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Uncertainty Multiple Criteria Decision Making Analysis for Stealth Combat Aircraft Selection

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***Abstract*—**Fuzzy set theory and its extensions (intuitionistic fuzzy sets, picture fuzzy sets, and neutrosophic sets) have been widely used to address imprecision and uncertainty in complex decision-making. However, they may struggle with inherent indeterminacy and inconsistency in real-world situations. This study introduces uncertainty sets as a promising alternative, offering a structured framework for incorporating both types of uncertainty into decision-making processes.

This work explores the theoretical foundations and applications of uncertainty sets. A novel decision-making algorithm based on uncertainty set-based proximity measures is developed and demonstrated through a practical application: selecting the most suitable stealth combat aircraft.

The results highlight the effectiveness of uncertainty sets in ranking alternatives under uncertainty. Uncertainty sets offer several advantages, including structured uncertainty representation, robust ranking mechanisms, and enhanced decision-making capabilities due to their ability to account for ambiguity.

Future research directions are also outlined, including comparative analysis with existing MCDM methods under uncertainty, sensitivity analysis to assess the robustness of rankings, and broader application to various MCDM problems with diverse complexities. By exploring these avenues, uncertainty sets can be further established as a valuable tool for navigating uncertainty in complex decision-making scenarios.

***Keywords*—**Uncertainty set, stealth combat aircraft selection multiple criteria decision-making analysis, MCDM, uncertainty proximity analysis.

I. Introduction

A fuzzy set (FS) Γ in a universe X is characterized by a membership function $\mu_\gamma(x): X \rightarrow [0,1]$ that maps each element x in X to a degree of membership between 0 and 1, $\Gamma = \left\{ < x, \mu_\gamma(x) > | \, x \in X \right\}$. The sum of these membership degrees over the entire universe is constrained to be 1: $\sum_{x \in X} \mu_\gamma(x) = 1$. This restricts the total uncertainty in the system to lie between 0 (complete certainty) and 1 (completely unknown) [1].

A direct extension of fuzz set (FS), intuitionistic fuzzy set (IFS) $\Gamma = \left\{ < x, \mu_\gamma(x), \nu_\gamma(x) > | \, x \in X \right\}$ with the condition $0 \leq \mu_\gamma(x) + \nu_\gamma(x) \leq 1$ incorporates hesitancy as a measure of uncertainty $\pi_\gamma(x) = 1 - \mu_\gamma(x) + \nu_\gamma(x)$, where the sum of membership degree $\mu_\gamma(x)$ and non-membership $\nu_\gamma(x)$ is constrained to be within the unitary interval $[0,1]$ [2].

While intuitionistic fuzzy sets (IFSs) do not explicitly account for indeterminacy, IFSs offer a means to represent incomplete information by allowing for partial membership and non-membership values. This representation enables decision-makers to express uncertainty when precise evaluations are not feasible.

Fuzzy set theory has been instrumental in representing uncertainty in various applications. However, it has limitations in capturing the nuances of real-world information. To address this challenge, extensions like intuitionistic fuzzy sets (IFS) and picture fuzzy sets (PFS) were introduced [3].

Basically, picture fuzzy sets (PFS) $\Gamma = \left\{ < x, \mu_\gamma(x), \eta_\gamma(x), \nu_\gamma(x) > | \, x \in X \right\}$ with the condition $0 \leq \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x) \leq 1$ are a direct extension of fuzzy sets (FS) and intuitionistic fuzzy sets (IFS). In a PFS, an element can belong to a set with varying degrees of membership $\mu_\gamma(x)$, neutrality $\eta_\gamma(x)$, and non-membership $\nu_\gamma(x)$. A key feature of PFS is the inclusion of a refusal membership degree $\rho_\gamma(x = 1 - \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x)$ alongside the positive and negative membership degrees.

However, to maintain consistency with classical logic principles, the sum of all three membership degrees $0 \leq \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x) \leq 1$ is constrained to be within the unitary interval $[0,1]$. This ensures that the representation of uncertainty remains interpretable and avoids contradictions within the framework.

Fuzzy set theory and its extensions have been successful in modeling uncertainty in various domains. However, these approaches may not fully capture the complexities of real-world information, particularly situations where indeterminacy plays a significant role.

To address these challenges, neutrosophic sets (NS) $\Gamma = \left\{ < x, \mu_\gamma(x), \eta_\gamma(x), \nu_\gamma(x) > | \, x \in X \right\}$ with the condition ${}^{-}0 \leq \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x) \leq 3^{+}$ emerge as a further extension of fuzzy sets (FS) and intuitionistic fuzzy sets (IFS), aiming to address this limitation [4]. Unlike prior extensions, neutrosophic sets introduce independent truth $\mu_\gamma(x)$, indeterminacy $\eta_\gamma(x)$, and falsity $\nu_\gamma(x)$ membership degrees. These degrees can vary independently within the interval $]^{-}0,1^{+}[$, allowing for a more nuanced representation of uncertainty. However, to maintain some level of interpretability, the sum of all three membership degrees $0 \leq \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x) \leq 3$ is constrained to be within a specific range $[0,3]$. However, this constraint ensures that the combined uncertainty representation exceeds the

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boundaries of classical logic entirely. A neutrosophic set explicitly accounts for inconsistency by allowing for truth, falsity, and indeterminacy values that can exceed classical boundaries. The expanded range accommodates scenarios where contradictory or ambiguous information exists simultaneously, reflecting a broader spectrum of uncertainty.

Classical set theory struggles with real-world problems characterized by inconsistency and indeterminacy, where data may be contradictory, information imprecise, and existing methods fail to capture the full range of uncertainty. Fuzzy set theory and its extensions (intuitionistic fuzzy sets, picture fuzzy sets, and neutrosophic sets) have offered valuable tools for representing uncertainty in certain scenarios.

To address real-world challenges effectively, the concept of an uncertainty set (US) has emerged as a standardized framework for precisely defining complex decision environments. Drawing upon key features from fuzzy sets, intuitionistic fuzzy sets, picture fuzzy sets, and neutrosophic sets, uncertainty sets offer a comprehensive approach to modeling uncertainty in decision-making processes. Additionally, by adhering to a specific condition $0 \le \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x) \le 1$, uncertainty sets provide a structured and coherent representation of uncertainty, allowing decision-makers to navigate intricate decision landscapes with clarity and precision.

