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Quantum-Like Approach for Deriving a Theory Describing the Concept of Interpretation

Yehuda Roth

Abstract—In quantum theory, a system's time evolution is predictable unless an observer performs measurement, as the measurement process can randomize the system. This randomness appears when the measuring device does not accurately describe the measured item, i.e., when the states characterizing the measuring device appear as a superposition of those being measured. When such a mismatch occurs, the measured data randomly collapse into a single eigenstate of the measuring device. This scenario resembles the interpretation process in which the observer does not experience an objective reality but interprets it based on preliminary descriptions initially ingrained into his/her mind. This distinction is the motivation for the present study in which the collapse scenario is regarded as part of the interpretation process of the observer. By adopting the formalism of the quantum theory, we present a complete mathematical approach that describes the interpretation process. We demonstrate this process by applying the proposed interpretation formalism to the ambiguous image "My wife and mother-in-law" to identify whether a woman in the picture is young or old.

Keywords—Interpretation, ambiguous images, data reception, state matching, classification, determination.

I. Preface Terminologies in Quantum Mechanics

This preface refers to the non-relativistic quantum mechanics. While implementing quantum mechanics, we distinguish two cases.

Case I. Quantum systems that before being subject to measurement, **spontaneously** evolve over time in accordance to the Scrödinger equation [1].

Case II. In this preface, we refer to quantum computers, for which it is necessary to define an algorithm that determines how the system progresses over time. This quantum algorithm needs to consider an end user, i.e., a quantum observer who needs to perform measurement to read the output data [2].

The difference between these cases relates to "the measurement problem," i.e.,, the interaction between a quantum system (except for quantum-low-temperature systems are microscopically small) and macroscopic surroundings. This preface recalls how each case offers different terminologies in terms of the "hard measurement" description [3] and justifies the terminologies to be used in this study.

Case Elaboration

[Case I] This system's time evolution is associated with two types [1]:

Deterministic time-evolution, described by the linear Schrödinger equation. Macro-objectivation, an alternative name for the measurement process. It describes a stochastic process generated by a nonlinear and stochastic term added to the Schrödinger equation. Describing both processes reduces the need for defining an observer concept. Moreover, as the description for those processes relates to spatial or momentum spaces, in several scenarios it is practical to implement the concept of the wave function ($\psi(\vec{r}) = \langle\vec{r}|\,|\psi\rangle$ or $\psi(\vec{p}) = \langle\vec{p}|\,|\psi\rangle$, i.e., the state projection extends over the spatial or momentum spaces) instead of settling for the fundamental concept of states. [Case II] Quantum computer's implement a two-state-system to represent a qubit such as the quantum states $|\uparrow\rangle$-$|\downarrow\rangle$ of a $1/2$-spin. In general, quantum information systems may consider spatial distances in entangled states, but the qubit's distribution along the spatial or momentum space is usually irrelevant. Therefore, most quantum computers do not need to consider the wave function description. In this scenario, the Schrödinger equation, which describes spontaneous time evolution, becomes less significant. Instead, the quantum system evolves according to the algorithm that operates a sequence of unitary operators (logical gates). Owing to this lack of spontaneity built into the system's definition, quantum computer systems are more likely to use the terminology of an observer representing a programmer or end-user. This study presents the possibility of building quantum machines with interpretation abilities. By considering this study as another branch of quantum computers and following this preface, we describe our system with states instead of wave functions and implement the observer terminology.

II. Introduction

Until the beginning of the 20th century, physicists espoused a philosophy in which physics describes an objective reality where measurements do not affect the results [4], [5]. In this context, the interpretation concept, which is related to the observer's personal view, was considered unscientific. Quantum theory's unique and challenging nature, particularly the collapse scenario [6], caused bifurcation in this approach. In this study we relate this measurement scenario into the subjectivity and interpretation [7], [8] concepts as follows:

Subjectivity The ability of selecting a particular measuring device is associated with the observer subjectivity.

Interpretation The fact that the same measuring device may provide different results is associated with an interpretation process.

