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Modeling Uncertainty in Multiple Criteria Decision Making Using the Technique for Order Preference by Similarity to Ideal Solution for the Selection of Stealth Combat Aircraft

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Abstract**—**Uncertainty set theory is a generalization of fuzzy set theory and intuitionistic fuzzy set theory. It serves as an effective tool for dealing with inconsistent, imprecise, and vague information. The technique for order preference by similarity to ideal solution (TOPSIS) method is a multiple-attribute method used to identify solutions from a finite set of alternatives. It simultaneously minimizes the distance from an ideal point and maximizes the distance from a nadir point. In this paper, an extension of the TOPSIS method for multiple attribute group decision-making (MAGDM) based on uncertainty sets is presented. In uncertainty decision analysis, decision-makers express information about attribute values and weights using uncertainty numbers to select the best stealth combat aircraft.

Keywords**—**Uncertainty set, stealth combat aircraft selection multiple criteria decision-making analysis, MCDM, uncertainty decision analysis, TOPSIS.

I. Introduction

Multiple attribute decision making (MADM) is a crucial component of decision science that plays a significant role in various domains such as economics, engineering, and social sciences. Decision-makers often face challenges in expressing their preferences accurately when dealing with MADM problems that involve imprecise, vague, or incomplete information. Over the past few decades, a variety of set theories have been introduced and widely utilized to address significant decision-making problems. These include fuzzy sets [1], interval-valued fuzzy sets [2], intuitionistic fuzzy sets [3-4], interval-valued intuitionistic fuzzy sets [5], type 2 fuzzy sets [6], hesitant fuzzy sets [7], picture fuzzy sets [8], neutrosophic sets [9], and uncertainty sets [10].

The uncertainty set theory, an extension of intuitionistic fuzzy sets, offers a practical tool for handling indeterminate and inconsistent information commonly encountered in real-world scenarios. An uncertainty set comprises three components: the truth membership $\mu_\gamma(x)$, the indeterminacy membership $\eta_\gamma(x)$, and the falsity membership $\nu_\gamma(x)$.

The Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) was initially developed to solve MADM problems [11]. It is based on the principle that the selected alternative should be closest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). In recent years, numerous MADM methods and multiple attribute group decision-making (MAGDM) methods based on extensions of the TOPSIS method have been proposed. Chen [12] introduced extensions of the TOPSIS method for group decision-making under a fuzzy environment. This work laid the foundation for incorporating fuzzy sets into the TOPSIS method to handle uncertainty in decision-making processes.

Jin et al. [13] conducted an evaluation study of human resources using intuitionistic fuzzy sets and the TOPSIS method. This work expanded the application of the TOPSIS method to handle intuitionistic fuzzy sets, providing a more comprehensive approach for decision-making.

Wei and Liu [14] proposed a risk evaluation method for high-technology based on uncertain linguistic variables and the TOPSIS method. This work addressed the need for handling linguistic variables in risk evaluation, showcasing the versatility of the TOPSIS method in diverse decision-making scenarios.

Liu [15] conducted research on a multi-attribute decision-making method based on interval vague sets and the TOPSIS method. This work contributed to the integration of interval vague sets into the TOPSIS method, enabling decision-makers to handle vague and imprecise information effectively.

Liu and Su [16] presented an extended TOPSIS method based on trapezoid fuzzy linguistic variables. By incorporating trapezoid fuzzy linguistic variables, this work enhanced the TOPSIS method's capability to deal with linguistic uncertainties in decision-making processes.

Verma et al. [17] presented a facility location selection approach using an interval-valued intuitionistic fuzzy TOPSIS method. This work demonstrated the application of the TOPSIS method with interval-valued intuitionistic fuzzy sets in facility location selection, highlighting its effectiveness in real-world decision-making problems.

Liu [18] proposed an extended TOPSIS method for multiple attribute group decision-making based on generalized interval-valued trapezoidal fuzzy numbers. By introducing generalized interval-valued trapezoidal fuzzy numbers, this work expanded the applicability of the TOPSIS method to a broader range of decision-making scenarios.

Chi and Liu [19] developed an extended TOPSIS method for multiple attribute decision-making problems based on interval neutrosophic sets. This work introduced the concept

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of neutrosophic sets to the TOPSIS method, offering a new perspective for handling complex decision-making situations.

Liu and Wang [20] developed a multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. This work introduced the concept of single-valued neutrosophic sets to the TOPSIS method, offering a new approach for decision-making under uncertainty.

Radulescu and Radulescu [21] introduced an extended TOPSIS approach for ranking cloud service providers. This work applied the TOPSIS method to the context of cloud service provider ranking, showcasing the versatility of the method in various domains.

Nafei et al. [22] proposed a group multiple attribute decision-making method based on interval neutrosophic sets. This work extended the TOPSIS method to handle group decision-making scenarios using interval neutrosophic sets, contributing to collaborative decision-making processes.

This study proposes an extension of the TOPSIS method for Multiple Attribute Group Decision Making (MAGDM) under uncertainty. The proposed approach utilizes uncertainty numbers (UNs) to represent both attribute values and weights provided by decision-makers. These UNs are then aggregated into a single decision matrix for subsequent analysis using TOPSIS with (L1), (L2), and (L∞) norms.

The remainder of this research is organized as follows: Section 2 provides a brief overview of uncertainty sets, their operational rules, and distance calculation methods. It then outlines an extension of the TOPSIS method for Multiple Attribute Group Decision Making (MAGDM) under uncertainty for ranking alternatives. Section 3 demonstrates the validity and effectiveness of the proposed approach through a numerical example for selecting stealth combat aircraft. Finally, Section 4 concludes the study with a summary of its key findings.

II. Methodology

This section provides a brief review of particular preliminaries regarding uncertainty sets, the distance between uncertainty sets (USs) and some other important concepts of multiple attribute decision making (MADM) and TOPSIS method [23-63].

Definition 1. [1] Let X be a fixed set in a universe of discourse. A fuzzy set Γ in X is an object having the form

$$\Gamma=\left\{< x,\mu_{\gamma}(x) >| \, x\in X\right\} \tag{1}$$

where $\mu_{\gamma}(x): X \rightarrow [0,1]$ represents a membership function of Γ with the condition $0 \le \mu_{\gamma}(x) \le 1$ for all $x \in X$.

Definition 2. [3] Let X be a fixed set in a universe of discourse. An intuitionistic fuzzy set Γ in X is an object having the form

$$\Gamma=\left\{< x,\mu_{\gamma}(x),\nu_{\gamma}(x) >| \, x\in X\right\} \tag{2}$$

where $\mu_{\gamma}(x): X \rightarrow [0,1]$ represents a membership function , a non-membership function $\nu_{\gamma}: X \rightarrow [0,1]$, and a hesitancy degree $\pi_{\gamma}(x) = 1-\mu_{\gamma}(x)+\nu_{\gamma}(x)$ of Γ with the condition $0 \le \mu_{\gamma}(x)+\nu_{\gamma}(x) \le 1$, for all $x \in X$.

