Passive and Active Spatial Pendulum Tuned Mass Damper with Two Tuning Frequencies

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***Abstract*—**The first bending modes of tall asymmetric structures in the two lateral X and Y-directions have two different natural frequencies. To add tuned damping to these bending modes, one needs to either a) use two pendulum-tuned mass dampers (PTMDs) with one tuning frequency, each PTMD targeting one of the bending modes, or b) use one PTMD with two tuning frequencies (one in each lateral directions). Option (a), being more massive, requiring more space, and being more expensive, is less attractive than option (b). Considering that the tuning frequency of a pendulum depends mainly on the pendulum length, one way of realizing option (b) is by constraining the swinging length of the pendulum in one direction but not in the other; such PTMD is dubbed passive Bi-PTMD. Alternatively, option (b) can be realized by actively setting the tuning frequencies of the PTMD in the two directions. In this work, accurate physical models of passive Bi-PTMD and active PTMD are developed and incorporated into the numerical model of a tall asymmetric structure. The model of PTMDs plus structure is used for a) synthesizing such PTMDs for particular applications and b) evaluating their damping effectiveness in mitigating the dynamic lateral responses of their target asymmetric structures, perturbed by wind load in X and Y-directions. Depending on how elaborate the control scheme is, the active PTMD can either be made to yield the same damping effectiveness as the passive Bi-PTMD of the same size or the passive Bi-TMD twice as massive as the active PTMD.

***Keywords*—**Active tuned mass damper, high-rise building, multi-frequency tuning, vibration control.

I. Introduction

When a tall structure blocks the wind, the air stream splits into two, passing along both sides and back of the structure. As the wind tries to circulate around the structure it separates from the structure while shedding vortices in a rather organized manner, creating a pressure pulsation on the structure in the direction perpendicular to the direction of the wind, commonly known as 'across-wind excitation'. The frequency of vortex shedding and thus the frequency of the force perturbing the structure, created by the above-mentioned pressure pulsation, depends on the velocity of the wind and the geometry of the structure [1]. This harmonic perturbation sets the structure in vibration with the same frequency as the vortex shedding frequency. Depending on the velocity and direction of wind, the vortex shedding frequency for a particular structure could match one of the lower natural frequencies of the structure resulting in resonant vibration of that structure.

PTMDs are commonly used for adding tuned damping to the lateral (bending) modes of vibration of tall structures with low natural frequencies. PTMDs are spatial (also known as spherical) pendulums with added viscous damping. The natural frequency of a pendulum, and thus the tuning frequency of a PTMD, is mainly dependent on its length. As such, a PTMD that swings with the same link length in all directions has the same tuning frequency in all directions and thus can only add tuned damping to one vibration mode of the structure it is appended to. The underlying dynamics of spatial (spherical) pendulums, with a fixed length, have been the subject of many studies including, but certainly not limited to, [2]-[4].

The paper presents how to model and examine the effectiveness of a passive pendulum tuned damping solution with more than one tuning frequency, in dampening lateral vibration of asymmetric tall structures. The first bending mode of such structures in one lateral direction and the 2nd bending mode in the other lateral direction, have two different natural frequencies. Adding tuned damping to such structures necessitates the use of either a) two PTMDs with different dynamic characteristics, each tuned to its corresponding target modes or b) one PTMD with two different tuning frequencies (in two orthogonal directions), i.e., a two degree-of-freedom PTMD. In addition to being economically unattractive the former alternative would have a large weight penalty and space-requirement. Passive and active realization of the more attractive latter alternative is the main subject of this paper.

II. Two Degree-of-Freedom Passive Pendulum Tuned Mass Damper

Multiple steel wire ropes (cables) are commonly used as the pendulum links of low-frequency PTMDs. The cables' length of a PTMD sets the natural frequency of the pendulum tuning the PTMD to its target mode of the structure. The common length of all cables renders the PTMD with the same natural frequency in both lateral (X and Y) directions.