In its entirety, the device output does not describe an objective reality but a match between the measured entity and one of the device states, i.e., it is the measuring device

Yehuda Roth is with Oranim College, Israel (e-mail: yudroth@gmail.com).

that interprets reality. Interpretation plays a key role in several aspects of life. We can say that "without interpretation, we cannot understand our data" [9]. In a few areas, e.g., science, the necessity to generate language as accurately as possible is mandatory. For example, for consistency in terminology, translators employ computer software that reuses previously translated text [10]. However, there is room for the translator's interpretation in the world of art, such as in literature; the same work may possess different versions depending on the translator's personality. Following this logic, if any software or machine provides machine-dependent translation, we may associate the devices with some level of individuality.

In this study, we associate a part of this interpretation process with a quantum-like measurement scenario, where different devices generate different results. Owing to the collapse scenario, even identical measuring devices can generate different results that yield various *self-interpretations*. Mathematical background of this approach toward the interpretation concept is presented in this study.

III. Interpretation in Visual Ambiguity and Recognition of Conceptual Collapse

With the term "conceptual collapse," we refer to a process that possesses similar characteristics to a quantum collapse, i.e., it is described with the same mathematical tools that are used in quantum mechanics but are not defined within the framework of the quantum theory [6]. Sharing the same mathematical formalism introduces the possibility of implementing quantum-like mathematical tools to design a real quantum-based device for realizing an interpretation process, i.e., a machine capable of performing self-interpretation.

In this study, we focus on ambiguous figures to demonstrate quantum-like interpretation formalism. These ambiguous figures cause visual interpretation between separated image forms [11], [12]. We focus on the ambiguous figure "my wife and my mother-in-law" shown in Fig. 1 [13], [14]. In a preliminary observation of Fig. 2, we recognize young (herein the observer recognizes as Yana) and old (named Olive) ladies. However, when we observe Fig. 1 (herein refereed to Yana-Olive), our perception spontaneously reverses between the two ladies' images [11]. Thus, like the quantum-collapse scenario, in which the output of the measurement collapses into a single option out of several states [6], we associate this "Yana-Olive" spontaneously reversed perception with the quantum-like collapse. In the following sections, as part of a model for describing the interpretation process, we provide the mathematical definition for this conceptual collapse.

Illustrations such as Olive+Yana (Fig. 1) have led researchers to trace the factors that influence the way we interpret images. Joseph Jastrow and other psychologists have suggested that vision is not a technical action like photography; i.e., it is not simply a context-free sensory perception but an interpretation influenced by various factors such as an emotional state [15]. That is to say, Fig. 1 shows only spots that appear in shades of black and white. It is the observer that provides the interpretation of Olive or Yana. The observer's interpretation is subject to the observer's history

Fig. 1 Example of an ambiguous image: Is it an old lady or a young lady?

Fig. 2 The image of Fig. 1 is "visually separated" into the interpretations when the illustrations show: a-the young woman, "Yana" and b-the older woman, "Olive"

[15]-[18]. For example, observers who view images of old ladies before viewing Fig. 1 will have higher probability of recognizing Olive. In this study, we show that the observer's biased interpretation of Yana or Olive is related to the coefficient magnitudes in a superposition expansion of a state that represents the ambiguous image of Fig. 1 in terms of the interpreted images of Yana and Olive.

IV. Categories of Interpretation Described by Spaces

A. Categories

Our interpretation scenarios comprise of three categories:

i) Event category: This category describes the event shared by all observers. The description of an event, before being detected in the interpretation system, has nothing to do with the space definition. It is the process of interpretation that associates the measured event with a space such as the Hilbert or Fock ones.
ii) Audience category: The term "audience" refers to observers detecting the same event. The audience space will be discussed in the future.
iii) Personal category: It represents a single observer who interprets the occurrence of activities, and this is the focus of our study.

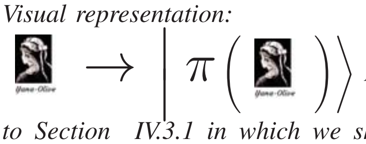
We now further elaborate on these categories.

B. Activities within Personal Space

The process of interpretation consists of four stages:

i) **State construction**: In state construction, the system transforms the items to be interpreted into a state in a Hilbert space. This state is denoted as $|\pi\,(Image)\rangle$. Details describing a transformation from an object (such as a simple image) into a state are not discussed in this study; however, a procedure for generating coherence was described in [19], where a nonlinear approach was implemented.