Definition 3. [8] Let X be a fixed set in a universe of discourse. A picture fuzzy set Γ in X is an object having the form

$$\Gamma=\left\{< x,\mu_{\gamma}(x),\nu_{\gamma}(x) >| \, x\in X\right\} \tag{3}$$

where $\mu_{\gamma}(x): X \rightarrow [0,1]$ represents a positive membership function , a neutral membership function $\eta_{\gamma}: X \rightarrow [0,1]$, a negative membership function $\nu_{\gamma}: X \rightarrow [0,1]$ and a refusal degree $\rho_{\gamma}(x = 1-\mu_{\gamma}(x)+\eta_{\gamma}(x)+\nu_{\gamma}(x)$ of Γ with the condition $0 \le \mu_{\gamma}(x)+\eta_{\gamma}(x)+\nu_{\gamma}(x) \le 1$ for all $x \in X$.

Definition 4. [10] Let X be a fixed set in a universe of discourse. An uncertainty set Γ in X is an object having the form

$$\Gamma=\left\{< x,\mu_{\gamma}(x),\nu_{\gamma}(x) >| \, x\in X\right\} \tag{4}$$

where $\mu_{\gamma}(x): X \rightarrow [0,1]$ represents a truth membership function, an indeterminacy membership function $\eta_{\gamma}: X \rightarrow [0,1]$, and a falsity membership function $\nu_{\gamma}: X \rightarrow [0,1]$ of Γ with the condition $0 \le \mu_{\gamma}(x)+\eta_{\gamma}(x)+\nu_{\gamma}(x) \le 1$ for all $x \in X$.

Definition 5. Given two uncertainty sets
$A=\left\{< x,\mu_A(x),\eta_A(x),\nu_A(x) >| \, \forall x\in X\right\}$ and
$B=\left\{< (x,\mu_B(x),\eta_B(x),\nu_B(x) >| \, \forall x\in X\right\}$, the relations are defined as follows:

a) $$A \subseteq B \textit{ if and only if } \mu_A(x) \le \mu_B(x), \quad \eta_A(x) \ge \eta_B(x), \nu_A(x) \ge \nu_B(x) \tag{5}$$

b) $$A \subseteq B \textit{ if and only if } \mu_A(x) = \mu_B(x), \quad \eta_A(x) = \eta_B(x), \nu_A(x) = \nu_B(x) \tag{6}$$

c) $$A \cap B = \begin{Bmatrix} < x,(\mu_A \wedge \mu_B)(x), \\ (\eta_A \vee \eta_B)(x),(\nu_A \vee \nu_B)(x) >| \, \forall x \in X \end{Bmatrix} \tag{7}$$

d) $$A \cup B = \begin{Bmatrix} < x,(\mu_A \vee \mu_B)(x), \\ (\eta_A \wedge \eta_B)(x),(\nu_A \wedge \nu_B)(x) >| \, \forall x \in X \end{Bmatrix} \tag{8}$$

e) $$\overline{A}=\left\{< x,\nu_A(x),\eta_A(x),\mu_A(x) >| \, \forall x\in X\right\} \tag{9}$$

where the symbol $\wedge$ represents the t-norm, while $\vee$ represents the t-conorm.

Definition 6. For $\lambda > 0$, the corresponding operations for three indeterminacy numbers (INs) $A_1 = (\mu_{A_1}, \eta_{A_1}, \nu_{A_1})$,

$A_2 = (\mu_{A_2}, \eta_{A_2}, \nu_{A_2})$, and $A = (\mu_A, \eta_A, \nu_A)$ are defined as follows:

a) $A_1 \oplus A_2 = (\mu_{A_1} + \mu_{A_2} - \mu_{A_1}\mu_{A_2}, \eta_{A_1}\eta_{A_2}, \nu_{A_1}\nu_{A_2})$ (10)

b) $A_1 \otimes A_2 = (\mu_{A_1}\mu_{A_2}, \eta_{A_1} + \eta_{A_2} - \eta_{A_1}\eta_{A_2}, \nu_{A_1} + \nu_{A_2} - \nu_{A_1}\nu_{A_2})$ (11)

c) $\lambda A = (1-(1-\mu_A)^\lambda, \eta_A^\lambda, \nu_A^\lambda)$ (12)

d) $A^\lambda = (\mu_A^\lambda, 1-(1-\eta_A)^\lambda, 1-(1-\nu_A)^\lambda)$ where $\lambda > 0$ (13)

e) $0_i^+ = \{<(x,1,0,0)>| \forall x \in X\}$ (14)

e) $0_i^- = \{<(x,0,1,1)>| \forall x \in X\}$ (15)

Definition 7. For any uncertainty number (UN) $A = (\mu_A, \eta_A, \nu_A)$, the score $s(A)$, the accuracy $h(A)$, and the certainty $c(A)$ functions of A are defined as:

$$s(A) = (\mu_A - \nu_A) \quad (16)$$

$$h(A) = \mu_A + \eta_A + \nu_A \quad (17)$$

$$c(A) = \mu_A \quad (18)$$

where $s(A) \in [-1,1]$ and $h(A) \in [0,1]$. For any UNs A_1 and A_2

1. If $s(A_1) > s(A_2)$, *then* $(A_1) > (A_2)$,

2. If $s(A_1) = s(A_2)$, *then*

i. If $\mathrm{h(A_1)>h(A_2)} \Rightarrow \mathrm{A_1 > A_2}$

ii. If $\mathrm{h(A_1)=h(A_2)}$, *then* $\mathrm{A_1 \approx A_2}$

Definition 8. Let $a_i (i = 1,2,...,n)$ be a collection of uncertainty numbers (UNs) and $\omega = [\omega_1, \omega_2, ..., \omega_n]^T$ is the weight vector of a_i with the condition $\omega_i > 0$ and $\sum_{i=1}^n \omega_i = 1$. Then, the uncertainty weighted averaging (UWA) operator is a mapping $A^n \to A$ such that

$$UWA(a_1, a_2, ..., a_n) = \oplus_{i=1}^n (\omega_i a_i)$$

$$UWA(a_1, a_2, ..., a_n) = \begin{pmatrix} 1 - \prod_{i=1}^n (1-\mu_{a_i})^{\omega_i}, \\ \prod_{i=1}^n (\eta_{a_i})^{\omega_i}, \prod_{i=1}^n (\nu_{a_i})^{\omega_i} \end{pmatrix} \quad (19)$$

