World Academy of Science, Engineering and Technology
International Journal of Computer and Systems Engineering
Vol:17, No:8, 2023

Intervention Targeting in Environmental Networks

Chukwudi Henry Dike

Abstract—We explore targeted subsidy in a set-up for which manufacturing firms in a waste-spillover network make endogenous production decisions. Here, games of substitution in digraphs arises where waste-producing firms internalise negative externality in a quadratic fashion. We find neutrality in intervention policies that create or reduce spillover links. Most importantly, we observe centrality distinction in asymmetric digraphs so that the dependence and power of each firm play unique roles. Here we see that in targeted subsidy, a firm with greater centrality guarantees optimal welfare improvement. This centrality however measures the weakness of each firm's Nash-based link to other neighbourhood firms i.e., lower negative externality.

Keywords—Centrality, externality, key-player, Nash-Equilibrium.

I. Introduction

In an increasingly environmentally conscious world, manufacturing behaviours and decisions are made to internalise so many cost aspects including pollution and other environmental hazards. A common example of such emission would be carbon which is seen as a key factor to the greenhouse effect. Pollution and environmental hazards a firm's business environment could arise not only from consumption of negative externality from other aspects of society but could also be a function of the firm's emissions and waste disposal techniques. In essence, decisions based on the presence of these spillovers can be argued to create an interaction fixed geographical network where proximity to other firms becomes material to the overall production and capacity utilisation of a manufacturing firm. We explore endogenous production rate determination in a fixed spatial network which takes the form of an "externality production/externality consumption" network. This is drawn from a growing literature in not just works on networks like [10] but also environmental-based papers including [15] and more recently, [25]. Here, firms account for administrative overheads associated with levels of total externality production and a percentage of externality consumption. Inter-firm externality production rate is the negative production externality linked to manufacturing. Additionally, firms are attentive to the cost arising from their waste management procedure including indirect cost accruing to the firm through the internalisation of pollution and environmental hazards associated with its activities (regardless of the source).

We study a scenario where this indirect cost arising from waste management is quadratic to a firm's aggregate pollution/environmental hazard. What we find is that the optimal externality production rate a firm undertakes is the result of an interaction of strategic substitution with other industries in its geographical network[1]. Such substitution behaviours are in line with renowned *public goods in network* literature including [11], [1], etc. Under mild conditions, Pure Strategy Nash Equilibrium (PSNE) externality production rates exist and are uniquely defined. We proceed to discuss some properties of our PSNE including the implications for the existence of corner solutions in which some firms may be inactive and as such refrains from engaging in manufacturing. A mild contribution found in our PSNE is such that it presents directed network conditions for equilibrium in strategic substitution games with linear best replies, an aspect surprisingly lacking in the literature.

For the rest of the paper, we discuss policy dynamics and optimal utilitarian welfare. Our first result highlights the welfare neutrality of firms, whose equilibrium externality production rate is positive, to policies which strengthen or weaken links between firms. The direct implication is that reducing/increasing a single or multiple firm externality production links to its neighbours leaves the firm's payoff, as well as aggregate payoff, unchanged. Subsidies directed at waste management expenses are, on the other hand, welfare improving.

Our final analysis constitutes our most valued results. Here, we discuss subsidy targeting within the system. While there are multiple ways of improving welfare through funds allocation, we are not able to establish which is the most optimal. However, we do show that in terms of individual targeting, firms with the least externality magnitude are the most likely to attain maximum outcomes. The rank of firms magnitude is based on its centrality measure which links is a modified form of the Bonacich centrality and one which is only the case given that our network is directed. Some key implications of this result are as follows: First one is the fact that this paper takes advantage of uni-directional properties in intervention targeting where agents (firms) are selected based on a one-way impact. The second aspect builds on this one-way implication in the sense that we see the importance in the distinction between incoming and outgoing based centrality in optimal intervention policies. Specific to this work, incoming centrality measures the independence of a firm while its outgoing measures the degree to which its actions harm other firms in the network.

