

Optimal Construction Using Multi-Criteria Decision-Making Methods

Masood Karamoozian, Zhang Hong

Abstract—The necessity and complexity of the decision-making process and the interference of the various factors to make decisions and consider all the relevant factors in a problem are very obvious nowadays. Hence, researchers show their interest in multi-criteria decision-making methods. In this research, the Analytical Hierarchy Process (AHP), Simple Additive Weighting (SAW), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) methods of multi-criteria decision-making have been used to solve the problem of optimal construction systems. Systems being evaluated in this problem include; Light Steel Frames (LSF), a case study of designs by Zhang Hong studio in the Southeast University of Nanjing, Insulating Concrete Form (ICF), Ordinary Construction System (OCS), and Precast Concrete System (PRCS) as another case study designs in Zhang Hong studio in the Southeast University of Nanjing. Crowdsourcing was done by using a questionnaire at the sample level (200 people). Questionnaires were distributed among experts in university centers and conferences. According to the results of the research, the use of different methods of decision-making led to relatively the same results. In this way, with the use of all three multi-criteria decision-making methods mentioned above, the PRCS was in the first rank, and the LSF system ranked second. Also, the PRCS, in terms of performance standards and economics, was ranked first, and the LSF system was allocated the first rank in terms of environmental standards.

Keywords—Multi-criteria decision making, AHP, SAW, TOPSIS.

I. INTRODUCTION

OPTIMAL construction refers to the process of making decisions and selecting the best alternatives in construction projects based on multiple criteria or objectives. Multi-criteria decision-making (MCDM) methods are used to facilitate this decision-making process by considering various factors and criteria simultaneously. MCDM methods provide a systematic approach to evaluate and compare different alternatives based on multiple criteria, which may include cost, time, quality, sustainability, safety, and other relevant factors. These methods help decision-makers consider the trade-offs and make informed decisions that align with their project goals and objectives.

The choice of an optimal construction system has always been one of the concerns of employers [1], because if chosen improperly, its consequences for several years will be felt by the beneficiaries of the project [2], [3]. The process of selecting the optimum system, due to the involvement of different criteria, is considered a multi-criteria decision. Some efforts

have been made for this purpose, such as a review of the application of such methods in construction [2], [4]. Other researches have been done for the similar topics such as selection of construction equipment, [5] and comparison of modern structural systems [6] or selecting optimal project delivery system [7] also choosing appropriate contracting method [8] or construction investments [3]. In this research, we are going to compare the results with three methods, including: AHP, TOPSIS, and SAW.

This research is an attempt to select the optimal construction system by using the following methods: (1) AHP [9], (2) SAW [10], [11], and (3) TOPSIS [12], [13]. Applying these methods has the following advantages over the previous research: (1) using TOPSIS and SAW decision-making methods in addition to the AHP method, (2) use of the following multiple criteria for a more comprehensive assessment of options, (3) a larger statistical community in order to obtain more reliable data, (4) evaluation of the most common building systems in the field of mass production, and (5) increasing the ranking of MCDM methods. This research aims to familiarize with MCDM practices such as AHP, TOPSIS, and SAW; familiarity with important criteria in evaluating and comparing optimal construction systems; and provision of solutions for the development of industrialization of buildings in the country with the approach of using modern construction materials.

This research is organized into parts as follows: The research methodology and the criteria and sub-criteria choosing are presented in Section I. The data processing according to the AHP method is discussed in Section II and the SAW data process is shown in Section III. Data process using the TOPSIS for the criteria and sub-criteria is presented in Section IV. Selection of options based on integration techniques is explained in Section V. Calculation of the inconsistency rate of pair comparisons is discussed in Section VI. Calculation of the sum of squared errors is presented in Section VII. And finally, in Section VIII, the conclusive remarks are presented.

II. THE RESEARCH METHODOLOGY

In this research, four modern construction systems have been evaluated and compared by experts. These systems include frame systems, LSF construction style [14], [15], ICF [16], OCS [17], and PRCS [18]. The reason for choosing these systems has been confirmed by the Iranian Research and Development Center for Building and Housing and their prevalence in mass production projects. To understand the

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effective criteria in choosing housing construction systems, by reviewing the internal and external research carried out in this background, various sources have been used to obtain criteria. The structure of criteria and sub-criteria for optimal construction systems is presented in Table I.