Definition 9. Let $a_i (i = 1,2,...,n)$ be a collection of uncertainty numbers (UNs) and $\omega = [\omega_1, \omega_2, ..., \omega_n]^T$ is the weight vector of a_i with the condition $\omega_i > 0$ and $\sum_{i=1}^n \omega_i = 1$. Then, the uncertainty weighted geometric (UWG) operator is a mapping $A^n \to A$ such that

$$UWG(a_1, a_2, ..., a_n) = \otimes_{i=1}^n (a_i^{\omega_i})$$

$$UWG(a_1, a_2, ..., a_n) = \begin{pmatrix} \prod_{i=1}^n (\mu_{a_i})^{\omega_i}, 1 - \prod_{i=1}^n (1-\eta_{a_i})^{\omega_i}, \\ 1 - \prod_{i=1}^n (1-\nu_{a_i})^{\omega_i} \end{pmatrix} \quad (20)$$

Definition 10. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their distance L_1 is defined as:

$$d_{L_1}(A,B) = \frac{1}{n} \sum_{i=1}^n \begin{pmatrix} |\mu_A(x_i) - \mu_B(x_i)| + \\ |\eta_A(x_i) - \eta_B(x_i)| + \\ |\nu_A(x_i) - \nu_B(x_i)| \end{pmatrix} \quad (21)$$

Definition 11. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their weighted distance L_1 is defined as:

$$d_{L_1}(A,B) = \frac{1}{n} \sum_{i=1}^n \omega_i \begin{pmatrix} |\mu_A(x_i) - \mu_B(x_i)| + \\ |\eta_A(x_i) - \eta_B(x_i)| + \\ |\nu_A(x_i) - \nu_B(x_i)| \end{pmatrix} \quad (22)$$

Definition 12. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their distance L_2 is defined as:

$$d_{L_2}(A,B) = \sqrt{\frac{1}{n} \sum_{i=1}^n \begin{bmatrix} (\mu_A(x_i) - \mu_B(x_i))^2 + \\ (\eta_A(x_i) - \eta_B(x_i))^2 + \\ (\nu_A(x_i) - \nu_B(x_i))^2 \end{bmatrix}} \quad (23)$$

Definition 13. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their weighted distance L_2 is defined as:

$$d_{L_2}(A,B) = \sqrt{\frac{1}{n} \sum_{i=1}^n \omega_i \begin{bmatrix} (\mu_A(x_i) - \mu_B(x_i))^2 + \\ (\eta_A(x_i) - \eta_B(x_i))^2 + \\ (\nu_A(x_i) - \nu_B(x_i))^2 \end{bmatrix}} \quad (24)$$

Definition 14. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their distance L_∞ is defined as:

$$d_{L_\infty}(A,B) = \sum_{i=1}^n \max \begin{pmatrix} |\mu_A(x_i) - \mu_B(x_i)|, \\ |\eta_A(x_i) - \eta_B(x_i)|, \\ |\nu_A(x_i) - \nu_B(x_i)| \end{pmatrix} \quad (25)$$

Definition 15. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their weighted distance L_∞ is defined as:

$$d_{L_\infty}(A,B) = \sum_{i=1}^n \max \left[\omega_i \begin{pmatrix} |\mu_A(x_i) - \mu_B(x_i)|,| \\ \eta_A(x_i) - \eta_B(x_i)|,| \\ \nu_A(x_i) - \nu_B(x_i)| \end{pmatrix} \right] \quad (26)$$

Definition 16. Given an uncertainty decision matrix $J_a = [J_{a\,ij}]_{mxn}$ with uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$.The positive ideal solution J^+ and the negative ideal solution J^- of the attributes $c_i \in C(i=1,2,...,m)$ from the aggregated group decision matrix $[A_{ij}]_{mxn}$;

$$J^+ = \left(\mu_j^+, \eta_j^+, \nu_j^+\right) = \begin{pmatrix} \max\limits_j \mu_{ij}, \min\limits_j \eta_{ij}, \min\limits_j \nu_{ij} \\ \mid j=1,2,...,n \end{pmatrix} \tag{27}$$

$$J^- = \left(\mu_i^-, \eta_i^-, \nu_i^-\right) = \begin{pmatrix} \min\limits_i \mu_{ij}, \max\limits_i \eta_{ij}, \max\limits_i \nu_{ij} \\ \mid i=1,2,...,m \end{pmatrix} \tag{28}$$

Definition 17. Let $X=\{x_1,x_2,...,x_m\}$ be a set of alternatives, $C=\{(c_1,c_2,...,c_m\}$ be the set of attributes. The ratings of alternatives $x_j \in X(j=1,2,...,n)$ on attributes $c_i \in C$ are expressed with uncertainty number $A_{ij}=\left\langle \mu_{ij},\eta_{ij},\nu_{ij}\right\rangle$. Then,

$$[A_{ij}]_{mxn} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & & X_n \end{array} \\ \begin{pmatrix} \langle \mu_{11},\eta_{11},\nu_{11}\rangle & \langle \mu_{12},\eta_{12},\nu_{12}\rangle & \cdots & \langle \mu_{1n},\eta_{1n},\nu_{1n}\rangle \\ \langle \mu_{21},\eta_{21},\nu_{21}\rangle & \langle \mu_{22},\eta_{22},\nu_{22}\rangle & \cdots & \langle \mu_{2n},\eta_{2n},\nu_{2n}\rangle \\ \vdots & \vdots & \cdots & \vdots \\ \langle \mu_{m1},\eta_{m1},\nu_{m1}\rangle & \langle \mu_{m2},\eta_{m2},\nu_{m2}\rangle & \cdots & \langle \mu_{mn},\eta_{mn},\nu_{mn}\rangle \end{pmatrix} \end{array}$$

$[A_{ij}]_{mxn}$ is called a multiple criteria decision-making matrix. The importance weight vector of attribute set $C=\{(c_1,c_2,...,c_m\}$ is given as:

$$\omega_i=(\omega_1,\omega_2,...,\omega_m)=\left(\langle\alpha_1,\beta_1,\gamma_1\rangle,\langle\alpha_2,\beta_2,\gamma_2\rangle,....,\langle\alpha_m,\beta_m,\gamma_m\rangle\right)$$

The initial decision matrix is normalized $[A_{ij}]_{mxn}$: benefit-type attributes remain the same, while the cost-type attributes are transformed into benefit-type according to Equation (9).

$$[A_{ij}]_{mxn} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & & X_n \end{array} \\ \begin{pmatrix} \langle \mu_{11},\eta_{11},\nu_{11}\rangle & \langle \mu_{12},\eta_{12},\nu_{12}\rangle & \cdots & \langle \mu_{1n},\eta_{1n},\nu_{1n}\rangle \\ \langle \mu_{21},\eta_{21},\nu_{21}\rangle & \langle \mu_{22},\eta_{22},\nu_{22}\rangle & \cdots & \langle \mu_{2n},\eta_{2n},\nu_{2n}\rangle \\ \vdots & \vdots & \cdots & \vdots \\ \langle \mu_{m1},\eta_{m1},\nu_{m1}\rangle & \langle \mu_{m2},\eta_{m2},\nu_{m2}\rangle & \cdots & \langle \mu_{mn},\eta_{mn},\nu_{mn}\rangle \end{pmatrix} \end{array}$$