An uncertainty set, denoted mathematically as $\Gamma = \left\{ < x, \mu_\gamma(x), \eta_\gamma(x), \nu_\gamma(x) > \mid x \in X \right\}$ with the condition $0 \le \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x) \le 1$, encompasses a collection of possible values or membership functions for a variable (x). Here, $\mu_\gamma(x)$ represents the degree of truth, $\eta_\gamma(x)$ represents the degree of indeterminacy, and $\nu_\gamma(x)$ represents the degree of falsity associated with variable (x). Unlike a traditional set with clear boundaries, an uncertainty set allows for a range of possibilities, acknowledging the inherent vagueness or ambiguity in information.

Uncertainty sets offer a unified and coherent framework for modeling complex situations where membership is not always clear-cut. By quantifying the degree of inconsistency or indeterminacy associated with each element, uncertainty sets provide valuable information for making informed decisions in uncertain scenarios.

Compared to other fuzzy set extensions like intuitionistic fuzzy sets, picture fuzzy sets, and neutrosophic sets, uncertainty sets strike a balance between inconsistency and indeterminacy. Uncertainty sets limit the combined effect of these factors to a unitary interval, ensuring a coherent and interpretable representation of uncertainty within classical logic principles.

In contrast to intuitionistic fuzzy sets, which require separate consideration of membership and non-membership degrees without explicit incorporation of indeterminacy, uncertainty sets offer a unified approach to representing uncertainty. By encompassing both indeterminacy and inconsistency within a single constraint, uncertainty sets provide a comprehensive and simplified representation of uncertain information.

Unlike neutrosophic sets, which allow for independent variation of truth, indeterminacy, and falsity values within an extended range, uncertainty sets adhere to classical logic by keeping the sum of these values within a conventional interval $[0,1]$. This adherence facilitates easier interpretation and integration with existing decision-making frameworks.

The restriction of uncertainty sets to a unitary interval enhances interpretability and transparency in decision-making processes. Decision-makers can easily grasp and interpret the degree of uncertainty associated with each element within a familiar range, promoting effective communication and informed decision-making.

The flexibility and adaptability of uncertainty sets in representing a wide range of uncertain information make them a promising framework for addressing complex decision-making problems. By accommodating both indeterminacy and inconsistency while adhering to classical logic principles, uncertainty sets can capture various degrees of uncertainty encountered in real-world scenarios.

Overall, uncertainty sets offer advantages in terms of balance, compatibility, simplicity, interpretability, and flexibility compared to other fuzzy set extensions. Their ability to provide a unified and coherent representation of uncertainty within classical logic boundaries makes them a powerful tool for decision-making in uncertain environments, effectively addressing different sources of uncertainty.

This study focuses on core fuzzy set extensions and deliberately excludes power-parameter based extensions like pythagorean fuzzy subsets [5], q-rung orthopair fuzzy sets [6], circular intuitionistic fuzzy sets [7], fermatean fuzzy sets [8], spherical fuzzy sets [9] and their extensions. Here's the reasoning behind excluding these sets:

Focus on core extensions: This study prioritizes exploring the fundamental principles of uncertainty representation through established fuzzy set extensions like intuitionistic fuzzy sets, picture fuzzy sets and neutrosohic sets. Power-parameter based extensions introduce additional complexity by incorporating an extra parameter that can potentially complicate interpretation and analysis.

Maintaining scope: Including all possible fuzzy set extensions could significantly broaden the scope of the study. Focusing on core extensions allows for a more in-depth exploration of fundamental concepts within the allotted timeframe or resource constraints.

Comparison with established methods: By focusing on core extensions, this study facilitates a clearer comparison with existing well-understood methods like intuitionistic fuzzy sets, highlighting the advantages of uncertainty sets in a more direct way. Power-parameter based extensions can be explored in future research for even more nuanced uncertainty representation.

The remainder of the paper is organized as follows: In Section 2 uncertainty sets are briefly summarized. The steps of uncertainty multiple criteria decision-making analysis are briefly summarized. In Section 3, a stealth combat aircraft selection application is carried out and, the results are analyzed. In Section 4, the paper concludes with a recommendation for further future work.

II. METHODOLOGY

This section presents some basic definitions of uncertainty sets, uncertainty numbers, relations, operation and theoretical concepts of multiple criteria decision analysis [1-50].

Definition 1. [1] Let X be a fixed set in a universe of discourse. A fuzzy set Γ in *X* is an object having the form

$$\Gamma = \left\{ < x, \mu_\gamma(x) > \mid x \in X \right\} \tag{1}$$

where $\mu_\gamma(x): X \to [0,1]$ represents a membership function of Γ with the condition $0 \le \mu_\gamma(x) \le 1$ for all $x \in X$.

Definition 2. [2] Let X be a fixed set in a universe of discourse. An intuitionistic fuzzy set Γ in *X* is an object having the form

$$\Gamma = \left\{ < x, \mu_\gamma(x), \nu_\gamma(x) > \mid x \in X \right\} \tag{2}$$

where $\mu_\gamma(x): X \to [0,1]$ represents a membership function , a non-membership function $\nu_\gamma : X \to [0,1]$, and a hesitancy degree $\pi_\gamma(x) = 1 - \mu_\gamma(x) + \nu_\gamma(x)$ of Γ with the condition $0 \le \mu_\gamma(x) + \nu_\gamma(x) \le 1$, for all $x \in X$.

Definition 3. [3] Let X be a fixed set in a universe of discourse. A picture fuzzy set Γ in *X* is an object having the form

$$\Gamma = \left\{ < x, \mu_\gamma(x), \nu_\gamma(x) > \mid x \in X \right\} \tag{3}$$

where $\mu_\gamma(x): X \to [0,1]$ represents a positive membership function , a neutral membership function $\eta_\gamma : X \to [0,1]$, a negative membership function $\nu_\gamma : X \to [0,1]$ and a refusal degree $\rho_\gamma(x = 1 - \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x)$ of Γ with the condition $0 \le \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x) \le 1$ for all $x \in X$.

