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A Neutral Set Approach for Applying TOPSIS in Maintenance Strategy Selection

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***Abstract*—**This paper introduces the concept of neutral sets (NSs) and explores various operations on NSs, along with their associated properties. The foundation of the Neutral Set framework lies in ontological neutrality and the principles of logic, including the Law of Non-Contradiction. By encompassing components for possibility, indeterminacy, and necessity, the NS framework provides a flexible representation of truth, uncertainty, and necessity, accommodating diverse ontological perspectives without presupposing specific existential commitments. The inclusion of Possibility acknowledges the spectrum of potential states or propositions, promoting neutrality by accommodating various viewpoints. Indeterminacy reflects the inherent uncertainty in understanding reality, refraining from making definitive ontological commitments in uncertain situations. Necessity captures propositions that must hold true under all circumstances, aligning with the principle of logical consistency and implicitly supporting the Law of Non-Contradiction. Subsequently, a neutral set-TOPSIS approach is applied in the maintenance strategy selection problem, demonstrating the practical applicability of the NS framework. The paper further explores uncertainty relations and presents the fundamental preliminaries of NS theory, emphasizing its role in fostering ontological neutrality and logical coherence in reasoning.

***Keywords*—**Uncertainty sets, neutral sets, maintenance strategy selection multiple criteria decision-making analysis, MCDM, uncertainty decision analysis, distance function, multiple attribute, decision making, selection method, uncertainty, TOPSIS.

I. INTRODUCTION

Fuzzy set theory, a significant advancement in uncertainty modeling, introduced the concept of membership degrees, which represent the partial belonging of an element to a set. This concept of graded membership has inspired the development of neutral sets, which take a more comprehensive approach to representing uncertainty [1-9].

In an era characterized by complex systems, diverse data sources, and unpredictable events, the need for a comprehensive framework to analyze uncertainty has become increasingly evident. The neutral set emerges as a response to this need, offering a novel approach that integrates multiple viewpoints and addresses the limitations of existing theories like probability and fuzzy logic [1-9]. These traditional approaches often focus on specific aspects of uncertainty, such as probability distributions in probability theory or linguistic variables in fuzzy logic [1-9]. They may overlook the holistic nature of uncertainty and fail to capture the dynamic interactions between optimistic and pessimistic perspectives.

Neutral sets address these limitations by offering a triadic framework that considers three independent components:

Possibility $\left(0 \le \mu_A(x) \le 1\right)$: This component represents the optimistic outlook, reflecting the potential feasibility or likelihood of a particular outcome.

Indeterminacy $\left(0 \le \eta_A(x) \le 1\right)$: This component acknowledges the inherent ambiguity and lack of complete information that often exists in complex situations.

Necessity $\left(0 \le v_A(x) \le 1\right)$: This component signifies the essential requirements or constraints that must be fulfilled for a successful decision.

This comprehensive perspective allows the neutral set to capture the nuances of uncertainty, moving beyond the limitations of traditional methods.

The neutral set framework, encompassing components for Possibility $\mu_A(\mathrm{x})$, Indeterminacy $\eta_A(x)$, and Necessity $v_A(x)$, establishes a foundation rooted in ontological neutrality and the principles of logic, including the Law of Non-Contradiction. Firstly, by incorporating Possibility $\mu_A(\mathrm{x})$, the neutral set acknowledges the spectrum of potential states or propositions, accommodating various ontological perspectives without presupposing the existence or non-existence of entities. This allows for a flexible representation of what might be possible under different ontological frameworks, promoting neutrality by accommodating diverse viewpoints. Secondly, the inclusion of Indeterminacy $\eta_A(x)$ recognizes the inherent uncertainty or ambiguity in our understanding of reality, reflecting situations where the truth status or ontological nature of propositions is not fully determined.

This aspect aligns with the principle of logical neutrality, as it refrains from making definitive ontological commitments in the face of uncertainty. Finally, Necessity $\eta_A(x)$ within the Neutral Set captures propositions that must hold true under all circumstances, reflecting a core tenet of logical reasoning. The Law of Non-Contradiction, a fundamental principle in logic, is implicitly supported by the Necessity component, as it rules out circumstances where contradictory statements are simultaneously true. Thus, by encompassing Possibility, Indeterminacy, and Necessity, the Neutral Set framework embodies ontological neutrality while remaining consistent with the principles of logic, including the Law of Non-Contradiction.

Example: Imagine using a neutral set to model the potential spread of a disease. Possibility could represent the most optimistic scenario for containment, considering factors like early detection and effective vaccination. Indeterminacy

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could account for unforeseen factors like mutations in the virus or changes in public behavior. Necessity could reflect the minimum resources required to achieve control, such as healthcare personnel and isolation facilities.

By considering these diverse aspects of uncertainty, the neutral set offers a powerful tool for complex decision-making in various scientific fields. This framework allows researchers and practitioners to integrate diverse viewpoints, ensure unbiased assessments, and develop robust and reliable predictions in the face of ambiguity. The mathematical novelty and potential for new computational algorithms further enhance the potential of uncertainty sets for diverse data analysis scenarios across various disciplines.

The remainder of this research is structured as follows. Section 2 provides a brief overview of the neutral set and its integration with the TOPSIS method for Multiple Attribute Group Decision Making (MAGDM) under uncertainty, enabling the ranking of alternatives. Section 3 showcases the proposed approach's validity and effectiveness through a numerical example for selecting a maintenance strategy. Finally, Section 4 concludes the study by summarizing the key findings.