Then, the weighted normalized multiple criteria decision-making matrix $[A_{ij}]_{mxn}=\omega_i[A_{ij}]_{mxn}$ is presented as:

$$[A_{ij}]_{mxn} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & & X_n \end{array} \\ \begin{pmatrix} \langle \mu_{11},\eta_{11},\nu_{11}\rangle & \langle \mu_{12},\eta_{12},\nu_{12}\rangle & \cdots & \langle \mu_{1n},\eta_{1n},\nu_{1n}\rangle \\ \langle \mu_{21},\eta_{21},\nu_{21}\rangle & \langle \mu_{22},\eta_{22},\nu_{22}\rangle & \cdots & \langle \mu_{2n},\eta_{2n},\nu_{2n}\rangle \\ \vdots & \vdots & \cdots & \vdots \\ \langle \mu_{m1},\eta_{m1},\nu_{m1}\rangle & \langle \mu_{m2},\eta_{m2},\nu_{m2}\rangle & \cdots & \langle \mu_{mn},\eta_{mn},\nu_{mn}\rangle \end{pmatrix} \end{array}$$

where

$$\left\langle \mu_{ij},\eta_{ij},\nu_{ij}\right\rangle = \omega_i A_{ij} = \langle\alpha_i,\beta_i,\gamma_i\rangle \left\langle \mu_{ij},\eta_{ij},\nu_{ij}\right\rangle$$
$$= \left\langle \alpha_i\mu_{ij}, \beta_i+\eta_{ij}-\beta_i\eta_{ij}, \gamma_i+\nu_{ij}-\gamma_i\nu_{ij}\right\rangle$$

Based on the obtained weighted normalized decision matrices D1, D2, D3, and equation (12), the aggregated group decision matrix $[A_{ij}]_{7x3}$ of all decision-makers is constructed as:

$$[A_{ij}]_{mxn} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & & X_n \end{array} \\ \begin{pmatrix} \langle \mu_{11},\widehat{\eta}_{11},\nu_{11}\rangle & \langle \mu_{12},\widehat{\eta}_{12},\widehat{\nu}_{12}\rangle & \cdots & \langle \mu_{1n},\widehat{\eta}_{1n},\widehat{\nu}_{1n}\rangle \\ \langle \mu_{21},\widehat{\eta}_{21},\nu_{21}\rangle & \langle \mu_{22},\widehat{\eta}_{22},\widehat{\nu}_{22}\rangle & \cdots & \langle \mu_{2n},\widehat{\eta}_{2n},\widehat{\nu}_{2n}\rangle \\ \vdots & \vdots & \cdots & \vdots \\ \langle \mu_{m1},\widehat{\eta}_{m1},\widehat{\nu}_{m1}\rangle & \langle \mu_{m2},\widehat{\eta}_{m2},\widehat{\nu}_{m2}\rangle & \cdots & \langle \mu_{mn},\widehat{\eta}_{mn},\widehat{\nu}_{mn}\rangle \end{pmatrix} \end{array}$$

Then, the positive ideal solution J^+ and the negative ideal solution J^- of the attributes $c_i \in C(i=1,2,...,m)$ from the aggregated group decision matrix $[A_{ij}]_{mxn}$ according to the Definition 16;

$$J^+ = \left(\mu_j^+, \eta_j^+, \nu_j^+\right) = \begin{pmatrix} \max\limits_j \mu_{ij}, \min\limits_j \eta_{ij}, \min\limits_j \nu_{ij} \\ \mid j=1,2,...,n \end{pmatrix}$$

$$J^- = \left(\mu_i^-, \eta_i^-, \nu_i^-\right) = \begin{pmatrix} \min\limits_i \mu_{ij}, \max\limits_i \eta_{ij}, \max\limits_i \nu_{ij} \\ \mid i=1,2,...,m \end{pmatrix}$$

The distances from ideal solutions (d_j^+ , d_j^-) are calculated according to Definitions 11, 13 and 15;

Finally, the alternatives are ranked using the relative closeness coefficients $d_j = \dfrac{d_j^-}{d_j^- + d_j^+}$ according to Definitions 11, 13 and 15. The higher the relative closeness coefficient d_j, the better the alternative.

Definition 19. Let $X=\{x_1,x_2,...,x_n\}$ be a set of alternatives, $C=\{(c_1,c_2,...,c_m\}$ be a set of attributes, and $\omega_i=\langle\alpha_i,\beta_i,\gamma_i\rangle_{mx1}$ be a weighted vector for attributes. Hence, the algorithm for ranking alternatives is presented:

Algorithm: Uncertainty TOPSIS (UTOPSIS)

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a widely used method for solving Multiple Attribute Decision Making (MADM) and Multi-Attribute Group Decision Making (MAGDM) problems. Its core principle lies in identifying the alternative that is closest to a hypothetical "positive ideal solution" (PIS) and farthest from a hypothetical "negative ideal solution" (NIS).

TOPSIS Theory: The traditional TOPSIS method follows these steps [11]:

Decision Matrix: A decision matrix is constructed where rows represent alternatives and columns represent criteria. Each cell contains a performance value for an alternative on a specific attribute.

Normalization: All criteria in the decision matrix are normalized to ensure a common scale for comparison.

Weighted Normalized Decision Matrix: Weights are assigned to each attribute, reflecting their relative importance. These weights are then multiplied with the corresponding normalized performance values.

Ideal Solutions: The positive ideal solution (PIS) is constructed by taking the highest normalized weighted value for each attribute across all alternatives. Conversely, the negative ideal solution (NIS) is constructed by taking the lowest normalized weighted value for each attribute.

Distances from Ideal Solutions: The distances between each alternative and both the PIS and NIS are calculated (often using Euclidean distance).

Similarity to Ideal Solution: A "closeness coefficient" is calculated for each alternative. This coefficient represents the relative closeness of an alternative to the PIS compared to its similarity to the NIS. Alternatives with the highest closeness coefficient are considered the most desirable solutions.

Ranking Alternatives: Alternatives are ranked in descending order based on their relative closeness coefficient.