As stated earlier, asymmetric tall structures having different natural frequencies in X and Y-directions require either two PTMDs or one PTMD with two different tuning frequencies (one in each lateral direction). PTMDs with two different tuning frequencies in two directions, also called Bi-PTMDs, are realized by constraining the swinging length of the pendulum cables in one direction but not in the other (orthogonal) direction. Fig. 1 shows one such PTMD with four-cable arrangement. Constraining can be done via placing two solid pieces, shown in red in Fig. 1, between the two parallel pairs of steel cables making up the pendulum links.

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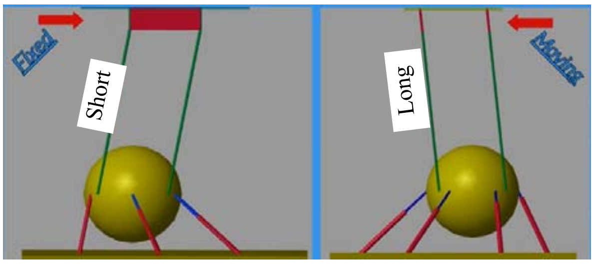

Fig. 1 A typical Bi-PTMD

III. Active Pendulum Tuned Mass Damper

PTMDs with multiple tuning frequencies can also be realized by introducing active elements in place of passive viscous dampers, along with the proper control strategies, into their make-up. Fig. 2 depicts the proposed active pendulum tuned mass damper (APTMD) with the mass suspended by multiple steel wire ropes of equal length and actuated by 6 hydraulic cylinders (actuators).

In addition to the main 'active multi-frequency control' scheme, an optional second layer of control namely 'active damping effectiveness enhancement' control can be added to the 'active multi-frequency tuning' control. This would make the proposed APTMD not only exhibit multi-frequency tuning but also provide up to twice as much damping effectiveness as that of an equally-sized passive Bi-PTMD. For more on this attribute of the control system, refer to [5].

Except for the use of hydraulic cylinders in place of passive viscous dampers, the active PTMD resembles a passive PTMD. With proper plumbing of hydraulic circuitry along with appropriate control logic, the hydraulic cylinders can be switched from active elements with actuation capacity to passive viscous dampers with energy dissipation capacity; refer to [6], [7] for configuring hydraulic cylinders as viscous dampers. The use of hydraulic cylinders, with this attractive feature, allows for the active PTMD to also act as a passive one degree-of-freedom PTMD. In its passive state, in which the hydraulic actuators are configured to act as passive viscous dampers, the pendulum length is selected so that the PTMD is optimally tuned to the lowest natural frequency (the first bending mode) of its target asymmetric structure. When the need arises (depending on the extent and frequency content of the structural vibration) the control logic switches the hydraulic cylinders from being passive viscous dampers to active actuators and turn the passive PTMD into an active PTMD with multi degree-of-freedom tuning capacity (capability) as well as enhanced damping effectiveness. Refer to [8] for more on the switchable hydraulic circuit and the switching logic of the active PTMD.

In its active state, while staying tuned to the first mode passively, the PTMD simultaneously tunes itself to 3 frequencies adding tuned damping to two bending (in X and Y lateral directions) and one torsional (γ, around Z direction) modes of target asymmetric structure. The hydraulic cylinders (legs) are the active elements of the active PTMD realizing the spatial motion of the pendulum mass, decided by the control algorithm.

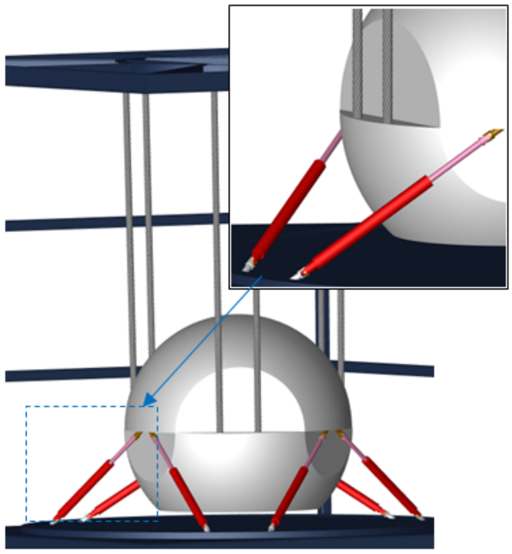

Fig. 2 The active PTMD

The actuation system of the active PTMD consists of six double-acting, double-rod hydraulic cylinders. Each hydraulic cylinder is equipped with a temperature-compensated flow control valve and a 4-way electrohydraulic servo-valve; the former is used when the device is in its passive state and the latter along with the hydraulic power supply are used when the device is in its active state. Refer to [5] for more on the hydraulic circuit and switching logic.

