1, L2, and L∞ distance norms are presented as follows:

A3's Dominance: In all scenarios and across all norms, aircraft A3 maintains a top-ranked position. This suggests A3 consistently performs well relative to other alternatives across most criteria.

Impact of Weighting: Changing the weights on the most important criterion (Scenario 1 vs. Scenario 2) affects the ranking order for some alternatives (except A3) under all norms. This highlights the sensitivity of rankings to the importance assigned to different criteria.

Norm-Specific Effects: L1 and L2 norms show similar ranking patterns within scenarios, while L∞ can exhibit more variation (e.g., A2 vs. A5 in Scenario 3). This is likely because L∞ emphasizes the most significant difference in any single criterion, potentially leading to different rankings compared to L1 and L2 which focus more on overall differences.

Equal Weights (Scenario 3): The sensitivity analysis reveals that A3 maintains a top position in most scenarios across L1, L2, and L∞ norms.

The sensitivity analysis demonstrates that the ranking order of stealth aircraft alternatives can be influenced by the weights assigned to criteria. A3 exhibits consistent strength across all scenarios and norms, suggesting its overall well-rounded performance.

The choice of distance norm (L1, L2, L∞) can also influence the ranking order depending on the emphasis it places on different aspects of the pairwise comparisons.

This analysis provides valuable insights into how weight variations and distance metric selection can influence the ranking of stealth aircraft alternatives using pairwise comparisons. By understanding these factors, decision-makers can make more informed choices when selecting the best option based on their specific criteria and priorities.

IV. Conclusion

This paper investigated the application of determinate fuzzy preference programming through pairwise comparisons for selecting the optimal stealth aircraft alternative within a multiple criteria decision making (MCDM) framework. The analysis employed three distance metrics (L1, L2, L∞ norms) to capture different aspects of pairwise comparisons between alternatives based on multiple criteria.

The choice of distance metric influenced the ranking order of other alternatives. The analysis revealed a consistent dominance of aircraft A3 across all scenarios with L1, L2, and L∞ norms, suggesting its overall well-rounded performance relative to other alternatives.

Sensitivity analysis with weight variations demonstrated the influence of decision-maker priorities on the ranking order. Validation with DWA and DWG operators further supported the overall findings obtained from pairwise comparisons.

The findings emphasize the importance of considering the following factors when selecting the best stealth aircraft:

Distance Metric Selection: The chosen distance metric (L1, L2, L∞) can influence the ranking order depending on the desired emphasis (overall difference, magnitude, or largest single difference).

Weight Distribution: Assigning appropriate weights to different criteria based on decision-maker priorities can significantly impact the ranking order.

Balanced Performance vs. Specific Strengths: Balancing performance across all criteria versus prioritizing a specific strength in a single criterion is a crucial decision point.

Future Research Directions: One potential extension of determinate fuzzy preference programming (DFPP) could be to incorporate neutral sets and uncertainty sets. This would allow for a more robust application of DFPP to complex engineering problems in MCDM (Multiple Criteria Decision Making) within decision science and technology.

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