TABLE I
 THE STRUCTURE OF CRITERIA AND SUB-CRITERIA FOR OPTIMAL CONSTRUCTION SYSTEM

Criteria	Sub-criteria
Executive criteria	Existence of executive regulations
	Compatibility with modular design
	Ability to diversify in architectural design
	Low execution stages
	The rigidity of the ceiling
	Dependence on the use of heavy machinery
	System status in terms of thermal insulation
	System status in terms of sound insulation
	The lack of seasonal constraints in the implementation process
	The pre-fabricated ability
	No need for skilled workforce
	Compatibility with Non-Structural Elements (Installations)
	Fire safety
	Ease of quality control
Durability and durability of materials and elements	
The ability to make later changes	
Economic criteria	Low construction costs
	Low build time
	Ease of supply of materials in the inside of the country
	Low maintenance costs
	Fast return on investment
Environmental Criteria	Low energy consumption in design, construction, and operation
	Recycling of materials
	The lack of environmental pollution
	Compliance with climate conditions
	Impact on the labor market
	No traffic disturbance during construction
	Providing worker's health and safety
	Providing visual beauty
Compatibility with Iranian Islamic culture	

The general formula for determining the sample size is [19], [20]:

$$n = \frac{Z_{\alpha/2}^2 pq}{e^2} \quad (1)$$

In this equation we will have, n: Minimum required sample size, $Z_{\alpha/2}^2$: Standard variable value (for a 95% confidence level, the value of that in the related table is equal to 1.96), e: an error that the researcher makes in a survey that is usually chosen in the research between 0.01 to 0.1, which in this study is equal to 0.08, p: the ratio of success among sample individuals, due to the uncertainty, (0.5) has been used for its maximum value (1-p): The proportion of unsuccessful outcomes among the sample individuals. Again, due to uncertainty, the maximum value of 0.5 is used. Use of this method causes the selected sample to be sufficiently large. By inserting the corresponding numbers in (1), the sample number of 150 will be achieved.

III. ANALYTICAL HIERARCHY PROCESS METHOD

At first, normalization was done on the matrix of comparing the options against the sub-criteria by the method of clock normalization [21], [22] and vector of options weight in the method numerical mean will be gained. Then, the rating of each option against the sub-criteria is obtained by observing the relative weight of the following criteria. By combining the last column of the matrices related to the performance of the options against the sub-criteria, the matrix of options rating is obtained from the main criteria, Table II.

TABLE II
 MATRIX OF OPTIONS RATING AGAINST THE MAIN CRITERIA

Option	Criteria		
	Executive criteria	Economic criteria	Environmental criteria
LSF	0.25	0.242	0.296
ICF	0.23	0.243	0.227
OCS	0.24	0.232	0.235
PRCS	0.28	0.283	0.242

At this stage, we need to get the weight of the main criteria. For this purpose, firstly the main comparing criteria matrix should be normalized with Saaty method [23] and then we get the vector of the main criteria by the method of the numerical mean, see Table III.

TABLE III
 COMPARATIVE WEIGHT VECTOR OF MAIN CRITERIA

Criteria	Comparative weight of criteria
Executive criteria	0.33
Economic criteria	0.43
Environmental Criteria	0.24

$$\begin{bmatrix} 0.25 & 0.242 & 0.296 \\ 0.23 & 0.243 & 0.227 \\ 0.24 & 0.232 & 0.235 \\ 0.28 & 0.283 & 0.242 \end{bmatrix} * \begin{bmatrix} 0.33 \\ 0.43 \\ 0.24 \end{bmatrix} = \begin{bmatrix} 0.2628 \\ 0.2333 \\ 0.2356 \\ 0.2682 \end{bmatrix} \quad (2)$$

From the multiplication of the matrix of options points against the main criteria in the matrix of the main criteria, (2), we obtain the final score of the options that are based on the ranking of options in the AHP method, see Table IV.