Definition 4. Let X be a fixed set in a universe of discourse. An uncertainty set Γ in *X* is an object having the form

$$\Gamma = \left\{ < x, \mu_\gamma(x), \nu_\gamma(x) > \mid x \in X \right\} \tag{4}$$

where $\mu_\gamma(x): X \to [0,1]$ represents a truth membership function, an indeterminacy membership function $\eta_\gamma : X \to [0,1]$, and a falsity membership function $\nu_\gamma : X \to [0,1]$ of Γ with the condition $0 \le \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x) \le 1$ for all $x \in X$.

Definition 5. [4] Let X be a fixed set in a universe of discourse. A single-valued neutrosophic set (SVNN) Γ in *X* is an object having the form

$$\Gamma = \left\{ < x, \mu_\gamma(x), \nu_\gamma(x) > \mid x \in X \right\} \tag{5}$$

where $\mu_\gamma(x): X \to [0,1]$ represents a truth membership function, an indeterminacy membership function $\eta_\gamma : X \to [0,1]$, and a falsity membership function $\nu_\gamma : X \to [0,1]$ of Γ with the condition $0 \le \mu_\gamma(x) + \eta_\gamma(x) + \nu_\gamma(x) \le 3$ for all $x \in X$.

Definition 6. Given two uncertainty sets $A = \left\{ < x, \mu_A(x), \eta_A(x), \nu_A(x) > \mid \forall x \in X \right\}$ and $B = \left\{ < (x, \mu_B(x), \eta_B(x), \nu_B(x) > \mid \forall x \in X \right\}$, the relations are defined as follows:

a) $$A \subseteq B \textit{ if and only if } \mu_A(x) \le \mu_B(x), \quad \eta_A(x) \ge \eta_B(x), \nu_A(x) \ge \nu_B(x) \tag{6}$$

b) $$A \subseteq B \textit{ if and only if } \mu_A(x) = \mu_B(x), \quad \eta_A(x) = \eta_B(x), \nu_A(x) = \nu_B(x) \tag{7}$$

c) $$A \cap B = \left\{ \begin{array}{l} < x, (\mu_A \wedge \mu_B)(x), \\ (\eta_A \vee \eta_B)(x), (\nu_A \vee \nu_B)(x) > \mid \forall x \in X \end{array} \right\} \tag{8}$$

d) $$A \cup B = \left\{ \begin{array}{l} < x, (\mu_A \vee \mu_B)(x), \\ (\eta_A \wedge \eta_B)(x), (\nu_A \wedge \nu_B)(x) > \mid \forall x \in X \end{array} \right\} \tag{9}$$

e) $$\bar{A} = \left\{ < x, \nu_A(x), \eta_A(x), \mu_A(x) > \mid \forall x \in X \right\} \tag{10}$$

where the symbol $\wedge$ represents the t-norm, while $\vee$ represents the t-conorm.

Definition 7. For $\lambda > 0$, the corresponding operations for three indeterminacy numbers (INs) $A_1 = (\mu_{A_1}, \eta_{A_1}, \nu_{A_1})$, $A_2 = (\mu_{A_2}, \eta_{A_2}, \nu_{A_2})$, and $A = (\mu_A, \eta_A, \nu_A)$ are defined as follows:

a) $$A_1 \oplus A_2 = (\mu_{A_1} + \mu_{A_2} - \mu_{A_1}\mu_{A_2}, \eta_{A_1}\eta_{A_2}, \nu_{A_1}\nu_{A_2}) \tag{11}$$

b) $$A_1 \otimes A_2 = (\mu_{A_1}\mu_{A_2}, \eta_{A_1} + \eta_{A_2} - \eta_{A_1}\eta_{A_2}, \nu_{A_1} + \nu_{A_2} - \nu_{A_1}\nu_{A_2}) \tag{12}$$

c) $$\lambda A = (1 - (1 - \mu_A)^\lambda, \eta_A^\lambda, \nu_A^\lambda) \tag{13}$$

d) $$A^\lambda = (\mu_A^\lambda, 1 - (1 - \eta_A)^\lambda, 1 - (1 - \nu_A)^\lambda) \text{ where } \lambda > 0 \tag{14}$$

e) $$0_i^+ = \left\{ < (x, 1, 0, 0) > \mid \forall x \in X \right\} \tag{15}$$

e) $$0_i^- = \left\{ < (x, 0, 1, 1) > \mid \forall x \in X \right\} \tag{16}$$

Definition 8. For any uncertainty number (UN) $A = (\mu_A, \eta_A, \nu_A)$, the score $s(A)$, the accuracy $h(A)$, and the certainty $c(A)$ functions of *A* are defined as:

$$s(A) = (\mu_A - \nu_A) \tag{17}$$

$$h(A) = \mu_A + \eta_A + \nu_A \tag{18}$$

$$c(A) = \mu_A \tag{19}$$

where $s(A) \in [-1,1]$ and $h(A) \in [0,1]$. For any UNs A_1 and A_2

1. If $s(A_1) > s(A_2)$, *then* $(A_1) > (A_2)$,

2. If $s(A_1) = s(A_2)$, *then*

i. If $\mathrm{h(A_1)>h(A_2)} \Rightarrow \mathrm{A_1 > A_2}$

ii. If $\mathrm{h(A_1)=h(A_2)}$, *then* $\mathrm{A_1 \approx A_2}$

Definition 9. Let $a_i (i = 1,2,...,n)$ be a collection of uncertainty numbers (UNs) and $\omega = [\omega_1, \omega_2, ..., \omega_n]^T$ is the weight vector of a_i with the condition $\omega_i > 0$ and $\sum_{i=1}^{n} \omega_i = 1$. Then, the uncertainty weighted averaging (UWA) operator is a mapping $A^n \to A$ such that

$$UWA(a_1, a_2, ..., a_n) = \oplus_{i=1}^{n} (\omega_i a_i)$$

$$UWA(a_1, a_2, ..., a_n) = \begin{pmatrix} 1 - \prod_{i=1}^{n} (1 - \mu_{a_i})^{\omega_i}, \\ \prod_{i=1}^{n} (\eta_{a_i})^{\omega_i}, \prod_{i=1}^{n} (\nu_{a_i})^{\omega_i} \end{pmatrix} \tag{20}$$