II. Methodology

This section introduces the concept of neutral sets with three independent components possibility $\left(0 \le \mu_A(x) \le 1\right)$, indeterminacy $\left(0 \le \eta_A(x) \le 1\right)$, and necessity $\left(0 \le \nu_A(x) \le 1\right)$ and its integration with the TOPSIS method for Multiple Attribute Group Decision Making (MAGDM) under uncertainty [10]. Neutral sets deal with uncertainty in situations or events, particularly relevant to decision-making processes. It emphasizes three independent components:

Possibility ($\mu_A(\mathrm{x})$): This represents the degree to which an outcome associated with a decision is achievable, ranging from 0 (impossible) to 1 (certain). It captures the likelihood of a particular outcome occurring based on available information: $0 \le Possibility\left(\mu_A(x)\right) \le 1$ (ranges from entirely impossible to absolutely certain).

Indeterminacy ($\eta_A\left(x\right)$): Indeterminacy represents the level of ambiguity or lack of information associated with the potential outcomes of a decision. It spans from 0 (fully determined with perfect knowledge) to 1 (completely indeterminate with no clear understanding of potential consequences). This component acknowledges situations where the decision-maker has limited knowledge about the future: $0 \le Indeterminacy\left(\eta_A(x)\right) \le 1$ (ranges from perfect information to complete uncertainty).

Necessity ($\nu_A\left(x\right)$): Necessity refers to the minimum level of belief needed for a specific outcome to be considered desirable or essential for achieving the overall goal. It also ranges from 0 (not necessary at all) to 1 (absolutely necessary). Essentially, it reflects how important a particular outcome is for the decision-maker: $0 \le Necessity\left(\nu_A(x)\right) \le 1$ (ranges from not necessary at all to absolutely necessary).

These three components are independent, meaning they can vary independently of each other for a given situation or event. For example, an event might have high Possibility (likely to occur) even with high Indeterminacy (lack of information about the specifics), and a low Necessity (not essential for achieving a desired outcome). This scenario could represent a potential risk with a high chance of happening but minimal consequences if it does occur.

Definition 1. **[**10-11**]** Let *X* be a domain of discourse. An uncertainty set *A* in the domain *X* is defined as

$$A = \{< x, \mu_A\left(x\right), \eta_A\left(x\right), \nu_A\left(x\right) > \mid\ x \in X\} \tag{1}$$

where $\mu_A(\mathrm{x}): X \to \left[0,1\right]$, $\eta_A(\mathrm{x}): X \to \left[0,1\right]$, and $\nu_A(\mathrm{x}): X \to \left[0,1\right]$ are three maps in *X* that satisfy the condition $0 \le \mu_A\left(x\right) + \eta_A\left(x\right) + \nu_A\left(x\right) \le 1 \mid\ x \in X$. The numbers $\mu_A(\mathrm{x})$, $\eta_A\left(x\right)$, and $\nu_A\left(x\right)$ are the degree of truth, indeterminacy, and falsity functions of element *x* to *A*, respectively.

Definition 2. Let *X* be a domain of discourse. A neutral set *A* in the domain *X* is denoted as

$$A = \{< x, \mu_A\left(x\right), \eta_A\left(x\right), \nu_A\left(x\right) > \mid\ x \in X\} \tag{2}$$

where $\mu_A(\mathrm{x}): X \to \left[0,1\right]$, $\eta_A(\mathrm{x}): X \to \left[0,1\right]$, and $\nu_A(\mathrm{x}): X \to \left[0,1\right]$ are three independent maps in *X* that satisfy the condition $0 \le \mu_A\left(x\right) + \eta_A\left(x\right) + \nu_A\left(x\right) \le 3 \mid\ x \in X$. The numbers $\mu_A(\mathrm{x})$, $\eta_A\left(x\right)$, and $\nu_A\left(x\right)$ are the degree of possibility, indeterminacy, and necessity functions of element *x* to *A*, respectively.

Definition 3. Let $A = \left(\mu_1, \eta_1, \nu_1\right)$ and $B = \left(\mu_2, \eta_2, \nu_2\right)$ be two neutral set numbers. The mathematical operations (union, intersection, complement) between *A* and *B* are defined as follows:

Union (⋃): Union of two neutral set numbers *x* and *y* is defined as:

$$\begin{aligned} A \cup B = \{(x, max(\mu_A(x), \mu_B(x)), min(\eta_A(x), \\ \eta_B(x)), max(\nu_A(x), \nu_B(x)) \mid x \in X\} \end{aligned} \tag{3}$$

Union (⋃): This definition prioritizes the higher possibility of achieving an outcome ($max(\mu_A(x), \mu_B(x))$ and the lower level of indeterminacy ($min(\eta_A(x), \eta_B(x))$, indicating a more favorable or certain scenario. It also prioritizes the higher necessity ($max(\nu_A(x), \nu_B(x))$.

Intersection (⋂): Intersection of two neutral set numbers *A* and *B* is defined as:

$$A \cap B = \{(x, min(\mu_A(x), \mu_B(x)), max(\eta_A(x), \eta_B(x)), min(\nu_A(x), \nu_B(x)) \mid x \in X\} \quad (4)$$

Intersection (⋂): This definition focuses on the more limited possibility ($min(\mu_A(x), \mu_B(x))$ and the higher level of indeterminacy ($max(\eta_A(x), \eta_B(x))$, reflecting a less promising or more uncertain scenario. It also emphasizes the lower necessity ($min(\nu_A(x), \nu_B(x))$. This is useful for identifying potential risks or less favorable options.