TABLE IV
 RANKING OPTIONS BASED ON THE AHP METHOD

Option	Options points	Ranking
LSF	0.2628	2
ICF	0.233315	4
OCS	0.235659	3
PRCS	0.268226	1

IV. SIMPLE ADDITIVE WEIGHTING

Step1. The Normalization of Decision-Making Matrix by Linear Normalization Method: The normalization of a decision-making matrix is a process used to standardize the values within the matrix, making it easier to compare and analyze the different criteria or alternatives. One method commonly used for normalization is the Linear Normalization Method. This normalization process

helps to eliminate any bias caused by differences in the magnitude or scale of the original values. It allows decision-makers to compare and evaluate the criteria or alternatives on a consistent basis.

TABLE V
NORMALIZED DECISION-MAKING MATRIX BY LINEAR NORMALIZATION METHOD

Option	Criteria		
	Executive criteria	Economic criteria	Environmental criteria
LSF	0.89	0.85	1
ICF	0.82	0.86	0.77
OCS	0.86	0.82	0.79
PRCS	1	1	0.82

Step2. Determining the Weight of the Criteria by Shannon Entropy Method [24]: We consider the matrix of the rating of options in terms of the main criteria, normalized by the Saaty normalization method [23], see Table VI.

TABLE VI
NORMALIZED MATRIX OF OPTIONS POINTS IN TERMS OF MAIN CRITERIA

Option	Criteria		
	Executive criteria	Economic criteria	Environmental criteria
LSF	0.25	0.242	0.296
ICF	0.23	0.243	0.227
OCS	0.24	0.232	0.235
PRCS	0.28	0.283	0.242

Step3. Entropy is calculated by (3):

$$E_j = -K \sum_{i=1}^m (p_{ij} \cdot \ln p_{ij}) \quad (3)$$

Step4. K is calculated as the constant value by (4) and holds the value of E_j between zero and one:

$$K = \frac{1}{\ln m} = \frac{1}{\ln 5} = 0.61 \quad (4)$$

Step5. Given the above relation, the value of E is obtained for the main criteria Table VII

TABLE VII
CALCULATION OF E_j

	Executive Criteria	Economic Criteria	Environmental Criteria
E_j	0.843958	0.843821	0.842178

Step6. Determining the degree of deviation or degree of deviation from the information generated by the j^{th} criteria, (5) and Table VIII:

$$d_j = 1 - E_j \quad (5)$$

TABLE VIII
CALCULATION OF d_j

	Executive Criteria	Economic Criteria	Environmental Criteria
d_j	0.156042	0.156179	0.157822

Step7. For weight indexes (W_j), we will have (6)-(8):

$$w_1 = \frac{d_j}{\sum_{i=1}^n d_j} = \frac{0.156042}{0.47} = 0.3319 \quad (6)$$

$$w_2 = \frac{d_j}{\sum_{i=1}^n d_j} = \frac{0.156179}{0.47} = 0.3323 \quad (7)$$

$$w_3 = \frac{d_j}{\sum_{i=1}^n d_j} = \frac{0.157822}{0.47} = 0.3358 \quad (8)$$

Step8. The most suitable option (A^*) is obtained by (9), Finally, options ranking of the research through saw method are as Table IX.

$$A^* = \{A_i | \max \sum w_j r_{ij}\} \quad (9)$$

TABLE IX
OPTIONS RANKING BASED ON THE SAW METHOD

Option	Options points of SAW method	Ranking
LSF	0.914	2
ICF	0.817	4
OCS	0.823	3
PRCS	0.939	1

V. USING TECHNIQUE FOR ORDER PREFERENCE BY SIMILARITY TO IDEAL SOLUTION

Step1. Calculating the Normalized Decision Matrix: First, we normalize the matrix of comparing the options in terms of the main criteria with the Euclidean normalization method [25], (11). In this method, every element of the matrix is divided by sum of the square squares of the elements of each column (normalized j^{th} column by the index xi), (10).

$$n_{ij} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^n r_{ij}^2}} \quad (10)$$

$$N_D = \begin{bmatrix} 0.497 & 0.483 & 0.589 \\ 0.459 & 0.485 & 0.451 \\ 0.477 & 0.462 & 0.467 \\ 0.558 & 0.564 & 0.481 \end{bmatrix} \quad (11)$$