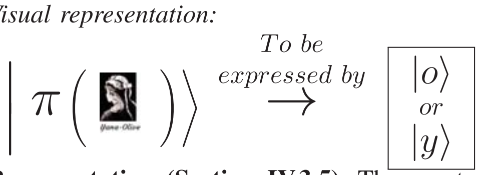
Visual representation:



For more details, refer to Section IV.3.1 in which we show how to associate the raw data with a state in a pixel representation and Section IV.3.2 in which we discuss orthogonality issues

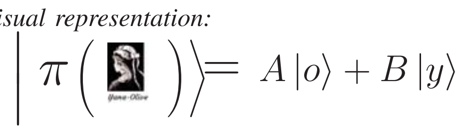
ii) **Classified representation**: The system defines the states (concepts) to be used to interpret the information received. Before the interpretation process, Fig. 1 is merely an unspecified collection of spots. Interpretation requires the definition of concepts to which the system translates the figure, and the classified states represent these. In our case, the spots collection will be translated into the concepts *old* and *young* with the corresponding states $|o\rangle$ and $|y\rangle$, respectively. To implement these $|o\rangle$ and $|y\rangle$ abstract concept as a concrete physical entity, one can assign states, e.g., $|\uparrow\rangle$ or $|\downarrow\rangle$ of a $1/2$-spin-particle, to define the *old* or *young* concepts. For example, spins pointing upward or downward can stand for old or young women, respectively. Identifying a spin's orientation with the age of a woman can be obtained if one associates the spin location with a specific area, representing the concept in the interpretation machine.

Visual representation:



iii) **Representation (Section IV.3.5)**: The constructed state is represented in terms of classification states.

Visual representation:



iv) **Determination section IV.3.6**: The state collapses into one of the classification states.

Visual representation:

$$A\,|o\rangle + B\,|y\rangle \;\begin{array}{l}\nearrow\ |o\rangle \\ \searrow\ |y\rangle\end{array}$$

C. Mathematical Details

1) Pixel basis of states: To clarify, the following definition of pixels and hue states is unrelated to the physical nature of electromagnetic radiation. It is a mathematical representation of images printed on paper or displayed on a screen. We recall that in the state construction stage, the received rough data are represented by a state $|\pi\,(Image)\rangle$ in the Hilbert space. This section aims to demonstrate the feasibility of translating an image (as on paper) into a state belonging to Hilbert space.

Our description consists of two sets spanning the following spaces:

i) *Hue set*: belongs to $1D$ space representing shade intensities.
ii) *Pixel basis of states*: Dividing an image into squares, we associate each square with a state $|i,j\rangle$ that defines a square location.

To build an image state, states from the hue space are projected on the pixels states to associate each pixel state with an amplitude. The Superposition of pixel states with the corresponding amplitude defines an image. For simplicity, we present a mathematical model describing black-and-white images.

The requirements for orthogonal states necessitate negative amplitudes. To allow this, we set a zero-valued amplitude in the hue space to be gray, so that darker and lighter states (compared with the background state) demonstrate positive and negative amplitudes, respectively (see Table I). The Hilbert space that we associate with the hue space consists of a single state, $|\eta\rangle$. The intensity of the hue is determined by the state amplitude, and for consistency with the Hilbert space, a state amplitude for $A\,|\eta\rangle$, the hue intensity is multiplied by the factor A^2 (there is no reason here to define complex amplitudes).

TABLE I
HUE STATES

Amplitude (A)	**Hue**
1	
0	
-1	

To define the pixel basis of states, we divide the image into squares (pixels), where each pixel is associated with a state $|i,j\rangle$, with i,j identifying a pixel position in an image matrix. By implementing the unity operator $\sum_{i,j} |i,j\rangle\,\langle i,j|$ over $|\eta\rangle$, we obtain the following image state $|\iota\rangle$:

$$|\iota\rangle = \sum_{i,j} P_{i,j}\,|i,j\rangle\,, \qquad P_{i,j} = \langle i,j|\,\eta\rangle \tag{1}$$

According to our definition, the strength of a shade in a i,j-pixel is determined by the corresponding factor $P_{i,j}^2$.