The steps of the extended uncertainty TOPSIS (UTOPSIS) approach are briefly outlined here:

Step 1: Input initial decision-making matrices $[A_{ij}]_{mxn}$;

Step 2: Determine the weighted vector for attributes $\omega_i = \langle \alpha_i, \beta_i, \gamma_i \rangle_{mx1}$;

Step 3: Normalize the initial decision-making matrices $[A_{ij}]_{mxn}$ according to the equation (10);

Step 4: Calculate the weighted normalized decision-making matrices $[A_{ij}]_{mxn}$;

Step 5: Calculate the aggregated group decision matrix $[A_{ij}]_{mxn}$

Step 6: Determine the positive ideal solution J^+ and the negative ideal solution J^- of the attributes $c_i \in C (i = 1, 2, ..., m)$ from the aggregated group decision matrix $[A_{ij}]_{mxn}$ according to the Definition 16;

$$J^+ = \left(\mu_i^+, \eta_i^+, \nu_i^+\right) = \begin{pmatrix} \max_i \mu_{ij}, \min_i \eta_{ij}, \min_i \nu_{ij} \\ \mid i = 1, 2, ..., m \end{pmatrix}$$

$$J^- = \left(\mu_i^-, \eta_i^-, \nu_i^-\right) = \begin{pmatrix} \min_i \mu_{ij}, \max_i \eta_{ij}, \max_i \nu_{ij} \\ \mid i = 1, 2, ..., m \end{pmatrix}$$

Step 7: Determine the distances from ideal solutions (d_j^+, d_j^-) according to Definitions 11, 13 and 15;

Step 8: Rank the alternatives using the relative closeness coefficients d_j according to Definitions 11, 13 and 15.

III. Application

This section presents a model for uncertainty in Multiple Criteria Decision Making (MCDM) applied to stealth combat aircraft selection. The model utilizes an uncertainty-based TOPSIS method employing (L1), (L2), and (L∞) norms.

A. Stealth Combat Aircraft Background

Stealth combat aircraft are designed with advanced technologies to reduce their radar cross-section and make them less visible to radar detection systems. These aircraft incorporate features such as special coatings, angular shapes, and internal weapon bays to minimize their radar signature. The goal of stealth technology is to enhance the aircraft's survivability and effectiveness in combat situations by reducing the likelihood of detection by enemy radar systems.

The dream of stealth aircraft, gliding through enemy airspace unseen and unheard, striking with surgical precision before vanishing into the night, began with the unveiling of the F-117 Nighthawk stealth combat aircraft. This revolutionary aircraft broke the mold of conventional fighter design, introducing an angular, faceted appearance that scattered radar waves, making the aircraft appear smaller and less distinct. Thus marked the birth of stealth technology, a suite of innovative methods to reduce an aircraft's signature across various spectrums. Stealth technology was developed in response to advancements in air defense systems. As radar technology advanced, traditional aircraft became increasingly vulnerable to detection and interception. Stealth suggested a solution, shifting the balance of power in favor of attackers.

An undetected aircraft could penetrate enemy air defenses with significantly higher survivability, deliver its payload with greater precision, and escape unscathed, offering a significant advantage in air-to-air and air-to-ground operations. However, stealth is a complex and expensive technology requiring specialized materials, intricate design, and operational limitations. Nonetheless, the need for air defense supremacy continues to drive innovation in stealth aircraft.

The B-2 Spirit, introduced as the successor to the F-117 Nighthawk, further pushed the boundaries of stealth technology with its advanced shaping, radar-absorbent materials, and heat management systems. This marked a significant leap forward in stealth capabilities, setting the stage for subsequent advancements. The F-22 Raptor and the F-35 Lightning II followed suit, each incorporating lessons learned from previous stealth aircraft designs while introducing their own advancements. These aircraft not only enhanced stealth features but also integrated cutting-edge sensor technologies to maintain superiority in an evolving battlespace.

As the landscape of aerial warfare continues to evolve, new players like the Chengdu J-20, the Sukhoi Su-57, and the TAI TF KAAN National Combat Aircraft (MMU-Milli Muharip Uçak) have entered the scene, raising the stakes in the race for air dominance. With adversaries constantly refining their detection capabilities, the imperative for stealth aircraft development intensifies. The quest for air defense supremacy propels the continual evolution of stealth technology. Operational strategies encompass not only the refinement of physical stealth attributes but also the integration of advanced sensors to counter emerging detection methods.

As nations vie for dominance in contested airspace, the pursuit of stealth becomes inexorably linked with the broader objective of ensuring aerial superiority. Stealth aircraft are meticulously engineered machines designed to minimize their detectability across multiple spectra, including radar, infrared, visible light, radio frequency, and audio emissions.

The core objective of stealth aircraft lies in evading enemy detection systems, thereby enhancing survivability and mission success rates. While achieving complete invisibility remains elusive, stealth aircraft substantially reduce their radar cross-section and other observable signatures, confounding conventional detection mechanisms. This feat is accomplished through a sophisticated blend of passive low observable features and active emitters, including low-probability-of-intercept radars and specialized radio and laser systems.

B. Stealth Combat Aircraft Design Characteristics

The stealth combat aircraft design characteristics cover a wide range of essential aspects involved in designing an aircraft with minimal detectability across various spectrums. These characteristics work together to create a complex and effective system for minimizing a combat aircraft's detectability, making it a valuable tool for various military operations:

Radar-Absorbent Materials (RAM): These specialized coatings, often comprising iron ferrite or carbon nanomaterials, are designed to absorb radar waves rather than reflect them. This feature significantly reduces the aircraft's Radar Cross Section (RCS), enhancing its stealth capabilities.

Shape Optimization: By incorporating specific geometric features such as leading-edge serrations and blended wing-body designs, stealth aircraft minimize radar wave reflections, thus adopting a stealthier shape that reduces detectability.

Heat Management: Effective heat management techniques are employed to mitigate the thermal signature of stealth aircraft. Engine exhaust, a significant source of infrared radiation, is carefully channeled and cooled using advanced materials and engine designs to minimize its detectability.

Acoustic Signature Reduction: Stealth aircraft utilize noise-dampening technologies such as engine enclosures and specialized inlets to minimize their acoustic signature. This reduces the aircraft's detectability by acoustic sensors, further enhancing its stealth capabilities.

Signature Management Coatings: Stealth aircraft may employ specialized coatings applied to external surfaces to further reduce their radar, infrared, and visual signatures. These coatings can contain materials that absorb or scatter electromagnetic radiation and reduce the aircraft's visibility to sensors and human observers.

Distributed Aperture Systems (DAS): DAS consists of multiple sensors distributed around the aircraft, providing 360-degree coverage for situational awareness and missile warning. These systems enhance the aircraft's ability to detect and track threats while minimizing its own detectability by reducing the need for traditional external sensors.

Spectral Signature Management: Stealth aircraft may incorporate design features and materials to manipulate their spectral signature across different wavelengths of electromagnetic radiation. This includes optimizing surface properties to reduce reflectivity and emissivity in specific spectral bands, such as visible light, infrared, and ultraviolet.

Stealth Coatings and Treatments: In addition to radar-absorbent materials (RAM), stealth aircraft may use coatings and treatments to reduce their visual and infrared signatures. These coatings can include materials that absorb or scatter light and heat, making the aircraft less visible and reducing its thermal emissions.

