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Numerical Simulation and Experimental Validation of the Tire-Road Separation in Quarter-Car Model

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***Abstract*—**The paper investigates vibration dynamics of tire-road separation for a quarter-car model; this separation model is developed to be close to the real situation considering the tire is able to separate from the ground plane. A set of piecewise linear mathematical models is developed and matches the in-contact and no-contact states to be considered as mother models for further investigations. The bound dynamics are numerically simulated in the time response and phase portraits. The separation analysis may determine which values of suspension parameters can delay and avoid the no-contact phenomenon, which results in improving ride comfort and eliminating the potentially dangerous oscillation. Finally, model verification is carried out in the MSC-ADAMS environment.

***Keywords*—**Quarter-car vibrations, tire-road separation, separation analysis, separation dynamics, ride comfort, ADAMS validation.

I. Introduction

VEHICLE ride analysis deals with the vibrational characteristics of the vehicle regarding either occupant comfort or tire-road contact [1]. In terms of vibration dynamics, the current studies focus on analyzing, designing and providing a good ride quality related to the tire-road contact assumption, which could be defined as the tire cannot reach the realistic situation [2]. Such a condition would cause an unsatisfying result and major shortcomings of the analytical methods. For more realistic analyses, indeed, the vibration dynamics of passenger vehicles must consider the tire-road separation progress [3]. In the literature on separation dynamics, it has been recently proved that the possibility of separation phenomenon between the wheel and the ground provides a system closer to the real system [1]-[5]. When the wheels lose contact with the surface, the system will be discontinuous and have new constraints with two different states namely in-contact and no-contact states [6]. Therefore, it is more complex to solve governing equations related to its discontinuous states for high degrees of freedom (DOF) systems such as a bicycle-car, a half-car, and a full-car. Furthermore, although a quarter-car model is the simplest, it is good enough to provide fundamental insight into vibration dynamics and capture the most significant vertical responses. Thus, this study looks into how the separation phenomenon affects vertical dynamics of a 2DOF quarter-car model and verifies the ride analysis of the system in the ADAMS medium regarding the separation condition.

II. Dynamic Equations of Motion

It can be seen from Fig. 1, a 2DOF quarter-car model examines numerically the vertical vibration considering tire-road separation [5]. The symbols m_s and m_u present for sprung and unsprung masses, respectively. The stiffnesses of main spring and tire are k_s, k_u. The damping value of the tire is extremely small compared to the main suspension damping; indeed, the simple system does not affect the accuracy of its dynamic response when the tire's damping is ignored [7]. The displacements away from the loaded equilibrium position of the masses are indicated by x_s and x_u, whilst the displacement of the road is shown by y, it can be also shown by x_r, interchangeably.

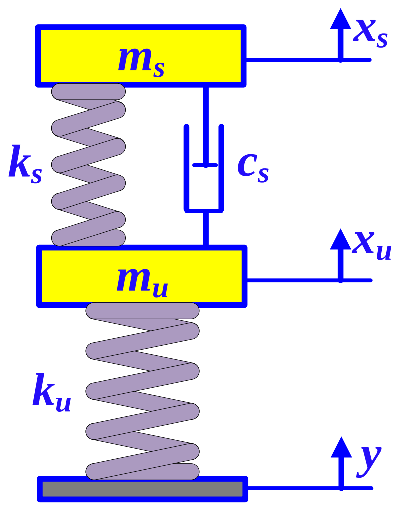

Fig. 1 A 2DOF quarter-car model

The governing equations of the sprung and unsprung masses in the contact state [8]:

$$m_s\ddot{x}_s + c_s(\dot{x}_s - \dot{x}_u) + k_s(x_s - x_u) = 0 \quad (1)$$

$$m_u\ddot{x}_u + c_s(\dot{x}_u - \dot{x}_s) + (k_u + k_s)x_u - k_s x_s = k_u y \quad (2)$$