Complement (¬): Complement of a neutral set number x is defined as:

$$\neg A = \{(1-\mu_A(x),\ \eta_A(x),\ 1-\nu_A(x)) \mid x \in X\} \quad (5)$$

Complement (¬): This definition essentially inverts the possibility ($1-\mu_A(x)$) and the necessity ($1-\nu_A(x)$) while keeping the indeterminacy ($\eta_A(x)$) the same.

These operations (Union (⋃), Intersection (⋂), Complement (¬)) enable addition, multiplication, scalar multiplication, and scalar exponentiation with neutral set numbers based on their Possibility (μ_A(x)), Indeterminacy ($\eta_A(x)$), and Necessity ($\nu_A(x)$) components.

Addition (⊕):

$$A \oplus B = (\mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \eta_A(x)\eta_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)) \quad (6)$$

Multiplication (⊗):

$$A \otimes B = (\mu_A(x)\mu_B(x), \eta_A(x) + \eta_B(x) - \eta_A(x)\eta_B(x), \nu_A(x)\nu_B(x)) \quad (7)$$

Scalar Multiplication (*kA*):

$$kA = (1-(1-\mu_A(x))^k, \eta_A(x)^k, 1-(1-\nu_A(x))^k \quad where\ k > 0 \quad (8)$$

Scalar Exponentiation (A^k):

$$A^k = (\mu_A(x)^k, 1-(1-\eta_A(x))^k, \nu_A(x)^k) \quad where\ k > 0 \quad (9)$$

Definition 4. Let $A = \{< x, \mu_A(x), \eta_A(x), \nu_A(x) > \mid x \in X\}$ and $B = \{< x, \mu_B(x), \eta_B(x), \nu_B(x) > \mid x \in X\}$ be two neutral sets, the L_1 , L_2, and L_∞ distances between *A* and *B* are defined as follows:

$$d_{L_1}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\begin{pmatrix} |\mu_A(x) - \mu_B(x)| + |\eta_A(x) - \eta_B(x)| \\ + |\nu_A(x) - \nu_B(x)| \end{pmatrix} \quad (10)$$

$$d_{L_2}(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\begin{pmatrix} (\mu_A(x) - \mu_B(x)^2 + \\ (\eta_A(x) - \eta_B(x))^2 + \\ (\nu_A(x) - \nu_B(x))^2 \end{pmatrix}} \quad (11)$$

$$d_{L_\infty}(A,B) = max\begin{pmatrix} |\mu_A(x) - \mu_B(x)|, \\ |\eta_A(x) - \eta_B(x)|, \\ |\nu_A(x) - \nu_B(x)| \end{pmatrix} \quad (12)$$

Definition 5. $d(A,B)$ is said to be a distance measure between neutral sets (NSs) if it satisfies the following properties:

P1: $d(A,B) \geq 0$.
P2: $d(A,B) = 0$ if and only if A=B for all A, B ∈ NSs.
P3: $d(A,B) = d(B,A)$.
P4: If $A \subseteq B \subseteq O$ where O ∈ NSs in *X* then: $d(A,O) \geq d(A,B)$ and $d(A,O) \geq d(B,O)$.

Philosophical Exploration: In the vast expanse of the cosmos, amidst the twinkling stars and swirling galaxies, there exists a realm known as the Cosmic Library. This fabled repository houses the sum of all knowledge, containing ancient tomes and celestial scrolls that hold the secrets of existence itself.

Within the Cosmic Library, there dwells a curious scholar named Lyra, whose insatiable thirst for knowledge knows no bounds. Lyra is on a quest for the fabled Cosmic Rosetta Stone – a legendary artifact said to unlock the deepest mysteries of the universe.

The Rosetta Stone is rumored to be hidden within the Labyrinth of Eternity, a labyrinthine maze of twisting corridors and shifting pathways guarded by enigmatic sentinels. To uncover the stone's secrets, Lyra must navigate the labyrinth's treacherous depths, braving its myriad challenges and deciphering its cryptic puzzles.

Yet, the path to the Rosetta Stone is shrouded in uncertainty. Ancient manuscripts speak of conflicting clues, and whispers of forgotten prophecies cast doubt upon the journey's outcome. The weight of the unknown bears down upon Lyra, threatening to thwart her quest before it even begins.

But Lyra is not deterred. Embracing the principles of the Neutral Sets, she approaches the labyrinth with a balanced perspective, acknowledging the interplay of possibility, indeterminacy, and necessity.

Possibility whispers tales of untold wonders and cosmic revelations, fueling Lyra's optimism and driving her forward into the unknown. Indeterminacy casts its shadow, veiling the true nature of the labyrinth's challenges and leaving Lyra to grapple with uncertainty at every turn.

And yet, necessity beckons – the necessity of uncovering the Cosmic Rosetta Stone to fulfill her quest and unlock the secrets of the universe. With determination in her heart and the wisdom of the Neutral Sets as her guide, Lyra presses onward, navigating the labyrinth with courage and conviction.

As she delves deeper into the labyrinth's depths, Lyra encounters trials and tribulations beyond imagining – riddles that test her intellect, obstacles that challenge her resolve, and mysteries that defy explanation. But with each challenge overcome, Lyra gains new insights and deeper understanding, drawing ever closer to her ultimate goal.

In the end, it is not blind luck or unwavering certainty that leads Lyra to the Cosmic Rosetta Stone, but rather a delicate balance of hope and humility, curiosity and caution. With a steady hand and an open mind, she uncovers the ancient artifact, unlocking the secrets of the universe and fulfilling her destiny as a seeker of truth.