Signature-Reconfigurable Design: Some stealth aircraft feature a signature-reconfigurable design, allowing them to adjust their radar and infrared signatures dynamically in response to changing operational requirements or threats. This capability enhances survivability by providing flexibility in signature management during mission execution.

Electromagnetic Compatibility (EMC): Stealth aircraft incorporate EMC design principles to minimize their electromagnetic emissions and susceptibility to electronic warfare. This includes shielding sensitive electronics, isolating electromagnetic interference sources, and employing techniques to reduce electromagnetic emissions during transmission.

Low-Observable External Stores: When external stores are necessary, stealth aircraft may utilize low-observable configurations and attachment methods to minimize their impact on the aircraft's overall signature. This includes streamlined shapes, radar-absorbent materials, and reduced radar cross-section designs for external weapons and fuel tanks.

C. Selection Criteria for Stealth Combat Aircraft

Selecting the optimal stealth combat aircraft entails a thorough evaluation of various factors to ensure alignment with the military's operational requirements. A critical aspect of this process involves defining the aircraft's primary purpose, whether it be air-to-air combat, ground attack, reconnaissance, or a multirole capability encompassing a combination of roles. This determination significantly influences the selection criteria, as different missions necessitate specific capabilities tailored to their unique demands. Moreover, political and international considerations may exert influence, with factors such as diplomatic relations and potential alliances potentially impacting the selection process. Collaboration with partner nations, technology transfer agreements, and geopolitical dynamics all play a role in shaping the decision-making framework.

Industrial and technological factors also weigh heavily in the selection process. Assessing the aircraft's contribution to the domestic aerospace industry, including its potential for technology transfer, job creation, and fostering research and development opportunities, is crucial for long-term strategic planning. Overall, the selection criteria for stealth combat aircraft encompass a comprehensive array of factors, each carefully weighed and considered to ensure the chosen aircraft effectively meets operational needs while also aligning with broader political, industrial, and technological objectives. Some of the key criteria for selecting stealth combat aircraft include:

Stealth Capability (C1): The primary criterion is the aircraft's ability to minimize its radar cross-section (RCS) and infrared signature to evade detection by enemy radar and sensors. This involves shaping the aircraft, using radar-absorbent materials, and reducing heat emissions to achieve low observability.

Performance Capability (C2): Evaluating speed, range, payload capacity, and maneuverability is essential. A stealth aircraft should excel in these areas while maintaining low observability.

Survivability (C3): This refers to the aircraft's ability to operate effectively in contested environments and withstand threats from sophisticated air defense systems and enemy aircraft. It involves assessing the aircraft's capability to evade detection, engage in combat, and endure in hostile environments. Factors considered include electronic warfare capabilities, countermeasures, and self-defense systems.

Avionics and Sensors (C4): These play a critical role in enabling effective target acquisition, tracking, and situational awareness. The aircraft's advanced avionics suite encompasses radar, electronic warfare systems, communication systems, and sensor fusion capabilities, all of which contribute to providing comprehensive situational awareness and enhancing mission effectiveness.

Interoperability (C5): This refers to the aircraft's ability to seamlessly integrate with other aircraft and military assets. This includes compatibility with existing and future military systems, such as command and control networks, data links, and other aircraft, enabling joint and coalition operations to be conducted effectively.

Operational Capability (C6): This encompasses tasks such as air superiority, ground attack, or reconnaissance, includes the aircraft's ability to effectively engage and destroy targets by carrying a variety of weapons, including air-to-air missiles, air-to-ground missiles, guided bombs, and other munitions. This also entails compatibility with a wide range of weapons, including air-to-air missiles, precision-guided munitions, and standoff weapons, ensuring that the aircraft can fulfill its mission requirements effectively.

Cost and Maintenance Affordability (C7): Balancing performance with cost-effectiveness is crucial, particularly considering the expense associated with stealth technology. Logistics and maintenance entail evaluating factors such as spare parts availability and ease of support to ensure operational efficiency.

In this study, the stealth combat aircraft selection problem was presented as an illustrative example to show its applicability and effectiveness in decision making problems.

Assume that $X = \{x_1, x_2, ..., x_n\}$ is a set of stealth combat aircraft alternatives, and $C = \{(c_1, c_2, ..., c_m\}$ is a set of attributes: Stealth Capability (C1), Performance Capability (C2), Survivability (C3), Avionics and Sensors (C4), Interoperability (C5), Operational Capability (C6), and Cost and Maintenance Affordability (C7). Attributes C1-C6 are of benefit type, while C7 is a cost type attribute.

In this group decision-making problem, a three-member decision-making committee with equal weights of importance from the National Ministry of Defense aims to select the best alternative stealth combat aircraft from three preselected alternatives, considering seven evaluation attributes. Utilizing criteria weights, such as the vector of importance weights for these attributes, enables decision-makers (Ds) to establish priorities in the decision-making process. Hence, the solution steps of the *Algorithm: Uncertainty TOPSIS (UTOPSIS)* according to the Definition 19 are presented as follows:

Step 1. Three initial decision-making matrices (D1,D2,D3) $[A_{ij}]_{7x3}$ are established as:

D1	X1	X2	X3
C1	<0.75,0.11,0.13>	<0.67,0.13,0.19>	<0.59,0.15,0.25>
C2	<0.85,0.07,0.07>	<0.45,0.09,0.45>	<0.57,0.05,0.37>
C3	<0.95,0.03,0.01>	<0.97,0.01,0.01>	<0.87,0.05,0.07>
C4	<0.79,0.07,0.13>	<0.77,0.07,0.15>	<0.67,0.15,0.17>
C5	<0.65,0.15,0.19>	<0.59,0.03,0.37>	<0.49,0.11,0.39>
C6	<0.95,0.01,0.03>	<0.55,0.15,0.29>	<0.77,0.05,0.17>
C7	<0.57,0.15,0.27>	<0.85,0.05,0.09>	<0.49,0.15,0.35>

D2	X1	X2	X3
C1	<0.71,0.15,0.13>	<0.65,0.15,0.19>	<0.95,0.01,0.03>
C2	<0.75,0.05,0.19>	<0.51,0.13,0.35>	<0.97,0.01,0.01>
C3	<0.85,0.01,0.13>	<0.73,0.03,0.23>	<0.77,0.05,0.17>
C4	<0.65,0.15,0.19>	<0.85,0.05,0.09>	<0.37,0.15,0.47>
C5	<0.91,0.03,0.05>	<0.47,0.13,0.39>	<0.79,0.05,0.15>
C6	<0.83,0.01,0.15>	<0.67,0.05,0.27>	<0.47,0.13,0.39>
C7	<0.69,0.11,0.19>	<0.89,0.05,0.05>	<0.65,0.13,0.21>