If the unsprung mass has a vertical displacement smaller than

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the relaxed radius of the tire, the vehicle suspension is in contact with the road. By contrast, the no-contact and free flight will occur if the unsprung mass has displaced a distance larger than the geometric radial distance [7]. Moreover, Kashyzadeh et al. suggested that if the sprung masses have a large vertical displacement, the tires will lose contact with the road [9]. Wong also concluded that that loss of contact is probable to happen approximately at the tire's peak dynamic deflection [10]. Based on both attitudes of the separation condition, Sabri considers a vibration model including two bodies with road surfaces, the relationship between tire force and vertical displacement of two bodies can be modelled in an in-contact zone [11], [12]. Therefore, the separation condition is the relative displacement between displacements of the unsprung mass and road excitation starting greater than the static tire compression x_{ST}.

$$x_u - y \geq x_{ST} \quad \text{no-contact} \qquad (3)$$
$$x_u - y < x_{ST} \quad \text{in-contact}$$

where,

$$x_{ST} = \frac{(m_u + m_s)g}{k_u}$$

In the no-contact state (3), the equation of the unsprung mass will be [2], [6]:

$$m_u\ddot{x}_u + c_s(\dot{x}_u - \dot{x}_s) + k_s(x_u - x_s) + (m_u + m_s)g = 0 \qquad (4)$$

The differential equation system is switched from (1) and (2) to (1) and (4) for the sprung and unsprung masses, respectively, when the separation condition (3) guaranteed. Thus, the system would be discontinuous and more complex.

III. Dynamic Responses

Being convenient for investigation and validation, assuming that the model vibrates harmonically with a cosine function $y = y_o cos\omega t$, where y_o is input amplitude of road, ω is the excitation frequency.

TABLE I
THE PARAMETERS OF A QUARTER-CAR MODEL

Parameters	Value [Unit]	Parameters	Value [Unit]
m_s	49.5 kg	k_s	15760 N/m
m_u	5.5 kg	k_u	197900 N/m
c_s	3000 Ns/m	x_{ST}	0.0027 m

Using data from a practical quarter-car model in Table I [2], [6], the time response displays in Fig. 2. Herein, the separation phenomenon happens at a high frequency $\omega = 30\, rad/s$. To observe the tire-road separation, the indicator variable I is introduced to identify which is the no-contact state (I = 0.02) or the in-contact state (I = 0). It can be noted that the separation interval remains approximately equal for each cycle and settle into a steady state after passing through the initial period. Therefore, the system, in this case, is called steady-state separation.

Transient separation can occur if tire-road separation conditions are guaranteed, however, it does converge immediately when the contact is regained. A transient separation might be viewed from Fig. 3 when the input frequency decreases to $\omega = 15\, rad/s$, it affects slightly the entire response of the system because there is a tiny deviation from those of the system considering no-separation assumption.