Definition 10. Let $a_i (i = 1,2,...,n)$ be a collection of uncertainty numbers (UNs) and $\omega = [\omega_1, \omega_2, ..., \omega_n]^T$ is the weight vector of a_i with the condition $\omega_i > 0$ and $\sum_{i=1}^{n} \omega_i = 1$. Then, the uncertainty weighted geometric (UWG) operator is a mapping $A^n \to A$ such that

$$UWG(a_1, a_2, ..., a_n) = \otimes_{i=1}^{n} (a_i^{\omega_i})$$

$$UWG(a_1, a_2, ..., a_n) = \begin{pmatrix} \prod_{i=1}^{n} (\mu_{a_i})^{\omega_i}, 1 - \prod_{i=1}^{n} (1 - \eta_{a_i})^{\omega_i}, \\ 1 - \prod_{i=1}^{n} (1 - \nu_{a_i})^{\omega_i} \end{pmatrix} \tag{21}$$

Definition 11. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their distance L_1 is defined as:

$$d_{L_1}(A,B) = \frac{1}{n} \sum_{i=1}^{n} \begin{pmatrix} |\mu_A(x_i) - \mu_B(x_i)| + \\ |\eta_A(x_i) - \eta_B(x_i)| + \\ |\nu_A(x_i) - \nu_B(x_i)| \end{pmatrix} \tag{22}$$

Definition 12. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their weighted distance L_1 is defined as:

$$d_{L_1}(A,B) = \frac{1}{n} \sum_{i=1}^{n} \omega_i \begin{pmatrix} |\mu_A(x_i) - \mu_B(x_i)| + \\ |\eta_A(x_i) - \eta_B(x_i)| + \\ |\nu_A(x_i) - \nu_B(x_i)| \end{pmatrix} \tag{23}$$

Definition 13. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their distance L_2 is defined as:

$$d_{L_2}(A,B) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \begin{bmatrix} (\mu_A(x_i) - \mu_B(x_i))^2 + \\ (\eta_A(x_i) - \eta_B(x_i))^2 + \\ (\nu_A(x_i) - \nu_B(x_i))^2 \end{bmatrix}} \tag{24}$$

Definition 14. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their weighted distance L_2 is defined as:

$$d_{L_2}(A,B) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \omega_i \begin{bmatrix} (\mu_A(x_i) - \mu_B(x_i))^2 + \\ (\eta_A(x_i) - \eta_B(x_i))^2 + \\ (\nu_A(x_i) - \nu_B(x_i))^2 \end{bmatrix}} \tag{25}$$

Definition 15. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their distance L_∞ is defined as:

$$d_{L_\infty}(A,B) = \max \begin{pmatrix} |\mu_A(x_i) - \mu_B(x_i)|, \\ |\eta_A(x_i) - \eta_B(x_i)|, \\ |\nu_A(x_i) - \nu_B(x_i)| \end{pmatrix} \tag{26}$$

Definition 16. Given two uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$, their weighted distance L_∞ is defined as:

$$d_{L_\infty}(A,B) = \max \left[\omega_i \begin{pmatrix} |\mu_A(x_i) - \mu_B(x_i)|,| \\ \eta_A(x_i) - \eta_B(x_i)|,| \\ \nu_A(x_i) - \nu_B(x_i)| \end{pmatrix} \right] \tag{27}$$

Definition 17. Given an uncertainty decision matrix $J_a = [J_{a\,ij}]_{mxn}$ with uncertainty numbers (UNs) $A = (\mu_A, \eta_A, \nu_A)$. Positive ideal solution J^+ is defined as:

$$J^+ = \left(\mu_j^+, \eta_j^+, \nu_j^+\right) = \begin{pmatrix} \max_j \mu_{ij}, \min_j \eta_{ij}, \min_j \nu_{ij} \\ |\ j = 1,2,...,n \end{pmatrix} \tag{28}$$

Definition 18. Let $X = \{x_1, x_2, ..., x_m\}$ be a set of alternatives, $C = \{(c_1, c_2, ..., c_m\}$ be the set of attributes. The ratings of alternatives $x_j \in X (j = 1,2,...,n)$ on attributes $c_i \in C$ are expressed with uncertainty number $A_{ij} = \langle \mu_{ij}, \eta_{ij}, \nu_{ij} \rangle$. Then,

$$[A_{ij}]_{mxn} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{array} \begin{pmatrix} X_1 & X_2 & & X_n \\ \langle \mu_{11}, \eta_{11}, \nu_{11} \rangle & \langle \mu_{12}, \eta_{12}, \nu_{12} \rangle & \cdots & \langle \mu_{1n}, \eta_{1n}, \nu_{1n} \rangle \\ \langle \mu_{21}, \eta_{21}, \nu_{21} \rangle & \langle \mu_{22}, \eta_{22}, \nu_{22} \rangle & \cdots & \langle \mu_{2n}, \eta_{2n}, \nu_{2n} \rangle \\ \vdots & \vdots & \cdots & \vdots \\ \langle \mu_{m1}, \eta_{m1}, \nu_{m1} \rangle & \langle \mu_{m2}, \eta_{m2}, \nu_{m2} \rangle & \cdots & \langle \mu_{mn}, \eta_{mn}, \nu_{mn} \rangle \end{pmatrix}$$

$[A_{ij}]_{mxn}$ is called a multiple criteria decision-making matrix. The importance weight vector of attribute set $C = \{(c_1, c_2, ..., c_m\}$ is given as:

$$\omega_i = (\omega_1, \omega_2, ..., \omega_m) = \left(\langle \alpha_1, \beta_1, \gamma_1 \rangle, \langle \alpha_2, \beta_2, \gamma_2 \rangle, ..., \langle \alpha_m, \beta_m, \gamma_m \rangle\right)$$

The initial decision matrix is normalized $[A_{ij}]_{mxn}$: benefit-type attributes remain the same, while the cost-type attributes are transformed into benefit-type according to Equation (10).

$$[A_{ij}]_{mxn} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & & X_n \end{array} \\ \left(\begin{array}{cccc} \langle \mu_{11}, \eta_{11}, \nu_{11} \rangle & \langle \mu_{12}, \eta_{12}, \nu_{12} \rangle & \cdots & \langle \mu_{1n}, \eta_{1n}, \nu_{1n} \rangle \\ \langle \mu_{21}, \eta_{21}, \nu_{21} \rangle & \langle \mu_{22}, \eta_{22}, \nu_{22} \rangle & \cdots & \langle \mu_{2n}, \eta_{2n}, \nu_{2n} \rangle \\ \vdots & \vdots & \cdots & \vdots \\ \langle \mu_{m1}, \eta_{m1}, \nu_{m1} \rangle & \langle \mu_{m2}, \eta_{m2}, \nu_{m2} \rangle & \cdots & \langle \mu_{mn}, \eta_{mn}, \nu_{mn} \rangle \end{array} \right) \end{array}$$