And as Lyra emerges from the Labyrinth of Eternity, the Cosmic Rosetta Stone in hand, she carries with her not only the knowledge of the ages but also a profound appreciation for the wisdom of the Neutral Sets – a timeless philosophy that guides the seekers of knowledge through the ever-shifting currents of uncertainty, towards the boundless horizons of possibility.

Definition 6. *a*) Traditional TOPSIS Method. TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method is a multiple criteria decision-making method that aims to determine the best alternative from a set of options based on their similarity to the ideal solution. It is based on the concept of an ideal point that has been perturbed or displaced. The compromise solution, in this context, corresponds to the alternative that exhibits the shortest distance from this perturbed ideal point. The TOPSIS method involves steps and mathematical formulations as follows [10]:

Step 1. Normalize the Decision Matrix

Let $X=[x_{ij}]_{mxn}$ be the decision matrix with *m* alternatives and *n* criteria. The normalized decision matrix, denoted as $R=[r_{ij}]_{mxn}$, is obtained by dividing each element of $X=[x_{ij}]_{mxn}$ by the square root of the sum of squares of all elements in the corresponding column:

$$r_{ij}=\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}},\, i=1,2,...,m;\;\; j=1,2,...,n \tag{13}$$

Step 2. Determine the Weighted Normalized Decision Matrix

Assign weights to each criterion to reflect their relative importance. Let $\omega=(\omega_1,\omega_2,...,\omega_n)$ be the vector of weights, where ω_j is the weight of criterion C_j and $\sum_{j=1}^{n}\omega_j=1$. Multiply each element of the normalized decision matrix by the corresponding weight to obtain the weighted normalized decision matrix, $V=[v_{ij}]_{mxn}$, denoted as

$$v_{ij}=\omega_j r_{ij},\; i=1,2,...,m;\;\; j=1,2,...,n \tag{14}$$

Step 3. Determine the Ideal and Anti-Ideal Solutions

Calculate the ideal solution A^+ and the anti-ideal solution A^- by selecting the maximum and minimum values for each criterion, respectively:

$$A^+=(a_1^+,a_2^-,...,a_n^+),\;\; a_j^+=\max_i v_{ij} \tag{15}$$

$$A^-=(a_1^-,a_2^-,...,a_n^-),\;\; a_j^-=\min_i v_{ij} \tag{16}$$

Step 4. Calculate the Similarity Scores

Calculate the similarity of each alternative to the ideal solution and the anti-ideal solution using the L_1, L_2 and L_∞ distances:

a) L_1 distance:

$$S^+(A_i)=\sum_{j=1}^{n}(|\, v_{ij}-a_j^+\,|) \tag{17}$$

$$S^-(A_i)=\sum_{j=1}^{n}(|\, v_{ij}-a_j^-\,|) \tag{18}$$

b) L_2 distance:

$$S^+(A_i)=\sqrt{\sum_{j=1}^{n}(v_{ij}-a_j^+)^2} \tag{19}$$

$$S^-(A_i)=\sqrt{\sum_{j=1}^{n}(v_{ij}-a_j^-)^2} \tag{20}$$

c) L_∞ distance:

$$S^+(A_i)=max(|\, v_{ij}-a_j^+\,|) \tag{21}$$

$$S^-(A_i)=max(|\, v_{ij}-a_j^-\,|) \tag{22}$$

Step 5. Calculate the Relative Closeness to the Ideal Solution

Calculate the relative closeness to the ideal solution for each alternative by dividing the distance to the anti-ideal solution by the sum of distances to the ideal and anti-ideal solutions:

$$C_i=\frac{S^-(A_i)}{S^+(A_i)+S^-(A_i)},\; i=1,2,...,m \tag{23}$$

Step 6. Rank the Alternatives

Rank the alternatives A_i based on their relative closeness values C_i. The alternative A_i^* with the highest relative closeness value C_i^* is considered the best choice.

b) The extended TOPSIS method for multiple attribute group decision-making

Suppose that $A=(A_1,A_2,...,A_n)$ be a set of alternatives, $C=(C_1,C_2,...,C_n)$ be a set of attributes, and $D=(D_1,D_2,...,D_k)$ be a set of decision-makers (DMs) in multiple attribute group decision-making [10-64].

Let $\omega_j^p=[\omega_1^p,\omega_2^p,...,\omega_n^p]$ be a vector of weights for attributes determined by DM (D_p) where ω_j^p is a neutral number denoting the weight of attribute C_j given by decision-maker (D_p). $1\le j\le n$ and $1\le p\le k$.

Assume that ω_p represents the weight of DM (D_p). If a decision group has k members, then $\omega_p=\frac{1}{k}$, where $\omega_p\in[0,1]$ and $\sum_{p=1}^{k}\omega_p=1$.

Let $X_p=[x_{ij}^p]_{mxn}$ be a decision matrix of the *m* alternatives in regard to the *n* attributes characterized by decision-maker D_p, shown as follows:

$$X_p=\begin{array}{c c} & \begin{array}{ccc} C_1 & & C_n \end{array} \\ \begin{array}{c} A_1 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} x^p{}_{11} & \cdots & x^p{}_{1n} \\ \vdots & \ddots & \vdots \\ x^p{}_{m1} & \cdots & x^p{}_{mn} \end{pmatrix} \end{array} \tag{24}$$

where $x_{ij}=(\mu_{ij},\eta_{ij},\nu_{ij})$ is a neutral number for the alternative A_i in regard to the attribute C_j.