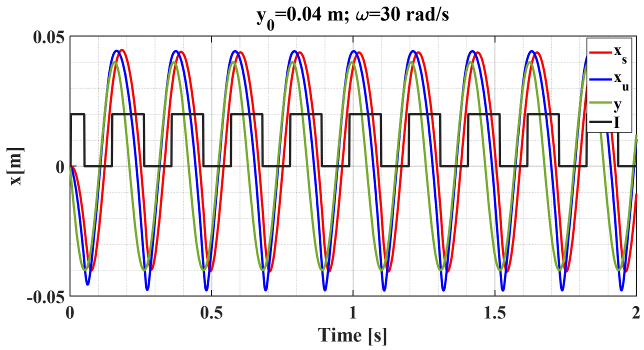

Fig. 2 Time response with steady-state separation

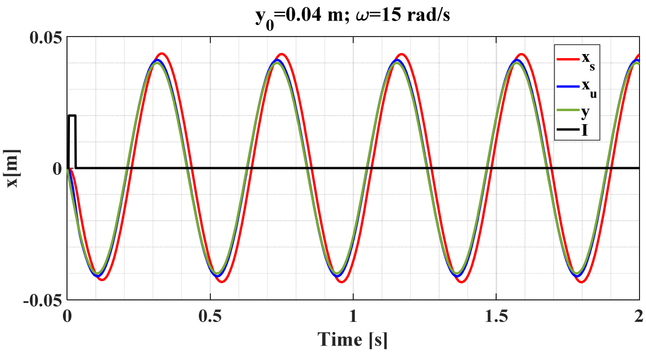

Fig. 3 Time response with transient separation

At a low value of excitation frequency $\omega = 10\, rad/s$, Fig. 4 denotes the time response in an in-contact zone. The system always is in contact with the ground, in other words, the tire is inseparable and cannot break the contact condition, it is also known as a continuous contact. With a no-separation period, both sprung and unsprung components will tend to follow the road profile including road amplitude and the indicator I coincides with the x axis.

The separation also happens at a high road amplitude and high input frequency as demonstrated in Fig. 5. Once the wheel loses its road contact, the vertical displacements of sprung and unsprung masses x_s, x_u are difficult to predict as a linear system, period (I = 0).

Keep using the input data from Fig. 5, an example of the change of x_s, x_u expresses in Fig. 6, the solid curves present the displacements of the real system considering the separation assumption compared to dotted curves without the separation consideration, they separate in the no-contact state (I ≠ 0) and only coincide in each contact.

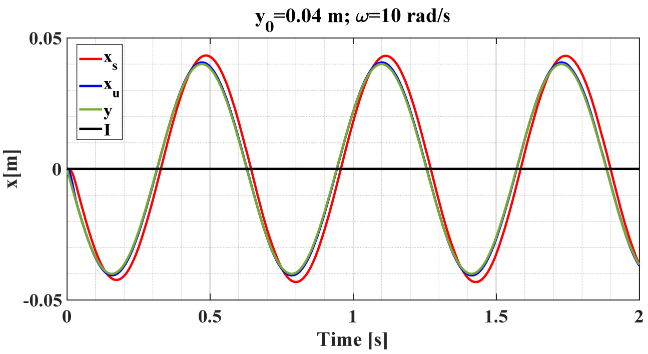

Fig. 4 Time response with no-separation

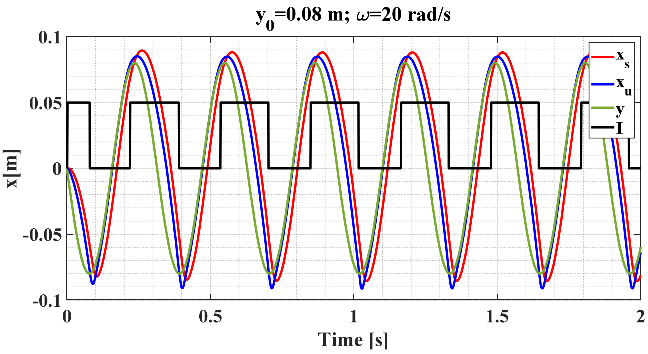

Fig. 5 Separation with high road amplitude

Among all factors affecting the ride comfort, vibrations of sprung mass play an important role as it generates vertical force upon the body of passengers. A given series of time response of the sprung mass x_s can be seen from Fig. 7. The multiple displacements are simultaneously in response to a high frequency excitation ω = 30 rad/s as illustrated in three planes as denoted by y_0, the value of road amplitude y_0 makes a significant increase for the sprung mass deflection from $y_0 = 0.04\ m$ to $y_0 = 0.08\ m$. During tire contact with the surface, all dynamic behaviors seem to be a harmonic response, but the phenomenon is not similarly real systems with separation consideration and the unexpectable acting on the vertical ride comfort has been reduced tremendously at high amplitudes of road excitation.