2) Orthogonality in the ambiguous images of Yana and Olive: In our perception, a lady can be either young or old; the two situations cannot exist simultaneously. From an algebraic perspective, this is the definition of orthogonality responsible for ambiguity. However, in the separate images of Olive and Yana, based on the definitions of the pixel states, no orthogonality exists. Moreover, as there are numerous similar shades overlapping between equivalent pixels in both images, we realize that the two images are far from being orthogonal. Orthogonality appears only at the interpretation stage, where the pixel collection is classified into the concept "ladies age." These concepts are orthogonal because a lady cannot be old and young simultaneously. The images are displayed only after they are classified as the states $|o\rangle$ for Olive and $|y\rangle$ for Yana; they are orthogonal, i.e., $\langle o|\, y\rangle = 0$. By defining $|\pi(\,Image\,)\rangle$ as an image state presented in the pixels representation, we obtain

The images presented in the pixels, representations are not orthogonal.

$\Big\langle \pi\Big($  $\Big) \Big| \pi\Big($  $\Big)\Big\rangle \neq 0$ (2)

The classified states are orthogonal.

$$\langle o\, |y\rangle = 0$$

3) Interpretation space within a personal category : According to the Merriam-Webster dictionary, classification is a systematic arrangement of groups or categories according to established criteria[20]. To the observer's perception, Fig. 1 should be classified according to the lady's age criterion. As the observer is unable to obtain an accurate distinction, the data "collapse" into a single concept: old or young. We provide the mathematical background for the **Data reception**, **Classification** and **Determination** process.

4) Data reception: The rough data (in our scenario, Fig. 1) arrive at the observer.

5) Classification and representation: The classified states for which the image will be interpreted are $|o\rangle$ and $|y\rangle$ for the old and young ladies, respectively. In the observer perception, these states are defined as orthogonal. To categorize $\Big| \pi\Big($  $\Big)\Big\rangle$ in terms of $|o\rangle$ and $|y\rangle$, we implement the operator $\mathbb{C} \stackrel{\text{def}}{=} |o\rangle\langle o| + |y\rangle\langle y|$, which is referred to as a classification operator. Thus, we obtain the following:

$\mathbb{C}\, \Big| \pi\Big($  $\Big)\Big\rangle = A\,|o\rangle + B\,|y\rangle$,

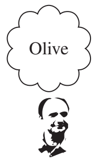
where,

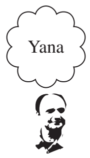
$A = \Big\langle o\, \Big| \pi\Big($  $\Big)\Big\rangle$ (3)

$B = \langle y\, \Big| \pi\Big($  $\Big)\Big\rangle$

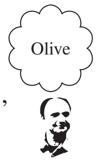
Note that the states $|o\rangle$, $|y\rangle$, and $\Big| \pi\Big($  $\Big)\Big\rangle$ belong to the same Hilbert space: *an observer's personal space*. As mentioned previously, the coefficients may vary according to changing circumstances [16]-[18]. For example, if an observer views Fig. 1 with no prior history, the probabilities of identifying Olive and Yana may be equal. By contrast, if he/she first views images of old women (not necessarily the woman from the right side of Fig. 2), the probability that he/she will recognize Olive increases. Thus, before the determination stage is activated, the determination system may possess many possible states that may determine the possibility of obtaining a specific interpretation. After the classification stage, when the image is represented as a superposition of the observer's classified states, the interpretation system determines the mechanism based on which the data should be interpreted. This is performed via a collapse-like procedure.

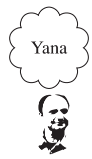
6) Determination—conceptual collapse: In [21] it was shown that the output of measurement should include not only numerical values but symbols, strings, or images. We implement this approach to consider the interpretation result in terms of such features. We proceed with the example of old and young ladies to demonstrate the last stage in the interpretation process. Given that $\Big| \pi\Big($  $\Big)\Big\rangle$ is represented in terms of the superposition of the classified states ($|o\rangle$ and $|y\rangle$), we apply the Born rule; accordingly, the probabilities of interpreting the image as Olive and Yana are $|A|^2$ and $|B|^2$, respectively. The corresponding observable is:

$\mathbb{D} =$ 

$|o\rangle\langle o| +$ 

$|y\rangle\langle y|$, (4)

where similar to [21], 

and 

serve as the measurement output. For numerical values, the eigenconcepts play the role of the eigenvalues. However, they can play the role of a visual output, sound, or any other concept that can serve as a measurement output.