The procedure of proposed method can be summarized as follows:

Step 1. According to the weighting vector ω_p, the decision matrix X_p and the multiplication operator of NSs presented in (7) calculate the weighted decision matrix $V_p=[v_{ij}^p]_{mxn}$ as follows:

$$V_p=\begin{array}{c c} & \begin{array}{ccc} C_1 & & C_n \end{array} \\ \begin{array}{c} A_1 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} x^p{}_{11}\otimes\omega_1^p & \cdots & x^p{}_{1n}\otimes\omega_n^p \\ \vdots & \ddots & \vdots \\ x^p{}_{m1}\otimes\omega_1^p & \cdots & x^p{}_{mn}\otimes\omega_n^p \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} C_1 & & C_n \end{array} \\ \begin{pmatrix} v^p{}_{11} & \cdots & v^p{}_{1n} \\ \vdots & \ddots & \vdots \\ v^p{}_{m1} & \cdots & v^p{}_{mn} \end{pmatrix} \end{array} \tag{25}$$

Step 2. Based on the obtained weighted decision matrices V_p and the weight of decision-makers one can get the aggregated group decision matrix *G* of all decision-makers D_1, D_2,.., D_k as follows:

$$G=\begin{array}{c c} & \begin{array}{ccc} D_1 & & D_k \end{array} \\ \begin{array}{c} A_1 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} g_{11} & \cdots & g_{1k} \\ \vdots & \ddots & \vdots \\ g_{m1} & \cdots & g_{mk} \end{pmatrix} \end{array} \tag{26}$$

where G_{ip} is a neutral value, representing the sum of alternatives in regard to DM D_p, and can be calculated as follows:

$$G_{ip}=\omega_p[v_{i1}^p\oplus v_{i2}^p\oplus...\oplus v_{in}^p] \tag{27}$$

where ω_p is the weight of decision-maker D_p and $\oplus$ is the addition operator presented in (6).

Step 3. Based on the obtained aggregated group decision matrix, the elements G_{ip} are NNs. The absolute neutral positive ideal solution (NPIS) J^+ and the neutral negative ideal solution (NNIS) J^- can be defined as follows:

$$\begin{aligned} J^+&=(G_1^+,G_1^+,...,G_k^+)\\ J^-&=(G_1^-,G_1^-,...,G_k^-)\end{aligned} \tag{28}$$

where $G_j^+=(1,0,1)$ and $G_j^-=(0,1,0)$, $j=1,2,...,k$. Also, one can select the virtual positive ideal solution and negative ideal solution by selecting the best values for each attribute from all alternatives as follows:

$$\begin{aligned} G_j^+&=(\max_i\mu_{ij},\min_i\eta_{ij},\max_i\nu_{ij})=(\mu_j^+,\eta_j^+,\nu_j^+)\\ G_j^-&=(\min_i\mu_{ij},\max_i\eta_{ij},\min_i\nu_{ij})=(\mu_j^-,\eta_j^-,\nu_j^-)\end{aligned} \tag{29}$$

where $1\le j\le k$.

Step 4. Based on the distance measures in *Definition 4*. calculate the distances between alternative A_i and the elements in the obtained positive ideal solution J^+ as follows:

$$\begin{aligned} d_{i,L_1}^+&=\frac{1}{n}\sum_{j=1}^{n}(|\mu_{ij}-\mu_j^+|+|\eta_{ij}-\eta_j^+|+|\nu_{ij}-\nu_j^+|)\\ d_{i,L_2}^+&=\sqrt{\frac{1}{n}\sum_{j=1}^{n}(\mu_{ij}-\mu_j^+)^2+(\eta_{ij}-\eta_j^+)^2+(\nu_{ij}-\nu_j^+)^2}\\ d_{i,L_\infty}^+&=\max(|\mu_{ij}-\mu_j^+|,|\eta_{ij}-\eta_j^+|,|\nu_{ij}-\nu_j^+|)\end{aligned} \tag{30}$$

also, the degree of distances between the alternative A_i and the elements in the obtained negative ideal solution J^- can be calculated as follows:

$$d_{i,L_1}^-=\frac{1}{n}\sum_{j=1}^{n}(|\mu_{ij}-\mu_j^-|+|\eta_{ij}-\eta_j^-|+|\nu_{ij}-\nu_j^-|)$$

$$d_{i,L_2}^{-} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(\mu_{ij}-\mu_j^{-})^2 + (\eta_{ij}-\eta_j^{-})^2 + (v_{ij}-v_j^{-})^2} \tag{31}$$

$$d_{i,L_\infty}^{-} = \max(|\mu_{ij}-\mu_j^{-}|,|\eta_{ij}-\eta_j^{-}|,|v_{ij}-v_j^{-}|)$$

where $1 \le i \le m$ and $1 \le j \le n$.

Step 5. Compute the relative closeness coefficient to choose the most appropriate and efficient decision by ranking the alternatives as follows:

$$C_i = \frac{d_i^{-}}{d_i^{+} + d_i^{-}},\ i = 1,2,...,m \tag{32}$$

Step 6. Utilize the relative closeness coefficients to rank the alternatives. The larger C_i is, the better alternative A_i is.

III. Application

In this section, an example based on TOPSIS method for MAGDM under the uncertainty environment is used as a demonstration of the applications and the effectiveness of the proposed decision-making method.