A. Stewart Platform

The six linear actuators (hydraulic cylinders/legs) along with the moving mass of the active PTMD form a 6-legged closed-chain mechanism with 6 degrees of freedom. This mechanism, commonly known as Stewart platform [9], is capable of manipulating the mass in any direction and orientation, generating controlled dynamic motions. A typical configuration of Stewart platform is shown in Fig. 3, having a base at one end and a moving platform at the other end. Similar to Stewart platforms actuated by six linear actuators (legs) with universal joint at both ends and a cylindrical joint in the middle, the active PTMD uses hydraulic cylinders with the same joint arrangement.

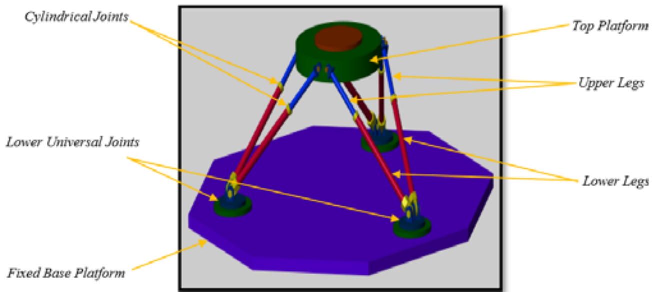

Fig. 3 Structural concept of a Stewart platform

B. The Control Scheme

In response to the vibratory motion of the structure, the active PTMD interacts with the structure in a feedback manner by applying a reactive damping plus stiffness force vector, to the structure, at its installation location (e.g., the upper floor of a building). The feedback control scheme is comprised of 6 decoupled force controllers implemented at the leg (hydraulic cylinder) level. Each hydraulic cylinder is equipped with either a force sensor or two pressure sensors across its piston, the measurements of which are used as the feedback signal. The next section discusses how reference inputs to these controllers are evaluated.

C. Reference Input to the Controller

The damping force plus the stiffness force vector required to have the active PTMD realize the multi-frequency tuning and the corresponding optimal damping is presented in (1):

$$U_2 = K\,P + C\,\dot{P} \tag{1}$$

In this equation, K and C are 6 × 6 matrices containing the required stiffness and damping coefficients, respectively. And $P = (X \quad Y \quad \gamma)^T$ and $\dot{P} = (\dot{X} \quad \dot{Y} \quad \dot{\gamma})^T$ are the position and velocity vectors of the active PTMD mass measured in the global Cartesian coordinate system placed on the upper floor of building where the PTMD is installed. The superscript T signifies the 'transpose'.

Considering that in PTMDs with multiple pendulum links, a) the pitch and yaw motion (rotations around the two lateral axes Y and X) are restrained and b) translation along the vertical axis is negligible, the number of directions in which the stiffness and damping are adjusted reduces to two lateral as well as roll (rotation around Z), i.e., X, Y, and γ directions. As such, the damping force vector U_2 is formulated in the global coordinate system, as shown in (2):

$$U_2 = \begin{pmatrix} K_X & 0 & 0 \\ 0 & K_Y & 0 \\ 0 & 0 & K_\gamma \end{pmatrix} \begin{pmatrix} X \\ Y \\ \gamma \end{pmatrix} + \begin{pmatrix} C_X & 0 & 0 \\ 0 & C_Y & 0 \\ 0 & 0 & C_\gamma \end{pmatrix} \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\gamma} \end{pmatrix} \tag{2}$$