Till now we considered the interpretation process as a mathematical formalism that can be implemented using a quantum-mechanics-based machine. Deviating from this description and allowing a possibility that our formalism does describe a real biological interpretation process, we refer to the eigenconcepts of (4) to symbolize a way the mind presents the observer with the concept of Olive or Yana. The cloud marks symbolize that this is a personal experience of the observer.

V. Observer's Role in the Process of Interpretation

The presentation of a process to describe interpretation does not obviate the need to define the observer concept. Earlier, the role of the observer was limited to performing a measurement and reading the results (i.e., reading the eigenvalues of the observable); now, the observer's role is to read the results of the interpretation process. In our formalism, this is the eigenconcept of the appropriate operator as shown in (4), for the old-young women. We can expand the current interpretation model to include an interpreting system that responds to an interpretation. From the mathematical perspective, following or replacing the eigenconcepts of (4) (such as  and ) with operators (eigenoperators) may help achieve the desired response. When these operators follow eigenconcept terms, it is known as a "conscious" response. By contrast, the absence of eigenconcepts with only a solitary standing operator represents an "unconscious reflexive reaction." In this context, we can associate the observer with the conscious concept. To summarize, we provide a table showing the complete analogy between the mathematical description of the interpretation process and the mathematical activities applied to the quantum theory and in the next section we illustrate a block diagram of the entire interpretation process.

Although the interpretation process presented in this study is purely mathematical, the analogy to the quantum theory raises the possibility of its realization by designing a device based on this theory. The following table presents the commonalities between these two areas.

TABLE II
ANALOGY BETWEEN THE INTERPRETATION PROCEDURE AND QUANTUM SYSTEM

General or mathematical description	Interpretation stage	Quantum analogy
State matching	The observer associates the arriving data with a state.	The observed item is represented by a coherent state.
Spanning set selection	Classification: the observer selects interpreting concepts, such as the ladies age.	The observer selects the measuring device.
States representation	The state of the data is expressed as a superposition of the possible interpretation states.	The observed physical state is expressed as a superposition of the states of the measuring device.
Collapse	Determination stage: The system determines the appropriate interpretation.	The physical state collapses into one of the device states.