Step2. Making the non-scale balanced matrix (V), (12):

$$V = N_D \cdot W_{n \times n} = \begin{bmatrix} 0.1654 & 0.207 & 0.141 \\ 0.1514 & 0.208 & 0.108 \\ 0.158 & 0.1989 & 0.112 \\ 0.184 & 0.2426 & 0.115 \end{bmatrix} \quad (12)$$

Step3. We calculate the positive ideal solution (13) and the negative ideal solution (14):

$$A^+ = \{(\max V_{ij}) = \{v_1^+, v_2^+, \dots, v_j^+, \dots, v_n^+\} \quad (13)$$

$$A^- = \{(\min V_{ij}) = \{v_1^-, v_2^-, \dots, v_j^-, \dots, v_n^-\} \quad (14)$$

Results will be (15) and (16):

$$A^+ = \{0.184, 0.2426, 0.141\} \quad (15)$$

$$A^- = \{0.1514, 0.1989, 0.108\} \quad (16)$$

Step4. Calculating the spacing in Euclidean normalization for the positive (17) and negative (18) ideal solution:

The i^{th} spacing of ideals using the Euclidean method is:

$$d_{i+} = \left\{ \sum_{j=1}^n (v_{ij} - v_j^+)^2 \right\}^{0.5}; i = 1,2,3, \dots m \quad (17)$$

$$d_{i-} = \left\{ \sum_{j=1}^n (v_{ij} - v_j^-)^2 \right\}^{0.5}; i = 1,2,3, \dots m \quad (18)$$

The results will be (19) and (20):

$$d_{i+} = \begin{bmatrix} 0.0403 \\ 0.0579 \\ 0.0588 \\ 0.0285 \end{bmatrix} \quad (19)$$

$$d_{i-} = \begin{bmatrix} 0.0365 \\ 0.0094 \\ 0.00761 \\ 0.0552 \end{bmatrix} \quad (20)$$

Step5. Calculating the relatively close space to the ideal solution, Table X.

TABLE X
 CALCULATION OF THE RELATIVE PROXIMITY OF AI TO THE IDEAL SOLUTION

Option	d_{i+}	d_{i-}	$C_i = \frac{d_{i-}}{(d_{i+} + d_{i-})}$
LSF	0.0403	0.0365	0.475
ICF	0.0579	0.0094	0.14
OCS	0.0588	0.00761	0.114
PRCS	0.0258	0.0552	0.682

Step6. Rank of options according to the order of the deviations.

TABLE XI
 RANKING OF OPTIONS BASED ON THE TOPSIS METHOD

Option	C_i	Ranking
LSF	0.475	2
ICF	0.14	4
OCS	0.114	3
PRCS	0.682	1

Step7. Selection of options based on integration techniques [26]: In the real world, decision-makers do not confine themselves to a decision-making method and may use different methods to achieve different results. In these circumstances, techniques have been proposed to combine the rankings of techniques such as Average ranking method, Copeland and Borda rule [27]-[29], see Table XII.

VI. CALCULATION OF THE INCONSISTENCY RATE OF PAIR COMPARISONS

The incompatibility rate of the paired comparison of the main criteria and the paired comparisons of options are calculated in

terms of the sub-criteria, all paired comparisons were confirmed in terms of inconsistency rates. These calculations prove the validity and reliability of the paired comparison of this research, Table XIII.

TABLE XII
 RANKING SYSTEMS BASED ON RANKING INTEGRATION METHODS

System	Average ranking method	Copeland method	Borda method
LSF	2	2	2
ICF	4	4	4
OCS	3	3	3
PRCS	1	1	1