Then, the weighted normalized multiple criteria decision-making matrix $[A_{ij}]_{mxn} = \omega_i [A_{ij}]_{mxn}$ is presented as:

$$[A_{ij}]_{mxn} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & & X_n \end{array} \\ \left(\begin{array}{cccc} \langle \mu_{11}, \eta_{11}, \nu_{11} \rangle & \langle \mu_{12}, \eta_{12}, \nu_{12} \rangle & \cdots & \langle \mu_{1n}, \eta_{1n}, \nu_{1n} \rangle \\ \langle \mu_{21}, \eta_{21}, \nu_{21} \rangle & \langle \mu_{22}, \eta_{22}, \nu_{22} \rangle & \cdots & \langle \mu_{2n}, \eta_{2n}, \nu_{2n} \rangle \\ \vdots & \vdots & \cdots & \vdots \\ \langle \mu_{m1}, \eta_{m1}, \nu_{m1} \rangle & \langle \mu_{m2}, \eta_{m2}, \nu_{m2} \rangle & \cdots & \langle \mu_{mn}, \eta_{mn}, \nu_{mn} \rangle \end{array} \right) \end{array}$$

where

$$\langle \mu_{ij}, \eta_{ij}, \nu_{ij} \rangle = \omega_i A_{ij} = \langle \alpha_i, \beta_i, \gamma_i \rangle \langle \mu_{ij}, \eta_{ij}, \nu_{ij} \rangle$$
$$= \langle \alpha_i \mu_{ij}, \beta_i + \eta_{ij} - \beta_i \eta_{ij}, \gamma_i + \nu_{ij} - \gamma_i \nu_{ij} \rangle$$

Based on the obtained weighted normalized decision matrices D1, D2, D3, and equation (13), the aggregated group decision matrix $[A_{ij}]_{7x3}$ of all decision-makers is constructed as:

$$[A_{ij}]_{mxn} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & & X_n \end{array} \\ \left(\begin{array}{cccc} \langle \mu_{11}, \widehat{\eta}_{11}, \nu_{11} \rangle & \langle \mu_{12}, \widehat{\eta}_{12}, \widehat{\nu}_{12} \rangle & \cdots & \langle \mu_{1n}, \widehat{\eta}_{1n}, \widehat{\nu}_{1n} \rangle \\ \langle \mu_{21}, \widehat{\eta}_{21}, \nu_{21} \rangle & \langle \mu_{22}, \widehat{\eta}_{22}, \widehat{\nu}_{22} \rangle & \cdots & \langle \mu_{2n}, \widehat{\eta}_{2n}, \widehat{\nu}_{2n} \rangle \\ \vdots & \vdots & \cdots & \vdots \\ \langle \mu_{m1}, \widehat{\eta}_{m1}, \widehat{\nu}_{m1} \rangle & \langle \mu_{m2}, \widehat{\eta}_{m2}, \widehat{\nu}_{m2} \rangle & \cdots & \langle \mu_{mn}, \widehat{\eta}_{mn}, \widehat{\nu}_{mn} \rangle \end{array} \right) \end{array}$$

Then, the positive ideal solution J^+ of the attributes $c_i \in C(i = 1, 2, ..., m)$ is determined from the weighted decision-making matrix $[A_{ij}]_{mxn}$ according to the Definition 17:

$$J^+ = \left(\mu_i^+, \eta_i^+, \nu_i^+ \right) = \left(\begin{array}{l} \max_i \mu_{ij}, \min_i \eta_{ij}, \min_i \nu_{ij} \\ | \, i = 1, 2, ..., m \end{array} \right)$$

The proximity scores $s(d_{L_1})$, $s(d_{L_2})$, and $s(d_{L_\infty})$ are calculated according to Definitions 11, 13 and 15.

Finally, the alternatives are ranked using the scores $s(d_{L_1})$, $s(d_{L_2})$, and $s(d_{L_\infty})$ according to Definitions 11, 13 and 15.

Definition 19. Sensitivity analysis is a crucial technique used in decision-making and optimization. It assesses how changes in input parameters (such as weights, coefficients, or constraints) impact the output of a model or system.

Sensitivity analysis is performed to measure the robustness of the results obtained from the proposed methodology. This analysis allows exploration of various scenarios related to decision-makers' priorities regarding criterion weights, which could potentially impact the outcome of the proposed methodology.

To achieve this, the uncertainty weights assigned by each expert to a specific criterion are modified while keeping the uncertainty weights of other criteria constant. Subsequently, the criteria weights are recalculated based on these new weights. The alternatives are then reordered using varying proximity values. Overall, the results are thoroughly analyzed by considering different scenarios.

Let $[A_{ij}]_{mxn}$ be a decision-making matrix,

$$\omega_i = (\omega_1, \omega_2, ..., \omega_m) = \left(\langle \alpha_1, \beta_1, \gamma_1 \rangle, \langle \alpha_2, \beta_2, \gamma_2 \rangle, ..., \langle \alpha_m, \beta_m, \gamma_m \rangle \right)$$

be a weighted vector and

$$\omega_i^{'} = (\omega_1, \omega_2, ..., \omega_m)$$
$$= \left(\begin{array}{l} \langle \alpha_1, \beta_1, \gamma_1 \rangle, \langle \alpha_2, \beta_2, \gamma_2 \rangle, ..., \\ \langle \alpha_k + \Delta a_k, \beta_k + \Delta \beta_k, \gamma_k + \Delta \gamma_k \rangle, ..., \langle \alpha_m, \beta_m, \gamma_m \rangle \end{array} \right)^T$$

be a changed weighted vector where Δa_k, $\Delta \beta_k$ and $\Delta \gamma_k$ are increments of α_k, β_k and γ_k, respectively.