The TOPSIS method is applied to rank the maintenance strategies for aircraft gas turbine engines, and their corresponding evaluation criteria are presented below:

A. Maintenance Strategies

Effective maintenance is crucial for ensuring the safety and reliability of aircraft gas turbine engines. An overview of the different maintenance strategies considered for this application is presented as follows [65-67]:

1. Traditional Approaches:

Preventive Maintenance (PM) (A1): This proactive approach involves replacing components at predetermined intervals regardless of their condition. This ensures equipment stays in good working order but can lead to unnecessary replacements.

Corrective Maintenance (CM) (A2): This reactive approach addresses failures only after they occur. While cost-effective initially, it can lead to unexpected downtime and potentially larger repair costs due to more extensive damage.

2. Modern Strategies:

These strategies utilize advanced techniques to optimize maintenance schedules:

Condition-Based Maintenance (CBM) (A3): This approach relies on sensors to continuously monitor equipment health. Maintenance actions are triggered based on real-time data exceeding predetermined thresholds, preventing failures before they happen. CBM can be cost-effective but requires investment in sensor technology and data analysis capabilities.

Predictive Maintenance (PDM) (A4): Similar to CBM, PDM utilizes sensor data. However, it goes a step further by employing data analysis techniques to predict the exact time of failure. This allows for even more precise planning of maintenance actions. PDM requires advanced data analytics expertise and robust infrastructure.

3. Specialized Approaches:

Risk-Based Maintenance (RBM) (A5): This strategy prioritizes equipment based on its potential consequences of failure. High-risk assets receive more frequent and intensive inspections compared to low-risk ones. RBM optimizes resources by focusing on areas with the greatest potential impact.

Design-out Maintenance (DOM) (A6): This strategy focuses on redesigning critical equipment components with high failure rates and maintenance costs. By addressing the root cause of failures through design changes, DOM can significantly reduce maintenance needs.

Opportunistic Maintenance (OM) (A7): This approach utilizes planned and unplanned shutdowns to perform maintenance activities on multiple pieces of equipment. For instance, if one engine requires repair during a scheduled shutdown, other engines can be inspected simultaneously. While efficient for maximizing uptime, OM is not suitable for time-sensitive operations.

The optimal maintenance strategy for aircraft gas turbine engines depends on various factors, including safety requirements, operational costs, engine type, and available resources. The proposed model provides a framework for evaluating these strategies and selecting the most suitable option considering multiple criteria.

B. Evaluation criteria for choosing the right strategy:

The proposed model aims to select the most suitable maintenance strategy based on three main criteria: damage, cost, and applicability [65-67].

Damage Criteria (C1): The damage criteria encompass sub-criteria such as production loss, personnel fatalities or injuries, environmental damage, and equipment damage. This criterion falls under the category of cost-type criteria

Cost Criteria (C2): The cost associated with each maintenance strategy is represented by four sub-criteria: cost of spare parts, cost of assurance spares, Mean Time Between Failures (MTBF), and Mean Time to Repair (MTTR). This criterion falls under the category of cost-type criteria

Applicability Criteria (C3): The applicability criteria assess the feasibility of implementing a particular maintenance strategy and consider two sub-criteria:

a. Investment Required: This sub-criterion evaluates the resources needed to implement the strategy, such as hardware, software, and personnel training. While an investment is required, it is typically considered negligible compared to other cost factors when evaluating per machine.

b. Technique Reliability: This sub-criterion assesses the reliability of the maintenance strategy itself and can be measured by two key aspects:

i. Failure Detection Capability: This evaluates how effectively the strategy identifies potential equipment failures before they occur. A good strategy should have a high success rate in detecting issues early on.

ii. Restoring Equipment Condition to "As New": This evaluates the strategy's ability to return the equipment to its optimal operational state after maintenance is performed. Ideally, the strategy should leave the equipment in a "like-new" condition, minimizing the risk of future failures. This criterion falls under the category of benefit-type criteria.

C. Multiple Attribute Group Decision Making (MAGDM) Problem

Suppose there is a panel tasked with comparing seven maintenance strategies for aircraft gas turbine engines: A1, A2, A3, A4, A5, A6, and A7, serving as alternatives. Additionally, assume three attributes: "Damage (C1)", "Cost (C2)", and "Applicability (C3)". A committee comprising three decision-makers, D1, D2, and D3, has been assembled within the Airline Technical Department to rank the alternatives and select the best maintenance strategies for aircraft gas turbine engines.

Assuming that the decision values of maintenance strategy alternatives A1, A2, A3, A4, A5, A6, and A7 concerning the attributes 'Damage (C1)', 'Cost (C2)', and 'Applicability (C3)' are provided by decision-makers D1, D2, and D3 based on neutral numbers, as displayed in Table 1, Table 2, and Table 3, respectively.

Step 1. Three initial decision-making matrices (D1,D2,D3) $X_p = [x_{ij}^p]_{mxn}$ are established as:

Table 1. The decision values given by D1

D1	C1	C2	C3
A1	<0.75,0.15,0.73>	<0.67,0.01,0.69>	<0.69,0.01,0.75>
A2	<0.85,0.01,0.87>	<0.65,0.05,0.75>	<0.77,0.05,0.87>
A3	<0.95,0.03,0.91>	<0.97,0.01,0.81>	<0.87,0.05,0.77>
A4	<0.79,0.05,0.63>	<0.77,0.01,0.75>	<0.67,0.05,0.67>
A5	<0.65,0.15,0.49>	<0.69,0.03,0.67>	<0.79,0.01,0.89>
A6	<0.95,0.01,0.83>	<0.75,0.15,0.79>	<0.77,0.05,0.87>
A7	<0.57,0.15,0.67>	<0.85,0.05,0.79>	<0.79,0.05,0.85>