TABLE XIII
 CALCULATION OF THE PAIRED COMPARISON MISMATCH RATE

Paired comparison	Inconsistency Rate
The main criteria comparison	0.021
Comparison of options in terms of implementing regulations	0.001143
Comparison of options in terms of compatibility with the modular design	0.00192
Comparison of options in terms of diversity in the architectural design	0.001039
Comparison of options in terms of low execution stages	0.005144
Comparison of options in terms of the rigidity of the ceiling	0.002892
Comparison of options in terms of non-dependence on the use of heavy machinery	0.002237
Comparison of options in terms of the system's thermal insulation status	0.042298
Comparison of options in terms of the system's sound insulation status	0.003264
Comparison of options in terms of lack of seasonal constraints in the execution method	0.037145
Comparison of options in terms of pre-fabric ability	0.007366
Comparison of options in terms of lack of skilled workers	0.088436
Comparison of options in terms of compatibility with non-structural elements (facilities)	0.00202
Comparison of options in terms of fire safety	0.03092
Comparison of options in terms of ease of quality control	0.003023
Comparison of options in terms of durability and reliability of materials and elements	0.043226
Comparison of options in terms of the ability to make later changes	0.049148
Comparison of options in terms of cost of construction	0.07995
Comparison of options in terms of build time	0.00064
Comparison of options in terms of ease of supply of materials within the country	0.008623
Comparison of options in terms of maintenance costs	0.018492
Comparison of options in terms of speed return on investment	0.08938
Comparison of options in terms of the ability to recover materials	0.000385
Comparison of options in terms of energy consumption in design, construction, and operation	0.085186
Comparison of options in terms of non-production of environmental pollution	0.003588
Comparison of options in terms of compliance with climatic conditions	0.065838
Comparison of options in terms of impact on the job market	0.042751
Comparison of options in terms of not causing traffic jams during construction	0.003058
Comparison of options in terms of providing workers' health and safety	0.004043
Comparison of options in terms of providing visual beauty	0.052823
Comparing options in terms of compatibility with Iranian culture	0.014859

VII. CALCULATION OF THE SUM OF SQUARED ERRORS

In order to obtain the closest method to the final result, we use the sum squared error method (21):

$$RSS = \sum_{i=1}^n (y_i - f(x_i))^2 \quad (21)$$

y_i is the final rank obtained and $f(x_i)$ is the rank obtained by any of the multi-index decision methods; the results are as in Table XIV.

TABLE XIV
 CALCULATION OF THE SUM OF SQUARED ERRORS

MCDM method	RSS
AHP	0
SAW	0
TOPSIS	1.41

VIII. CONCLUSION

In this paper, the optimal construction system is investigated by using three methods. Using MCDM methods for selecting optimal construction system results in relatively similar results in the ranking of construction systems; therefore, based on all three methods used in this research, AHP, SAW and TOPSIS, concrete PRCS and the LSF system was ranked first and second, respectively.

Based on the combination of all three methods (Average ranking method, Borda method, Copeland method), construction system rankings, were completely identical and PRCS, LSF, (3D panel system) and ICF were ranked first to fourth, respectively. Given the similarity of the results of the combined methods, this ranking can be considered as the final ranking of the new construction systems discussed in this study.

According to the sum of squared errors, the results of the two methods of AHP and SAW were identical with the results of the combined methods; but the TOPSIS method, compared to the combined methods, has a percentage error. Therefore, in this research, AHP and SAW methods are better than TOPSIS and have yielded more satisfactory results.

Due to the fact that the best option in terms of performance criteria was PRCS, it is suggested that contractors use a prefabricated concrete construction system in projects that are subject to operational constraints and require specific operational considerations.

Due to the fact that the best option in terms of environmental criteria was LSF, it is suggested that contractors in areas sensitive and vulnerable to environmental degradation, use a LSF system in order to implement massive projects.

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The optimal method presented in this research for the implementation of industrial construction projects is a PRCS, which not only allows for its industrial production, but also offers such advantages as architectural flexibility, installation compatibility, high speed installation of parts, rapid return of investment, energy saving, proper performance in terms of

thermal insulation and sound, etc. Therefore, it can be used as an appropriate method for the implementation of building and manufacturing of industrial housing. Finally, MCDM methods can be favorably applied to determine the optimal system selection. MCDM methods are used in this context, which provides a platform for transforming complex issues into simpler issues within which the planner can evaluate options with the help of criteria and sub-criteria. Given the fact that there are a lot of criteria in selecting the optimal construction system for the construction industry, and the need for a multi-criteria decision, the use of MCDM methods can be helpful in choosing the best possible solution.

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