Definition 20. Let $X = \{x_1, x_2, ..., x_n\}$ be a set of alternatives, $C = \{(c_1, c_2, ..., c_m\}$ be a set of attributes, and $\omega_i = \langle \alpha_i, \beta_i, \gamma_i \rangle_{mx1}$ be a weighted vector for attributes. Hence, the algorithm for ranking alternatives is presented:

Algorithm 1:

Step 1: Input initial decision-making matrices $[A_{ij}]_{mxn}$;
Step 2: Determine the weighted vector for attributes $\omega_i = \langle \alpha_i, \beta_i, \gamma_i \rangle_{mx1}$;
Step 3: Normalize the initial decision-making matrices $[A_{ij}]_{mxn}$ according to the equation (10);
Step 4: Calculate the weighted normalized decision-making matrices $[A_{ij}]_{mxn}$;
Step 5: Calculate the aggregated group decision matrix $[A_{ij}]_{mxn}$
Step 6: Determine the positive ideal solution J^+ of the attributes $c_i \in C(i = 1, 2, ..., m)$ from the aggregated group decision matrix $[A_{ij}]_{mxn}$ according to the Definition 17;

$$J^+ = \left(\mu_i^+, \eta_i^+, \nu_i^+ \right) = \left(\begin{array}{l} \max_i \mu_{ij}, \min_i \eta_{ij}, \min_i \nu_{ij} \\ | \, i = 1, 2, ..., m \end{array} \right)$$

Step 7: Calculate the proximity scores $s(d_{L_1})$, $s(d_{L_2})$, and $s(d_{L_\infty})$ according to Definitions 11, 13 and 15;

Step 8: Rank the alternatives using the scores $s(d_{L_1})$, $s(d_{L_2})$, and $s(d_{L_\infty})$ according to Definitions 11, 13 and 15.

III. Application

In this section, the stealth combat aircraft selection problem was presented as an illustrative example to show its applicability and effectiveness in decision making problems.

Assume that $X=\{x_1,x_2,...,x_n\}$ is a set of stealth combat aircraft alternatives, and $C=\{(c_1,c_2,...,c_m\}$ is a set of attributes: Stealth Capability (C1), Performance Capability (C2), Survivability (C3), Avionics and Sensors (C4), Interoperability (C5), Operational Capability (C6), and Cost and Maintenance Affordability (C7). Attributes C1-C6 are of benefit type, while C7 is a cost type attribute.

In this group decision-making problem, a three-member decision-making committee with equal weights of importance from the National Ministry of Defense aims to select the best alternative stealth combat aircraft from three preselected alternatives, considering seven evaluation attributes. Utilizing criteria weights, such as the vector of importance weights for these attributes, enables decision-makers (Ds) to establish priorities in the decision-making process. Hence, the solution steps of the *Algorithm 1* according to the Definition 20 are presented as follows:

Step 1. Three initial decision-making matrices (D1,D2,D3) $[A_{ij}]_{7x3}$ are established as:

D1	X1	X2	X3
C1	<0.71,0.15,0.13>	<0.77,0.03,0.19>	<0.15,0.25,0.59>
C2	<0.85,0.09,0.05>	<0.35,0.03,0.61>	<0.57,0.01,0.41>
C3	<0.95,0.01,0.03>	<0.87,0.05,0.07>	<0.47,0.25,0.27>
C4	<0.78,0.07,0.14>	<0.71,0.09,0.19>	<0.17,0.15,0.67>
C5	<0.81,0.03,0.15>	<0.37,0.03,0.59>	<0.39,0.11,0.49<
C6	<0.95,0.01,0.03>	<0.87,0.03,0.09>	<0.67,0.03,0.29>
C7	<0.69,0.13,0.17>	<0.79,0.05,0.15>	<0.25,0.35,0.39>

D2	X1	X2	X3
C1	<0.61,0.15,0.23>	<0.67,0.13,0.19>	<0.55,0.25,0.19>
C2	<0.75,0.09,0.15>	<0.35,0.13,0.51>	<0.57,0.01,0.41>
C3	<0.85,0.01,0.13>	<0.77,0.05,0.17>	<0.47,0.15,0.37>
C4	<0.67,0.17,0.15>	<0.61,0.19,0.19>	<0.87,0.05,0.07>
C5	<0.81,0.03,0.15>	<0.37,0.13,0.49>	<0.79,0.07,0.13>
C6	<0.85,0.01,0.13>	<0.87,0.09,0.03>	<0.57,0.13,0.29>
C7	<0.67,0.13,0.19>	<0.79,0.15,0.05>	<0.65,0.15,0.19>

D3	X1	X2	X3
C1	<0.51,0.15,0.33>	<0.57,0.13,0.29>	<0.65,0.15,0.19>
C2	<0.65,0.19,0.15>	<0.45,0.13,0.41>	<0.67,0.01,0.31>
C3	<0.85,0.11,0.03>	<0.67,0.15,0.17>	<0.57,0.13,0.29>
C4	<0.57,0.17,0.25>	<0.51,0.19,0.29>	<0.93,0.01,0.05>
C5	<0.71,0.13,0.15>	<0.47,0.13,0.39>	<0.79,0.11,0.09>
C6	<0.75,0.11,0.13>	<0.77,0.09,0.13>	<0.67,0.13,0.19>
C7	<0.79,0.07,0.13>	<0.69,0.15,0.15>	<0.65,0.17,0.17>

Step 2. The importance weight vector of attributes is determined as:

$$\omega=\begin{pmatrix}\overset{C1}{\langle 0.43,0.29,0.23\rangle},\overset{C2}{\langle 0.61,0.13,0.21\rangle},\overset{C3}{\langle 0.71,0.11,0.13\rangle},\overset{C4}{\langle 0.59,0.17,0.19\rangle},\\ \overset{C5}{\langle 0.55,0.13,0.27\rangle},\overset{C6}{\langle 0.49,0.13,0.33\rangle},\overset{C7}{\langle 0.75,0.05,0.15\rangle}\end{pmatrix}$$

Step 3. Using the equation (10), the normalized decision matrices (D1,D2,D3) $[A_{ij}]_{7x3}$ are established as the basis for further analysis as:

D1	X1	X2	X3
C1	<0.71,0.15,0.13>	<0.77,0.03,0.19>	<0.15,0.25,0.59>
C2	<0.85,0.09,0.05>	<0.35,0.03,0.61>	<0.57,0.01,0.41>
C3	<0.95,0.01,0.03>	<0.87,0.05,0.07>	<0.47,0.25,0.27>
C4	<0.78,0.07,0.14>	<0.71,0.09,0.19>	<0.17,0.15,0.67>
C5	<0.81,0.03,0.15>	<0.37,0.03,0.59>	<0.39,0.11,0.49<
C6	<0.95,0.01,0.03>	<0.87,0.03,0.09>	<0.67,0.03,0.29>
C7	<0.17,0.13,0.69>	<0.15,0.05,0.79>	<0.39,0.35,0.25>

D2	X1	X2	X3
C1	<0.61,0.15,0.23>	<0.67,0.13,0.19>	<0.55,0.25,0.19>
C2	<0.75,0.09,0.15>	<0.35,0.13,0.51>	<0.57,0.01,0.41>
C3	<0.85,0.01,0.13>	<0.77,0.05,0.17>	<0.47,0.15,0.37>
C4	<0.67,0.17,0.15>	<0.61,0.19,0.19>	<0.87,0.05,0.07>
C5	<0.81,0.03,0.15>	<0.37,0.13,0.49>	<0.79,0.07,0.13>
C6	<0.85,0.01,0.13>	<0.87,0.09,0.03>	<0.57,0.13,0.29>
C7	<0.19,0.13,0.67>	<0.05,0.15,0.79>	<0.19,0.15,0.65>

D3	X1	X2	X3
C1	<0.51,0.15,0.33>	<0.57,0.13,0.29>	<0.65,0.15,0.19>
C2	<0.65,0.19,0.15>	<0.45,0.13,0.41>	<0.67,0.01,0.31>
C3	<0.85,0.11,0.03>	<0.67,0.15,0.17>	<0.57,0.13,0.29>
C4	<0.57,0.17,0.25>	<0.51,0.19,0.29>	<0.93,0.01,0.05>
C5	<0.71,0.13,0.15>	<0.47,0.13,0.39>	<0.79,0.11,0.09>
C6	<0.75,0.11,0.13>	<0.77,0.09,0.13>	<0.67,0.13,0.19>
C7	<0.13,0.07,0.79>	<0.15,0.15,0.69>	<0.17,0.17,0.65>

Step 4. Using the equation (12), the weighted normalized decision-making matrices (D1,D2,D3) $[A_{ij}]_{mxn}$ are found as:

D1	X1	X2	X3
C1	<0.31,0.40,0.33>	<0.33,0.31,0.38>	<0.06,0.47,0.68>
C2	<0.52,0.21,0.25>	<0.21,0.16,0.69>	<0.35,0.14,0.53>
C3	<0.67,0.12,0.16>	<0.62,0.15,0.19>	<0.33,0.33,0.36>
C4	<0.46,0.23,0.30>	<0.42,0.24,0.34>	<0.10,0.29,0.73>
C5	<0.45,0.16,0.38>	<0.20,0.16,0.70>	<0.21,0.23,0.63>
C6	<0.47,0.14,0.35>	<0.43,0.16,0.39>	<0.33,0.16,0.52>
C7	<0.13,0.17,0.74>	<0.11,0.10,0.82>	<0.29,0.38,0.36>

D2	X1	X2	X3
C1	<0.26,0.40,0.41>	<0.29,0.38,0.38>	<0.24,0.47,0.38>
C2	<0.46,0.21,0.33>	<0.21,0.24,0.61>	<0.35,0.14,0.53>
C3	<0.60,0.12,0.24>	<0.55,0.15,0.28>	<0.33,0.24,0.45>
C4	<0.40,0.31,0.31>	<0.36,0.33,0.34>	<0.51,0.21,0.25>
C5	<0.45,0.16,0.38>	<0.20,0.24,0.63>	<0.43,0.19,0.36>
C6	<0.42,0.14,0.42>	<0.43,0.21,0.35>	<0.28,0.24,0.52>
C7	<0.14,0.17,0.72>	<0.04,0.19,0.82>	<0.14,0.19,0.70>

D3	X1	X2	X3
C1	<0.22,0.40,0.48>	<0.25,0.38,0.45>	<0.28,0.40,0.38>
C2	<0.40,0.30,0.33>	<0.27,0.24,0.53>	<0.41,0.14,0.45>
C3	<0.60,0.21,0.16>	<0.48,0.24,0.28>	<0.40,0.23,0.38>
C4	<0.34,0.31,0.39>	<0.30,0.33,0.42>	<0.55,0.18,0.23>
C5	<0.39,0.24,0.38>	<0.26,0.24,0.55>	<0.43,0.23,0.34>
C6	<0.37,0.23,0.42>	<0.38,0.21,0.42>	<0.33,0.24,0.46>
C7	<0.10,0.12,0.82>	<0.11,0.19,0.74>	<0.13,0.21,0.70>

Step 5. Using equation (13), the aggregated group decision matrix (AM) $[A_{ij}]_{7x3}$ based on the obtained weighted normalized decision matrices (D1, D2, D3) of all decision-makers is constructed as:

AM	X1	X2	X3
C1	<0.26,0.40,0.40>	<0.29,0.36,0.40>	<0.20,0.44,0.46>
C2	<0.46,0.23,0.30>	<0.23,0.21,0.61>	<0.37,0.14,0.51>
C3	<0.63,0.14,0.18>	<0.55,0.18,0.25>	<0.36,0.26,0.40>
C4	<0.40,0.28,0.33>	<0.36,0.30,0.37>	<0.42,0.22,0.35>
C5	<0.43,0.18,0.38>	<0.22,0.21,0.62>	<0.37,0.21,0.43>
C6	<0.42,0.16,0.39>	<0.41,0.19,0.38>	<0.31,0.21,0.50>
C7	<0.12,0.15,0.76>	<0.09,0.15,0.79>	<0.19,0.25,0.56>