where K_i and C_i are additional stiffness and damping coefficient matrices needed in i = X, Y and γ directions, over what the pendulum itself provides. Note that the two matrices are assumed symmetric which could only be true if the lower modes were nearly decoupled. To meet the dimensional compatibility of the matrices in conducting matrix algebra required for the computation of the forces of the 6 legs, the 3x3 stiffness and damping coefficient matrices in (2) are padded with enough zeros to 6x6 matrices as shown in (3):

$$U_2 = \begin{pmatrix} K_X & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_Y & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_\gamma \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ \alpha \\ \beta \\ \gamma \end{pmatrix} + \begin{pmatrix} C_X & 0 & 0 & 0 & 0 & 0 \\ 0 & C_Y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_\gamma \end{pmatrix} \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{pmatrix} \tag{3}$$

where α and β are the constrained yaw and pitch of the pendulum mass.

For a more practical and workable control system, the U_2 force is transformed from the global coordinate system to the local, leg coordinate system. The stiffness plus viscous damping force vector of the six legs u_2 in the local coordinate systems of the legs is shown in (4):

$$u_2 = k\,p + c\,\dot{p} \tag{4}$$

where k and c are the stiffness and damping coefficient matrices of the legs and p and $\dot{p}$ are the position and velocity vectors of the legs. As described below the transformation is done using, a) *the principle of virtual work* relating the actuation in the local and global coordinate systems, and, b) *the Jacobian matrix relating the motions* in the local and global coordinate systems.

In the global coordinate system, the force and moment vectors that the legs collectively impart on the mass of PTMD are represented by (5):

$$U_2 = [U_{2X} \quad U_{2Y} \quad U_{2Z} \quad \dots \quad U_{2\gamma}]^T \tag{5}$$

and, in the local leg coordinate systems, the force vectors of 6 actuated legs 1 thru 6 are represented by (6):

$$u_2 = [u_{21} \quad u_{22} \quad u_{23} \quad \dots \quad u_{26}]^T \tag{6}$$

Furthermore, the virtual displacement vector of legs 1 thru 6 is represented by (7):

$$\delta p = [\delta l_1 \quad \delta l_2 \quad \delta l_3 \quad \dots \quad \delta l_6]^T \tag{7}$$

Also, the virtual displacement vector associated with the mass are represented by (8):

$$\delta P = [\delta X \quad \delta Y \quad 0 \quad 0 \quad 0 \quad \delta\gamma]^T \tag{8}$$

Assuming friction in the joints of the legs and the effects of gravity are negligible, the virtual work done by the leg forces at the local coordinate system would be equal to the virtual work done at the global coordinate system by the forces imparted on the mass, i.e.

$$u_2^T\,\delta p - U_2^T\,\delta P = 0 \tag{9}$$

where δp and δP are the vectors of local and global virtual displacements.

A relationship between the virtual displacements δp and δP can be defined by (10):

$$\delta p = J\,\delta P \tag{20}$$

where J is 6x6 the Jacobian matrix. We substitute the expression $J\,\delta P$ for δp in (9) yields

$$(u_2^T J - U_2^T)\,\delta P = 0$$

$$\Rightarrow u_2^T J - U_2^T = 0 \tag{31}$$

$$\Rightarrow u_2 = J^{-T} U_2 \tag{42}$$

Combining (3), (4), (11) and (12) results in (13) and (14) relating the stiffness and damping coefficients in the legs space to that in the global space.

$$k = J^{-T} K J^{-1} \tag{53}$$

$$c = J^{-T} C J^{-1} \tag{64}$$

where superscript $-T$ signifies the 'inverse of transpose', k and c are the corresponding proportional and derivative feedback gain matrices.

Having the stiffness and damping coefficient matrices in the local coordinate systems of the legs (hydraulic cylinders) i.e., k and c, allows for the evaluation of the instantaneous desired force vector of the legs. This is done by a) feeding the measured displacement vector of the legs to a centralized Proportional + Derivative (PD) scheme in which k and c are the corresponding proportional and derivative gain matrices and b) evaluating u_2 vector according to (12). The evaluated u_2 is then used as the reference input vector to 6 distributed Proportional (P) controllers generating the control signals to the servo-valves. The feedback signals to the distributed controllers are the measured force of each leg. The 'active multi-frequency tuning' control strategy is depicted in the block diagram of Fig. 4.