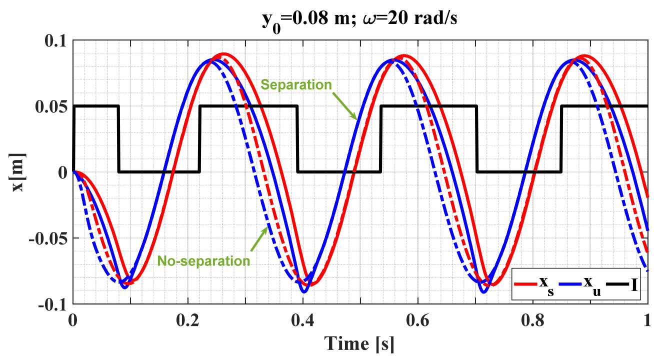

Fig. 6 Comparison of displacements between in-contact and no-contact assumptions

A very useful and popular strategy to observe behaviors of nonlinear systems is phase plane analysis. A phase portrait could typically be a two-dimensional graph or a three-dimensional graph in the configuration space of the dynamic system, which presents the trajectories in terms of two systemic state variables when the system releases from a set of distinct initial conditions. Using parametric inputs in Figs. 6 and 8 shows phase portraits of displacements of sprung and unsprung masses corresponding to their velocity. For the in-contact assumption, Fig. 8 (c) containing three directions of x_s, v_s, and the investigation time creates a $3-D$ helix of the geometric displacement of sprung mass corresponding to an ellipse in the two-dimensional plane as revealed in Fig. 8 (a), while there is a significant deviation in the separation response from the ellipse. In terms of the unsprung mass, Figs. 8 (b) and (d) demonstrate the correlation between the displacement x_u and its velocity v_u in phase portraits; the trajectories of $x_u - v_u$ curve deviate from the in-contact to no-contact consideration.