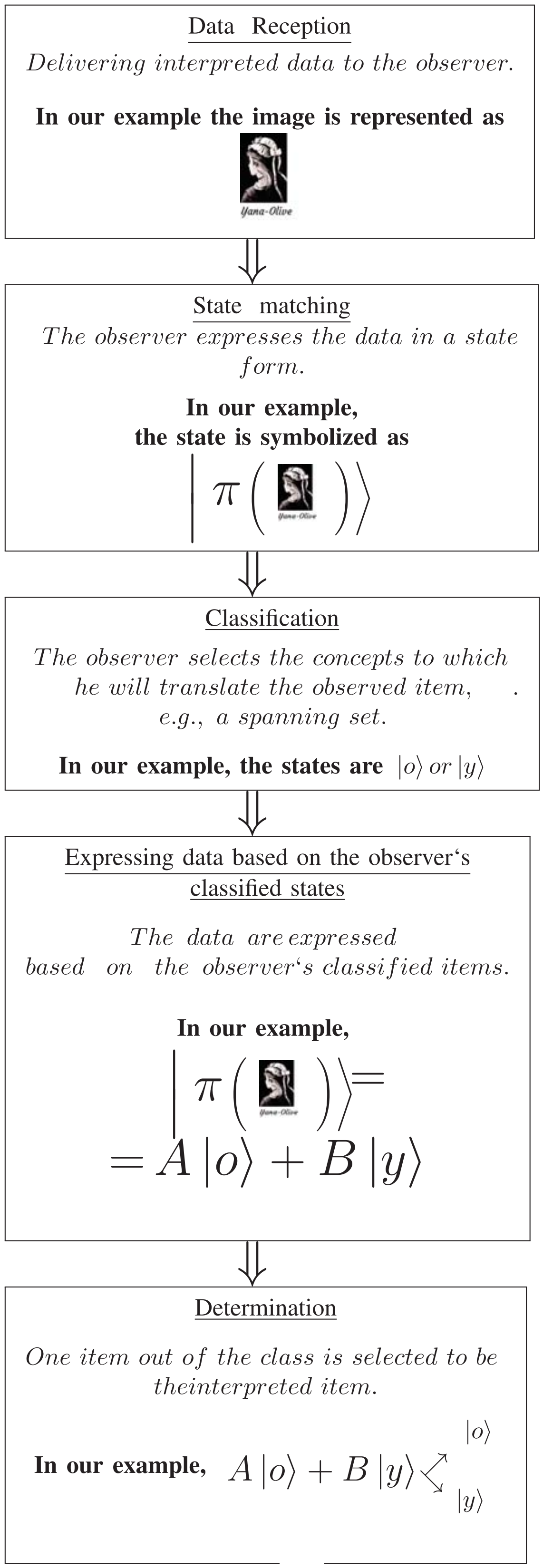


Fig. 3 Flowchart of the interpretation process

VI. Summary

The conventional description for quantum mechanics is that while solving the linear part of the Schrödinger equation, predictable time evolution is obtained. It is the collapse scenario that introduces randomness [1], [22]. Herein, we present an additional perspective on this randomness; we suggest treating it as part of a comprehensive process that can be implemented into an interpretation process. We do not rule out the possibility that the proposed mathematical description, especially the collapse process that exclusively characterizes quantum systems, describes the interpretation process that is done in biological systems. In other words, one can argue that our study implies that quantum processes are responsible for the interpretive activities in biological systems. However, in our formalism, we found no evidence to support the idea, so we preferred to introduce a concept of an interpretation machine instead of explaining how the brain performs this type of operation. And as evidence supporting our caution in linking brain activity to quantum behavior, we recall that we did not define quantum entities, such as spin-like neurons, in applying superposition relations as presented in Section IV.3.5. The study did present a complete analogy between our mathematical framework describing interpretation processes with quantum systems as summarized in Table II. Indeed, this analogy supports the possibility of associating brain activities with the quantum theory. But in our opinion, this analogy is not a sufficient justification for defining the interpretation process performed in the brain as quantum. Let us suggest another approach to dealing with the aforementioned "dilemma." Specific laws and concepts in physics can define a general criterion that determines the behavior of different systems. This behavior occurs without referring to the technical details of its occurrence. An example of this is the second law of thermodynamics which determines the spontaneous development of a macroscopic system's tendency to increase entropy. This law is so fundamental that it applies to physics and other fields such as biology. Note that the concept of entropy is common in all scientific areas.

This study implies that the concept of the observer is not necessarily unique only to the quantum theory but may influence a broader scientific context. Therefore, for the idea of the observer to be appropriate to serve as a scientific concept, it is necessary in the future to determine a unified definition that will be appropriate for quantum theory and other scientific fields.

In dealing with an interpretation machine, as suggested here, we must analyze the interactions between the two environments: A machine's internal environment defining the measuring device, and an external environment, is the one that the interpretation-device observes and interpret. According to measurement theory, both environments share the same Hilbert space. Suppose that we generate controlled environment's for interpreting machines and external surroundings and set each interpretation with "behaving" rules. For example, the magnitude of an amplitude of a constructed state depends on the interpretation result of another measurement (refer to (3) that describes the old-young ladies' example). This may induce a system that evolves according to the environments feedback interactions. Such relations may possess a nonlinear time evolution that yields diverse behaviors [23], [24]. If for instance, this evolution diverges into a single value, we will have a single interpretation- a certainty in the interpretation. Bifurcation scenario, i.e., a system that converges into two values leads to two possible interpretations such as the young-old-women presented here. A complete uncertainty may occur for systems that reach a chaotic result. Thus, we can explore the interpretation evolution of controlled systems which can simulate various scenarios for real interpretation evolution.

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This article is about interpretation. This idea is given by Lior, a guy with special needs with whom the author lived for 34 years. The author thanks him for that.

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