D3	X1	X2	X3
C1	<0.61,0.15,0.23>	<0.59,0.09,0.31>	<0.69,0.05,0.25>
C2	<0.75,0.09,0.15>	<0.45,0.03,0.51>	<0.87,0.01,0.11>
C3	<0.85,0.03,0.11>	<0.37,0.15,0.47>	<0.67,0.03,0.29>
C4	<0.37,0.17,0.45>	<0.31,0.19,0.49>	<0.91,0.01,0.07>
C5	<0.81,0.03,0.15>	<0.57,0.13,0.29>	<0.79,0.03,0.17>
C6	<0.77,0.09,0.13>	<0.79,0.11,0.09>	<0.37,0.13,0.49>
C7	<0.79,0.03,0.17>	<0.39,0.15,0.45>	<0.75,0.01,0.23>

Step 2. The importance weight vector of attributes is determined as:

$$\omega = \begin{pmatrix} \overset{C1}{\langle 0.45,0.27,0.25\rangle}, \overset{C2}{\langle 0.65,0.13,0.19\rangle}, \overset{C3}{\langle 0.73,0.11,0.13\rangle}, \overset{C4}{\langle 0.61,0.17,0.19\rangle}, \\ \overset{C5}{\langle 0.57,0.13,0.27\rangle}, \overset{C6}{\langle 0.51,0.13,0.33\rangle}, \overset{C7}{\langle 0.77,0.05,0.15\rangle} \end{pmatrix}$$

Step 3. Using the equation (10), the normalized decision matrices (D1,D2,D3) $[A_{ij}]_{7x3}$ are established as the basis for further analysis as:

D1	X1	X2	X3
C1	<0.75,0.11,0.13>	<0.67,0.13,0.19>	<0.59,0.15,0.25>
C2	<0.85,0.07,0.07>	<0.45,0.09,0.45>	<0.57,0.05,0.37>
C3	<0.95,0.03,0.01>	<0.97,0.01,0.01>	<0.87,0.05,0.07>
C4	<0.79,0.07,0.13>	<0.77,0.07,0.15>	<0.67,0.15,0.17>
C5	<0.65,0.15,0.19>	<0.59,0.03,0.37>	<0.49,0.11,0.39>
C6	<0.95,0.01,0.03>	<0.55,0.15,0.29>	<0.77,0.05,0.17>
C7	<0.27,0.15,0.57>	<0.09,0.05,0.85>	<0.35,0.15,0.49>

D2	X1	X2	X3
C1	<0.71,0.15,0.13>	<0.65,0.15,0.19>	<0.95,0.01,0.03>
C2	<0.75,0.05,0.19>	<0.51,0.13,0.35>	<0.97,0.01,0.01>
C3	<0.85,0.01,0.13>	<0.73,0.03,0.23>	<0.77,0.05,0.17>
C4	<0.65,0.15,0.19>	<0.85,0.05,0.09>	<0.37,0.15,0.47>
C5	<0.91,0.03,0.05>	<0.47,0.13,0.39>	<0.79,0.05,0.15>
C6	<0.83,0.01,0.15>	<0.67,0.05,0.27>	<0.47,0.13,0.39>
C7	<0.19,0.11,0.69>	<0.05,0.05,0.89>	<0.21,0.13,0.65>

D3	X1	X2	X3
C1	<0.61,0.15,0.23>	<0.59,0.09,0.31>	<0.69,0.05,0.25>
C2	<0.75,0.09,0.15>	<0.45,0.03,0.51>	<0.87,0.01,0.11>
C3	<0.85,0.03,0.11>	<0.37,0.15,0.47>	<0.67,0.03,0.29>
C4	<0.37,0.17,0.45>	<0.31,0.19,0.49>	<0.91,0.01,0.07>
C5	<0.81,0.03,0.15>	<0.57,0.13,0.29>	<0.79,0.03,0.17>
C6	<0.77,0.09,0.13>	<0.79,0.11,0.09>	<0.37,0.13,0.49>
C7	<0.17,0.03,0.79>	<0.45,0.15,0.39>	<0.23,0.01,0.75>

Step 4. Using the equation (11), the weighted normalized decision-making matrices (D1,D2,D3) $[A_{ij}]_{mxn}$ are found as:

D1	X1	X2	X3
C1	<0.34,0.35,0.35>	<0.30,0.36,0.39>	<0.27,0.38,0.44>
C2	<0.55,0.19,0.25>	<0.29,0.21,0.55>	<0.37,0.17,0.49>
C3	<0.69,0.14,0.14>	<0.71,0.12,0.14>	<0.64,0.15,0.19>
C4	<0.48,0.23,0.30>	<0.47,0.23,0.31>	<0.41,0.29,0.33>
C5	<0.37,0.26,0.41>	<0.34,0.16,0.54>	<0.28,0.23,0.55>
C6	<0.48,0.14,0.35>	<0.28,0.26,0.52>	<0.39,0.17,0.44>
C7	<0.21,0.19,0.63>	<0.07,0.10,0.87>	<0.27,0.19,0.57>

D2	X1	X2	X3
C1	<0.32,0.38,0.35>	<0.29,0.38,0.39>	<0.43,0.28,0.27>
C2	<0.49,0.17,0.34>	<0.33,0.24,0.47>	<0.63,0.14,0.20>
C3	<0.62,0.12,0.24>	<0.53,0.14,0.33>	<0.56,0.15,0.28>
C4	<0.40,0.29,0.34>	<0.52,0.21,0.26>	<0.23,0.29,0.57>
C5	<0.52,0.16,0.31>	<0.27,0.24,0.55>	<0.45,0.17,0.38>
C6	<0.42,0.14,0.43>	<0.34,0.17,0.51>	<0.24,0.24,0.59>
C7	<0.53,0.15,0.31>	<0.69,0.10,0.19>	<0.50,0.17,0.33>

D3	X1	X2	X3
C1	<0.27,0.38,0.42>	<0.27,0.34,0.48>	<0.31,0.31,0.44>
C2	<0.49,0.21,0.31>	<0.29,0.16,0.60>	<0.57,0.14,0.28>
C3	<0.62,0.14,0.23>	<0.27,0.24,0.54>	<0.49,0.14,0.38>
C4	<0.23,0.31,0.55>	<0.19,0.33,0.59>	<0.56,0.18,0.25>
C5	<0.46,0.16,0.38>	<0.32,0.24,0.48>	<0.45,0.16,0.39>
C6	<0.39,0.21,0.42>	<0.40,0.23,0.39>	<0.19,0.24,0.66>
C7	<0.13,0.08,0.82>	<0.35,0.19,0.48>	<0.18,0.06,0.79>

Step 5. Using equation (12), the aggregated group decision matrix (AM) $[A_{ij}]_{7x3}$ based on the obtained weighted normalized decision matrices (D1, D2, D3) of all decision-makers is constructed as:

AM	X1	X2	X3
C1	<0.31,0.37,0.37>	<0.29,0.36,0.42>	<0.34,0.32,0.37>
C2	<0.51,0.19,0.30>	<0.31,0.20,0.54>	<0.53,0.15,0.30>
C3	<0.65,0.13,0.20>	<0.54,0.16,0.29>	<0.57,0.15,0.27>
C4	<0.38,0.28,0.38>	<0.41,0.25,0.36>	<0.41,0.25,0.36>
C5	<0.45,0.19,0.36>	<0.31,0.21,0.52>	<0.40,0.18,0.44>
C6	<0.43,0.16,0.40>	<0.34,0.22,0.47>	<0.28,0.22,0.56>
C7	<0.31,0.13,0.55>	<0.42,0.12,0.43>	<0.33,0.13,0.53>

Step 6: The positive ideal solution J^+ and the negative ideal solution J^- of the attributes $c_i \in C(i=1,2,...,m)$ from the weighted normalized decision-making matrix $[A_{ij}]_{mxn}$ are determined according to the Definition 16 as:

$$J^+ = \left(\mu_i^+, \eta_i^+, \nu_i^+\right) = \begin{pmatrix} \max_i \mu_{ij}, \min_i \eta_{ij}, \min_i \nu_{ij} \\ \mid i = 1,2,...,m \end{pmatrix}$$

$$J^+ = \begin{pmatrix} \overset{C1}{\langle 0.34,0.32,0.37\rangle}, \overset{C2}{\langle 0.53,0.15,0.30\rangle}, \overset{C3}{\langle 0.65,0.13,0.20\rangle}, \overset{C4}{\langle 0.41,0.25,0.36\rangle}, \\ \overset{C5}{\langle 0.45,0.18,0.36\rangle}, \overset{C6}{\langle 0.43,0.16,0.40\rangle}, \overset{C7}{\langle 0.42,0.12,0.43\rangle} \end{pmatrix}$$

$$J^- = \left(\mu_i^-, \eta_i^-, \nu_i^-\right) = \begin{pmatrix} \min_i \mu_{ij}, \max_i \eta_{ij}, \max_i \nu_{ij} \\ \mid i = 1,2,...,m \end{pmatrix}$$

$$J^+ = \begin{pmatrix} \overset{C1}{\langle 0.29,0.37,0.42\rangle}, \overset{C2}{\langle 0.31,0.20,0.54\rangle}, \overset{C3}{\langle 0.54,0.16,0.29\rangle}, \overset{C4}{\langle 0.38,0.28,0.38\rangle}, \\ \overset{C5}{\langle 0.31,0.21,0.52\rangle}, \overset{C6}{\langle 0.28,0.22,0.56\rangle}, \overset{C7}{\langle 0.31,0.13,0.55\rangle} \end{pmatrix}$$

Step 7: The distances from ideal solutions (d_j^+, d_j^-) of alternatives A_j are calculated according to Definitions 11, 13 and 15 as:

d_{L_1}	$d_{L_1}^+$	$d_{L_1}^-$	d_J	R_J
X1	0,066	0,209	0,759	1
X2	0,209	0,067	0,243	3
X3	0,125	0,151	0,548	2

d_{L_2}	$d_{L_2}^+$	$d_{L_2}^-$	d_J	R_J
X1	0,069	0,180	0,724	1
X2	0,172	0,074	0,301	3
X3	0,115	0,142	0,553	2

d_{L_∞}	$d^{+}_{L_\infty}$	$d^{-}_{L_\infty}$	d_J	R_J
X1	0,066	0,209	0,759	1
X2	0,209	0,067	0,243	3
X3	0,125	0,151	0,548	2

Additionally:

- The L_1 norm is calculated as the sum of the absolute values of the vector.
- The L_2 norm is calculated as the square root of the sum of the squared vector values.
- The L_∞ norm is calculated as the maximum vector value.

In the context of ranking alternatives and decision-making, the proposed uncertainty set-based TOPSIS approach employs a relative closeness coefficient to assess and rank alternatives. Higher coefficients indicate a stronger preference. Alternatives are then ranked in descending order, with the highest coefficient corresponding to the most desirable option.

Step 8: In the context of the presented stealth combat aircraft selection problem, the ranking orders (R_j) of alternatives (A_j) obtained through the uncertainty-based TOPSIS analysis using (L1), (L2), and (L∞) norms according to Definitions 11, 13, and 15 are as follows:

R_j	Ranking orders of alternatives
$R(L_1)$	$X_1 \succ X_3 \succ X_2$
$R(L_2)$	$X_1 \succ X_3 \succ X_2$
$R(L_\infty)$	$X_1 \succ X_3 \succ X_2$

The ranking results show that alternative (X_1) has the highest relative closeness coefficient to the ideal solution within the uncertainty set. This indicates that (X_1) is the most suitable choice for the given decision scenario.

In conclusion, this study introduces an uncertainty set-based TOPSIS approach for ranking alternatives in the presence of ambiguity and imprecision. The application to a real-world stealth combat aircraft selection problem demonstrated the effectiveness of the proposed methodology in handling inconsistencies or indeterminacies often present in such evaluations. Furthermore, the inherent flexibility and adaptability of uncertainty sets suggest their potential as a powerful tool for navigating diverse uncertainties encountered in MCDM problems.

IV. Conclusion

This study addresses the growing need for robust methodologies in Multiple Criteria Decision Making (MCDM) by introducing uncertainty sets and extending the TOPSIS method with (L1), (L2), and (L∞) norms. The pervasiveness of ambiguity and imprecision in real-world decision-making necessitates frameworks that can effectively handle such complexities, particularly within fuzzy environments.

This work establishes uncertainty sets as a vital mathematical framework by elucidating their basic concepts and exploring their application in decision-making processes. Uncertainty sets offer a structured approach to navigate imprecise information, providing a promising avenue for tackling the intricacies of decision-making under uncertainty.

Furthermore, the study addresses the challenge of incorporating uncertainty information into MCDM problems. It proposes a extension of TOPSIS method within the uncertainty set framework to establish robust ranking orders for alternatives. This method empowers confident decision-making even amidst inherent ambiguity.

The effectiveness of the proposed approach is demonstrated by applying uncertainty-based TOPSIS analysis with (L1), (L2), and (L∞) norms to the real-world problem of stealth combat aircraft selection. The results highlight the potency of uncertainty sets in informing decision-making processes within complex scenarios.

The applicability of this approach extends beyond aircraft selection. Researchers and practitioners across diverse fields can leverage uncertainty-based TOPSIS analysis to tackle a wide range of real-life problems, from resource allocation and project prioritization to risk assessment. The inherent flexibility and adaptability of this method position it as a valuable tool for navigating uncertainty and making informed decisions in dynamic environments.

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