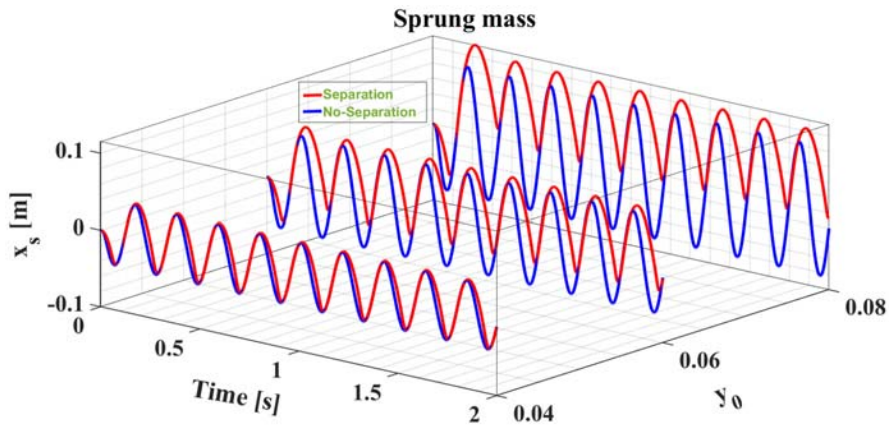

Fig. 7 Series of sprung mass's displacement

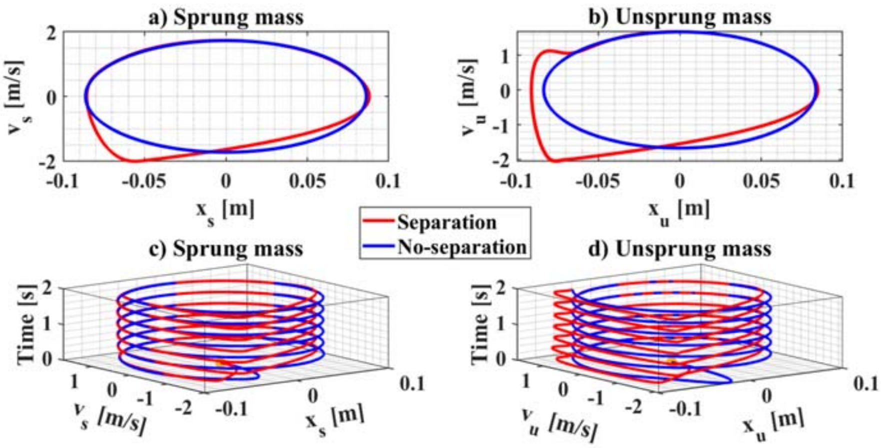

Fig. 8 Phase portrait of separation phenomenon

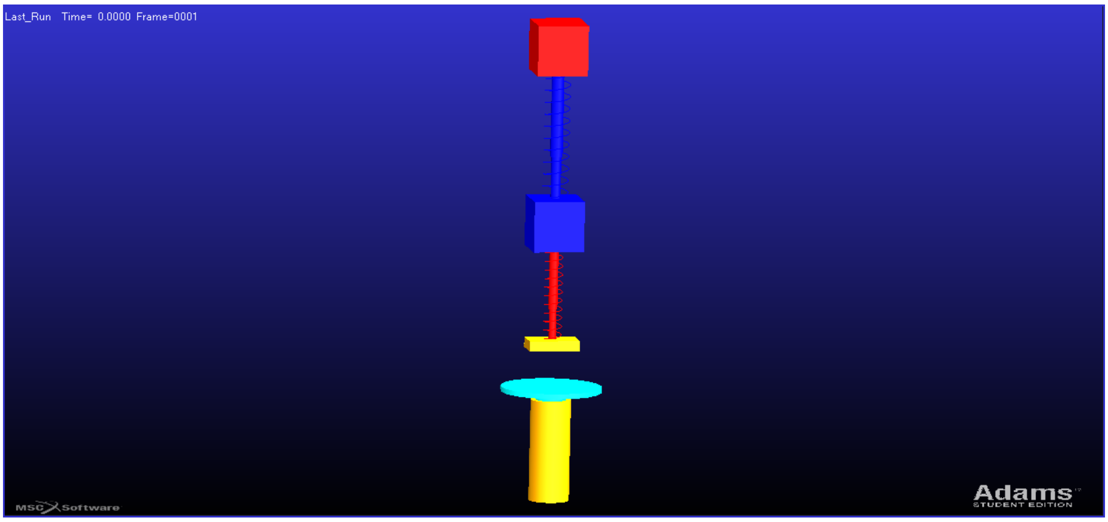

Fig. 9 The experimental setup in ADAMS medium

Generally, the vibration analysis shows that the real response has a significant difference compared to the model without the separation assumption. Therefore, the vertical dynamics of the vehicle must be re-examined to improve the accuracy of the modeling approach and the work in engineering projects.

IV. Experimental Validation

Comparing analytic solutions with experimental results on the vertical response of the system considering tire-road separation to be the most convincing stage for verification purposes. In fact, the experimental validation of the separation dynamics of the suspension systems through either experiments or an alternative industry-standard multibody dynamics software will be validated in this section. To ensure a fair comparison, the simulation test model in the ADAMS platform has to use the same vibration analysis parameters as mentioned above. For this reason, the suspension parameters of the practical model are extracted from Table I in Section III.

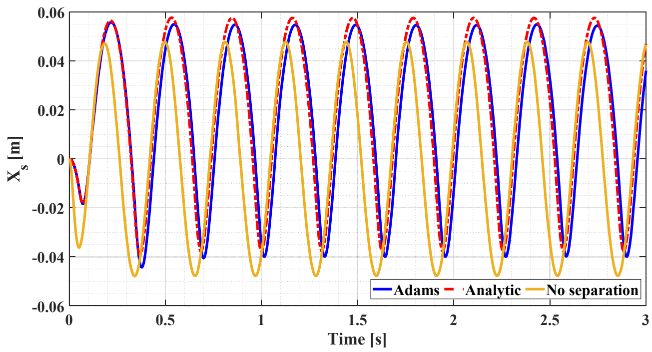

Fig. 10 Comparison of ADAMS and analytical solutions for sprung mass

In ADAMS, each sprung mass has assembled to an unsprung mass through corresponding joints, adjust redundant constraints, as well as applied forces. To perform the validation experiment, the external excitation is a single-component force generated by a motor, $y = y_o cos\omega t$ to vibrate the quarter-car model in the time domain. The experimental setup of the quarter-car model, which is a one-dimensional system of two masses, two springs and one damper, has been shown in Fig. 9. Herein, two block-shaped elements constrained to the ground by using frictionless translational joints, indeed, the model has two DOF. The experimental scenario is recorded to save numerically data into MATLAB file to compare with the analytical outcome.