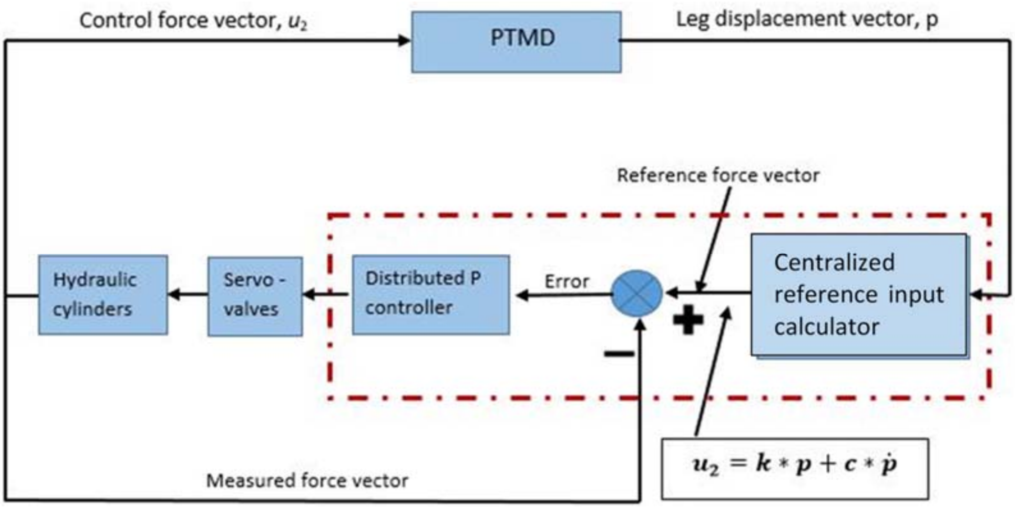

Fig. 4 Block diagram of the APTMD in its active state

With the desired force vector of the legs u_2 realized, the PTMD mass experiences the desired global force vector U_2. This results in the PTMD in its active state, experiencing the desired global stiffness K and damping coefficient C which in turn leads to the PTMD (in its active state) realizing the desired tuning frequencies along with the corresponding damping, in different directions.

As stated earlier, the 'active multi-frequency tuning' control scheme shown in Fig. 4 can be cascaded by another controller dubbed 'active damping effectiveness enhancement' controller, enhancing the damping effectiveness of the active PTMD [8]. In addition, a high-level supervisory controller oversees the vibration attributes of the structure both in terms of severity as well as frequency and decides to bring none, one ('active multi-frequency tuning') or both ('active multi-frequency tuning' plus 'active damping effectiveness enhancement') controllers online. That is when the perturbation in the first mode of the asymmetric building has low to moderate severity, the active PTMD will automatically configures itself as passive PTMD by disconnecting the actuators from the power source, disabling the active controllers, and switching the hydraulic cylinders (the legs) to viscous dampers. When the perturbation becomes large, the passive PTMD will switch to active PTMD by reconnecting the hydraulic cylinders (the legs) to the hydraulic power supply and enabling the active controllers. The supervisory control scheme continuously calculates the instantaneous reference force inputs to the decoupled feedback legs' force controllers and readjusts the parameters of an active PTMD so that the unit is always optimally tuned [8].

IV. Comparison of Passive Bi-PTMD and Active PTMD

The effectiveness of the passive Bi-PTMD system and that of the active PTMD under 'multi-frequency tuning control' is numerically compared by introducing them into the model of a multi-degree of freedom asymmetric, high-rise building and having them tuned to the first two natural frequencies of 0.18 and 0.29 Hz corresponding to its first two bending modes of the building in Y and X directions, simultaneously. The building has 41 stories with each floor having three degrees of freedom, two translational in X and Y directions and one rotational around the Z axis. Due to the uniformity in geometry and mass distribution in each floor, vibration of the lower modes in those three directions is nearly decoupled from each other.

The numerical model of the building, in which the first 15 modes of vibration are included, is developed. The nearly decoupled modes of vibration of the building allows for associating each mode with the vibration in one direction, only. Refer to [5] for constructing the state space model of the building and its interface with the PTMDs.