Table 2. The decision values given by D2

D2	C1	C2	C3
A1	<0.71,0.15,0.73>	<0.65,0.15,0.69>	<0.95,0.01,0.63>
A2	<0.75,0.05,0.59>	<0.51,0.13,0.65>	<0.97,0.01,0.71>
A3	<0.85,0.01,0.73>	<0.73,0.03,0.83>	<0.77,0.05,0.67>
A4	<0.65,0.15,0.69>	<0.85,0.05,0.79>	<0.67,0.15,0.77>
A5	<0.91,0.03,0.75>	<0.87,0.13,0.59>	<0.79,0.05,0.85>
A6	<0.83,0.01,0.65>	<0.67,0.05,0.77>	<0.77,0.03,0.69>
A7	<0.69,0.11,0.79>	<0.89,0.05,0.75>	<0.65,0.15,0.71>

Table 3. The decision values given by D3

D3	C1	C2	C3
A1	<0.69,0.15,0.73>	<0.79,0.09,0.61>	<0.69,0.05,0.75>
A2	<0.75,0.09,0.65>	<0.81,0.03,0.71>	<0.87,0.01,0.71>
A3	<0.85,0.03,0.71>	<0.77,0.15,0.67>	<0.67,0.03,0.79>
A4	<0.87,0.15,0.65>	<0.81,0.15,0.69>	<0.91,0.01,0.77>
A5	<0.81,0.03,0.75>	<0.77,0.05,0.79>	<0.79,0.03,0.77>
A6	<0.77,0.01,0.73>	<0.79,0.15,0.89>	<0.87,0.15,0.59>
A7	<0.79,0.03,0.67>	<0.69,0.15,0.75>	<0.75,0.01,0.83>

Step 2. The importance weight vector ω_j of attributes C_j is determined as:

$$\omega = \left(\overset{C1}{\langle 0.75,0.05,0.85 \rangle}, \overset{C2}{\langle 0.89,0.01,0.75 \rangle}, \overset{C3}{\langle 0.77,0.03,0.69 \rangle} \right)$$

Step 3. Using the equation (7), the weighted normalized decision-making matrices (V1,V2,V3) ,$V_p = [v_{ij}^p]_{mxn}$, are constructed as:

V1	C1	C2	C3
A1	<0.56,0.19,0.62>	<0.60,0.02,0.52>	<0.53,0.04,0.52>
A2	<0.64,0.06,0.74>	<0.58,0.06,0.56>	<0.59,0.08,0.60>
A3	<0.71,0.08,0.77>	<0.86,0.02,0.61>	<0.67,0.08,0.53>
A4	<0.59,0.10,0.54>	<0.69,0.02,0.56>	<0.52,0.08,0.46>
A5	<0.49,0.19,0.42>	<0.61,0.04,0.50>	<0.61,0.04,0.61>
A6	<0.71,0.06,0.71>	<0.67,0.16,0.59>	<0.59,0.08,0.60>
A7	<0.43,0.19,0.57>	<0.76,0.06,0.59>	<0.61,0.08,0.59>

V2	C1	C2	C3
A1	<0.53,0.19,0.62>	<0.58,0.16,0.52>	<0.73,0.04,0.43>
A2	<0.56,0.10,0.50>	<0.45,0.14,0.49>	<0.75,0.04,0.49>
A3	<0.64,0.06,0.62>	<0.65,0.04,0.62>	<0.59,0.08,0.46>
A4	<0.49,0.19,0.59>	<0.76,0.06,0.59>	<0.52,0.18,0.53>
A5	<0.68,0.08,0.64>	<0.77,0.14,0.44>	<0.61,0.08,0.59>
A6	<0.62,0.06,0.55>	<0.60,0.06,0.58>	<0.59,0.06,0.48>
A7	<0.52,0.15,0.67>	<0.79,0.06,0.56>	<0.50,0.18,0.49>

V3	C1	C2	C3
A1	<0.52,0.19,0.62>	<0.70,0.10,0.46>	<0.53,0.08,0.52
A2	<0.56,0.14,0.55>	<0.72,0.04,0.53>	<0.67,0.04,0.49
A3	<0.64,0.08,0.60>	<0.69,0.16,0.50	<0.52,0.06,0.55
A4	<0.65,0.19,0.55>	<0.72,0.16,0.52	<0.70,0.04,0.53
A5	<0.61,0.08,0.64>	<0.69,0.06,0.59	<0.61,0.06,0.53
A6	<0.58,0.06,0.62>	<0.70,0.16,0.67	<0.67,0.18,0.41
A7	<0.59,0.08,0.57>	<0.61,0.16,0.56	<0.58,0.04,0.57

Step 4. Using equation (8), the aggregated group decision matrix (G), $[G_{ij}]_{7x3}$, based on the obtained weighted normalized decision-making matrices (V1, V2, V3) of all decision-makers is constructed as:

G	C1	C2	C3
A1	<0.54,0.19,0.62>	<0.63,0.07,0.50>	<0.61,0.05,0.49>
A2	<0.59,0.09,0.61>	<0.60,0.07,0.53>	<0.68,0.05,0.53>
A3	<0.66,0.07,0.68>	<0.75,0.05,0.58>	<0.60,0.07,0.51>
A4	<0.58,0.15,0.56>	<0.72,0.06,0.56>	<0.59,0.08,0.51>
A5	<0.60,0.11,0.58>	<0.70,0.07,0.52>	<0.61,0.06,0.58>
A6	<0.64,0.06,0.63>	<0.66,0.11,0.61>	<0.62,0.09,0.50>
A7	<0.52,0.13,0.61>	<0.73,0.08,0.57>	<0.56,0.08,0.55>