Step 6: The positive ideal solution J^+ of the attributes $c_i \in C(i=1,2,...,m)$ from the aggregated group decision matrix (AM) $[A_{ij}]_{7x3}$ is determined according to the Definition 17 as:

$$J^+ = \left(\mu_i^+, \eta_i^+, \nu_i^+\right) = \begin{pmatrix} \max\limits_i \mu_{ij}, \min\limits_i \eta_{ij}, \min\limits_i \nu_{ij} \\ \mid i = 1,2,...,m \end{pmatrix}$$

$$J^+ = \begin{pmatrix} \overset{C1}{\langle 0.29,0.36,0.40\rangle}, \overset{C2}{\langle 0.46,0.14,0.30\rangle}, \overset{C3}{\langle 0.63,0.14,0.18\rangle}, \overset{C4}{\langle 0.42,0.22,0.33\rangle}, \\ \overset{C5}{\langle 0.43,0.18,0.38\rangle}, \overset{C6}{\langle 0.42,0.16,0.38\rangle}, \overset{C7}{\langle 0.19,0.15,0.56\rangle} \end{pmatrix}$$

Step 7: The proximity scores $s(d_{L_1})$, $s(d_{L_2})$, and $s(d_{L_\infty})$ of alternatives A_j are calculated according to Definitions 11, 13 and 15 as:

Measure	X1	X2	X3
$s(d_{L_1})$	0.073	0.257	0.237
R_j	1	3	2
$s(d_{L_2})$	0.091	0.220	0.188
R_j	1	3	2
$s(d_{L_\infty})$	0.263	0.606	0.608
R_j	1	2	3

Additionally:

- The L_1 norm is calculated as the sum of the absolute values of the vector.
- The L_2 norm is calculated as the square root of the sum of the squared vector values.
- The L_∞ norm is calculated as the maximum vector value.

Ranking Alternatives and Decision Making: The proposed uncertainty set-based MCDM approach utilizes a proximity measure to rank alternatives. This measure calculates the closeness of each alternative to an ideal solution within the uncertainty set framework. Lower proximity scores indicate a greater preference for that alternative. Based on the calculated proximity scores, the alternatives are ranked in ascending order, with the lowest score representing the most desirable option.

Step 8: In the context of the presented stealth combat aircraft selection problem, the ranking orders (R_j) of alternatives (A_j) obtained through the uncertainty-based MCDM analysis using (L1), (L2), and (L∞) norms according to Definitions 11, 13, and 15 are as follows:

R_j	Ranking orders of alternatives
$R(L_1)$	$X_1 \succ X_3 \succ X_2$
$R(L_2)$	$X_1 \succ X_3 \succ X_2$
$R(L_\infty)$	$X_1 \succ X_2 \succ X_3$

These ranking orders indicate that alternative (X_1) exhibits the lowest proximity to the ideal solution within the defined uncertainty set. Therefore, according to the analysis, (X_1) is the most suitable choice for the given decision scenario. The correlation analysis of the ranking orders of alternatives is found as follows:

	$R(L_1)$	$R(L_2)$	$R(L_\infty)$
$R(L_1)$	1		
$R(L_2)$	1	1	
$R(L_\infty)$	0,5	0,5	1

Strengths of Uncertainty Sets: Uncertainty sets offer several advantages for tackling MCDM problems characterized by ambiguity and imprecision. Here are some key points:

Structured representation of uncertainty: Uncertainty sets provide a structured framework for incorporating uncertainty information into both criteria weights and attribute evaluations. This allows for a more nuanced and realistic representation of the decision-making environment.

Robust ranking mechanism: The proximity measure employed within the uncertainty set framework facilitates the establishment of a robust ranking order for alternatives. By considering the inherent ambiguity in the data, the method avoids overly simplistic rankings based on single-point estimations.

Enhanced decision-making: By accounting for uncertainty, uncertainty set-based MCDM analysis empowers decision-makers with a more comprehensive understanding of the trade-offs involved in selecting the most suitable alternative.

Future Considerations: While the current study demonstrates the effectiveness of the proposed approach for the stealth combat aircraft selection problem, further research can explore potential extensions:

Comparative analysis: Comparisons with existing MCDM methods under uncertainty could provide valuable insights into the relative strengths and weaknesses of the uncertainty set-based approach.

Sensitivity analysis: Investigating the sensitivity of the ranking order to variations in the initial uncertainty parameters would further enhance the robustness of the methodology.

Generalization:Exploring the application of the uncertainty set-based MCDM framework to a wider range of MCDM problems with diverse complexities would solidify its broad applicability.

In conclusion, this study introduces an uncertainty set-based MCDM approach for ranking alternatives in the presence of ambiguity and imprecision. The application to a real-world stealth combat aircraft selection problem demonstrates the effectiveness of the proposed methodology in providing a robust ranking order for decision-making. The inherent flexibility and adaptability of uncertainty sets position them as a valuable tool for navigating uncertainty in various MCDM scenarios.

IV. Conclusion

This study introduces uncertainty sets as a vital mathematical framework for confronting the pervasive ambiguity and imprecision inherent in real-world decision-making scenarios. The increasing recognition of complexities in multiple criteria decision-making analysis (MCDM), particularly within fuzzy environments, necessitates robust methodologies for handling uncertainty. This work addresses this need by introducing uncertainty sets and exploring their applications in decision-making processes.

The introduction of uncertainty sets establishes a foundation for understanding and managing uncertainty systematically. These sets provide a structured approach to represent and navigate imprecise information, offering a promising avenue for grappling with the intricacies of decision-making under uncertainty. Basic concepts surrounding uncertainty sets are elucidated, setting the stage for subsequent development and analysis.

The focus then shifts towards employing uncertainty sets in the context of MCDM problems. The challenge of incorporating uncertainty information into criteria weights and attribute evaluations is addressed, aiming to rank alternatives while accounting for inherent ambiguity. A novel proximity method is developed within the uncertainty set framework to establish a robust ranking order for alternatives, navigating the complexity of decision-making with confidence.

To illustrate the applicability of this approach, uncertainty-based MCDM analysis is applied to a practical problem: stealth combat aircraft selection. By carefully considering seven evaluation attributes and three candidate alternatives, the effectiveness of this methodology is demonstrated in a real-world scenario. The results underscore the potency of uncertainty sets in informing decision-making processes, offering valuable insights into complex decision landscapes.

Beyond aircraft selection, the proposed uncertainty-based MCDM analysis holds broad applicability across diverse fields. Researchers and practitioners can leverage this approach to tackle a myriad of real-life problems, from resource allocation and project prioritization to risk assessment. The flexibility and adaptability of this method position it as a valuable tool for navigating uncertainty and making informed decisions in an ever-changing environment.

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