The simulation results show that the passive Bi-PTMD exhibits similar performance in reducing the primary structure's vibration levels as its equally sized (in terms of the weight of the moving mass) active PTMD under 'active multi-frequency tuning' control only.

The three traces in Fig. 5 show the frequency response functions as well as the resonant time traces of the structure's acceleration measured at the top floor along *Y* direction, with no tuned damping (blue/dotted-line), tuned damping using Bi-PTMD (black/solid-line), and active tuned damping (red/ center-line). Harmonic perturbation in *Y* direction with spatially varying amplitude along the height of the structure is used to perturb all the floors, simultaneously. The severity of the harmonic perturbations is selected corresponding to the 10 year return period wind (where the building is located) inducing 39 milli-g peak accelerations in *Y* direction.

Comparison of the black/solid-line traces with red/center-line traces in Fig. 5 shows that the Bi-PTMD and active PTMD under 'active multi-frequency tuning' control only, have similar damping effectiveness.

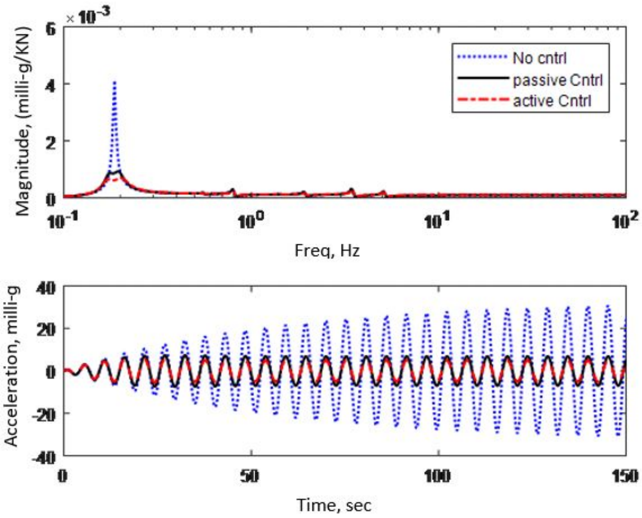

Fig. 5 FRFs and resonant time traces of the structure's top floor acceleration along Y direction, without (blue/dotted-line trace) and with Bi-PTMD (black/solid-line trace) and active PTMD (red/dashed-line trace)

Passive Bi-PTMD, being less complex in their make-up and not requiring energy to operate, has less operating cost than active PTMD. Considering that the proposed active PTMD is passively tuned to the frequency of the first mode and designed to operate passively when the structure mainly vibrates in that mode, the control efforts of the passive/on-demand active PTMD for quieting that mode would be minimal. Moreover, if a structure cannot take the weight of a passive Bi-PTMD then the active PTMD under parallel control of 'active multi-frequency tuning' as well as 'active damping effectiveness enhancement', having half as much weight of the passive Bi-PTMD but equivalent damping effectiveness, is certainly an attractive alternative.

V. Summary

The functionality and operation of a passive PTMD with two tuning frequencies in two different lateral directions, dubbed Bi-PTMD, and an active PTMD capable of switching to passive are presented and their damping effectiveness compared. Bi-PTMDs are realized by constraining the swinging length of the pendulum cables in one direction but not in the other (orthogonal) direction. The active PTMD, switchable to passive, is passively tuned to the first mode of the structure with the lowest natural frequency and acts as a traditional passive PTMD, but it can switch back to its active mode in which a parallel cascade of two control schemes a) increases the damping effectiveness of a small APTMD to the same level as a larger passive PTMD tuned to the same frequency and b) tunes the APTMD to multiple frequencies.

The physical models of the two multi-frequency PTMDs targeting the first two lateral, low-frequency bending modes (with two different natural frequencies) of an asymmetric 41 story high-rise building are developed and interfaced with the state space model of that building. These models are used to compare the damping effectiveness of the passive Bi-PTMD with that of the active PTMD of comparable size tuned to the same frequencies. The advantages and disadvantages of these two tuned damping schemes are listed.

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