Step 5. Using equations (28) - (29), the positive ideal solution J^+ and the negative ideal solution J^- of the attributes $c_j \in C(j=1,2,...,n)$ from the aggregated group decision-making matrix (G), $[G_{ij}]_{7x3}$, are determined as:

$$J^+ = \left(\mu_j^+, \eta_j^+, \nu_j^+\right) = \begin{pmatrix} \max_i \mu_{ij}, \min_i \eta_{ij}, \max_i \nu_{ij} \\ \mid j=1,2,...,n \end{pmatrix}$$

$$J^+ = \left(\overset{C1}{\langle 0.52, 0.19, 0.56\rangle}, \overset{C2}{\langle 0.60, 0.11, 0.50\rangle}, \overset{C3}{\langle 0.68, 0.05, 0.58\rangle} \right)$$

$$J^- = \left(\mu_j^-, \eta_j^-, \nu_j^-\right) = \begin{pmatrix} \min_i \mu_{ij}, \max_i \eta_{ij}, \min_i \nu_{ij} \\ \mid i=1,2,...,n \end{pmatrix}$$

$$J^- = \left(\overset{C1}{\langle 0.66, 0.06, 0.68\rangle}, \overset{C2}{\langle 0.75, 0.05, 0.61\rangle}, \overset{C3}{\langle 0.56, 0.09, 0.49\rangle} \right)$$

Step 6. Using equations (30)-(31), the distances from ideal solutions (d_i^+, d_i^-) of alternatives A_i are calculated as:

d_{L_1}	$d_{L_1}^+$	$d_{L_1}^-$	C_i	R_i
A1	0,104	0,221	0,679	1
A2	0,117	0,208	0,640	2
A3	0,283	0,041	0,128	7
A4	0,178	0,146	0,451	5
A5	0,141	0,183	0,565	3
A6	0,228	0,097	0,299	6
A7	0,172	0,153	0,470	4

d_{L_2}	$d_{L_2}^+$	$d_{L_2}^-$	C_i	R_i
A1	0,080	0,152	0,656	1
A2	0,089	0,139	0,611	2
A3	0,178	0,034	0,159	7
A4	0,118	0,107	0,476	5
A5	0,102	0,116	0,530	3
A6	0,149	0,079	0,347	6
A7	0,121	0,114	0,485	4

d_{L_∞}	$d_{L_\infty}^+$	$d_{L_\infty}^-$	C_i	R_i
A1	0,153	0,315	0,674	1
A2	0,226	0,259	0,533	4
A3	0,385	0,078	0,169	7
A4	0,240	0,293	0,549	3
A5	0,186	0,211	0,531	5
A6	0,331	0,159	0,324	6
A7	0,238	0,290	0,549	2

In the context of ranking alternatives and decision-making, the proposed TOPSIS approach, extended to incorporate neutral sets (NSs), utilizes a relative closeness coefficient to evaluate and prioritize alternatives. This coefficient quantifies the degree of proximity of each alternative to the ideal solution within the neutral set framework. Notably, higher coefficients signify a stronger preference for a particular alternative. Subsequently, alternatives are ranked in descending order based on these coefficients, with the highest coefficient corresponding to the most desirable option. This adaptation of TOPSIS to neutral sets enhances its capability to handle uncertainty and imprecision, providing a more nuanced and flexible approach to decision-making under diverse conditions.

Step 7. Using equation (32), in the context of the presented maintenance strategy selection problem, the ranking orders (R_i) of alternatives (A_i) obtained through the TOPSIS analysis using (L1), (L2), and (L∞) norms are as follows:

R_i	Ranking orders of alternatives
$R(L_1)$	$A_1 \succ A_2 \succ A_5 \succ A_7 \succ A_4 \succ A_6 \succ A_3$
$R(L_2)$	$A_1 \succ A_2 \succ A_5 \succ A_7 \succ A_4 \succ A_6 \succ A_3$
$R(L_\infty)$	$A_1 \succ A_7 \succ A_4 \succ A_2 \succ A_5 \succ A_6 \succ A_3$

The ranking results highlight that alternative (A_1) boasts the highest relative closeness coefficient to the ideal solution within the neutral set, signifying its suitability for the given decision scenario. This study presents an extended Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) approach based on neutral sets (NSs), aimed at ranking alternatives amidst ambiguity and imprecision.

The application of this methodology to a real-world maintenance selection problem underscores its efficacy in handling the inconsistencies or indeterminacies often inherent in such evaluations. Moreover, the inherent flexibility and adaptability of neutral sets suggest their potential as a potent tool for navigating the diverse uncertainties encountered in Multiple Criteria Decision Making (MCDM) problems.

IV. Conclusion

In conclusion, this paper has introduced a novel mathematical tool, the Neutral Set (NS) with three independent components for Possibility, Indeterminacy, and Necessity, as an effective means to handle uncertain and imprecise information, particularly in the context of multiple attribute group decision-making (MAGDM). By extending the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method to incorporate NSs, a framework has been provided to address decision-making problems under neutral environments, where decision-makers express attribute weights and values using neutral numbers.

The application of this extended TOPSIS method to the maintenance strategy selection problem exemplifies its utility in real-world scenarios, showcasing its adaptability to various domains including information project selection, material selection, and other complex engineering and management decision-making contexts. Moving forward, further research and application of the proposed method hold promise for enhancing decision-making processes by accommodating the inherent uncertainties and imprecisions inherent in real-world data.

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