For data from Table I and the road excitation $y = 0.04cos20t$, the experiment is carried out in 500 seconds to match the reality, and to present results clearly, the graphs are plotted with suitable periods. Fig. 10 reveals an excellent match of the displacements of sprung mass x_s between the ADAMS validation (solid curve) and analytic solution (dotted curve). In contrast, a considerable discrepancy between the results and the no-separation curve is made.

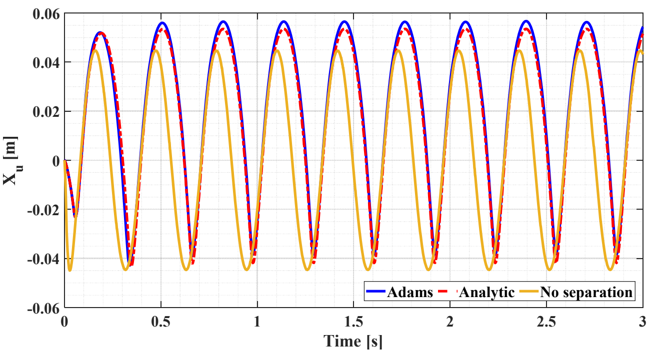

Fig. 11 Comparison of ADAMS and analytical solutions for unsprung mass

Similarly, there is also a great agreement between the analytical and experimental results for the component x_u of unsprung as could be inferred from Fig. 11.

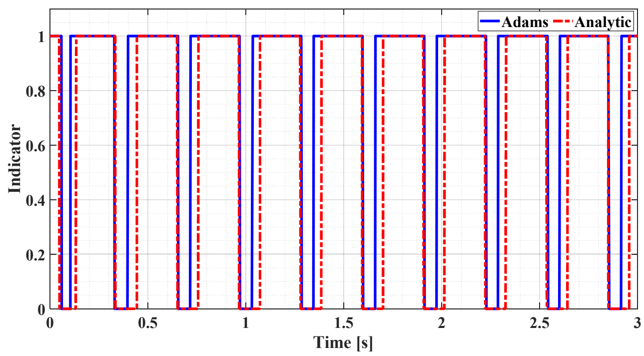

Fig. 12 Comparison of ADAMS and analytical solutions for indicators

In further detail, the indicators demonstrate the tire-road separation period corresponding to value 1 as shown in Fig. 12. The ADAMS indicator (solid curve) is not entirely consistent with the analytical indicator (dashed curve); nevertheless, they are reasonably close. The deviation may be caused by the elasticity at the contact point between the sprung mass and the piston surface. Overall, the analytical results considering the tire-road separation are much closer to the realistic ADAMS model compared to the traditional one without accepting the tire separation.

V. CONCLUSION

The paper shows the tire-road separation to present what part of the time responses and phase portraits must be re-investigated in the vibrating system. The governing equation system of motion is solved numerically in both states involving in-contact and no-contact. The analytic solutions conduct a wide range of road excitation from low to high input amplitudes. A comparison of dynamic responses between the real system and the system without separation consideration is analyzed in detail. Finally, the vertical vibration considering no-contact regimes has been validated in the ADAMS platform.

NOTATION

Sprung mass	m_s [kg]
Unsprung mass	m_u [kg]
Suspension spring stiffness	k_s [N/m]
Tire stiffness	k_u [N/m]
Main suspension damper	c_s [Ns/m]
Excitation frequency	ω [rad/s]
Amplitude of road	y_0 [m]
Road excitation	y [m]
Time	t [s]
Displacement of sprung mass	x_s [m]
Displacement of unsprung mass	x_u [m]
Static tire deflection	x_{ST} [m]
The acceleration of gravity	g [m/s^2]

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