A. Related Literature

As stated in the introduction, firms in this paper are primarily involved in individual production/manufacturing rate determination. This is as the extent of capacity utilisation here determines the profit of firms as well as their inter-firm

C.H. Dike is with the School of Economics, University of Kent, Canterbury, Kent, CT2 7NP, United Kingdom (e-mail: chd@kent.ac.uk).

[1]An implied assumption here is that all firms have full knowledge about their entire geographical network and their key parameters.

production externality production in the given period. As such, activities from linked firms are captured in such decision making. The issues surrounding environmental pollution and its effect on third parties is well established within literature involving environmental as well public economics, for example [18] discusses the impact of inadequate waste management (specifically with poor plastic disposal) on the quality of water in an environment (due to factors such as difficulty and prolonged time taken to degrade such materials). Further challenges with disposal of physical waste have been discussed in works including [24], [23], [28] (where they paid close attention to difficulties involved in informal waste recycling), [21] to mention but a few. In terms of the direct economic impact, we see works such as [29] where significant financial resources devoted towards waste disposal are broken down concerning the brewing sector in the Russian economy. Other than direct financial harm, poor disposal and a high degree of waste products are also detrimental to health [20] as well as other aspects of quality of life [22].

Arguments for the internalisation of production externality is common within environmental economics, especially in line with the Coasean Theorem. Specific works including [14] discusses a life-cycle based model is used in the energy sector. Others such as [16] and [17] estimate the cost-effectiveness of imposing carbon prices in contrast to taxes so as to reduce the carbon intensity to regulate the emissions to residents and the transport sector. In an attempt to study this internalisation, a branch of network-based environmental economics has emerged. This is based on *ecological interconnectedness* and has been studied in works including [25]. Additionally, we see another link to geographical network in [15][2]. However, few attempts have been given to link interactions in environmental networks to public good games which yield best reply functions that result in strategic substitution. This is in contrast to the broader field of behavioural economics where strategic substitution is common. This is more so within the topic of private-public good provision/consumption games most notably found in [1], [10], and [11] as well as key extensions including [2](which shows equilibrium in a fully bounded action profile). Such games of public good provision and more specifically private-public good provision can materialize in different ways in financial networks. That being said, the underlying ideas have not been aimed at identifying such behaviours in financial network games. A contrast of this work from seminal works including [10], and [1] is the close attention paid to interactions to an undirected network (with [11] providing initial intuitions as to weighted and direct network). While there are observable differences, intuitions are very useful in observing such behaviours in networks (including environmental and financial networks) which are uni-directional and weighted.

Our results on neutrality are in contrast to the invariance of private and public good consumption to income redistribution as initiated by [7] and extended to [27] as well as [1] (to networks). In these works, neutrality is described as the instance in which wealth transfer between agents[3] in a public good game leads to no change in the aggregate public good provision and individual consumption. A notable reason for the divergence in this paper arises from the fact that the intervention we consider is not particularly redistributive. Lastly, our targeting criterion has a lot of similarities to works like key player concepts in works like [4], [13], [6] as well as [5].

II. The Model

We assume a three-period industry consisting of $\hat{\mathcal{N}} = \{1, \ldots, n\}$ set of manufacturing firms. We denote the set of periods as $\mathcal{T}$ such that $\mathcal{T} = \{t-1, t, t+1\}$. For every firm $i \in \hat{\mathcal{N}}$, their neighborhood is denoted as $\mathcal{N}_i$ and $\mathcal{N}_i = \{\mathcal{N}_i^{out} \cup \mathcal{N}_i^{in}\} \subset \hat{\mathcal{N}}$ where $\mathcal{N}_i^{out}$ represents firms for which firm $i \in \mathcal{N}$ consumes negative production externality which we call *Producers* and $\mathcal{N}_i^{in}$ represents firms who receive firm $i \in \mathcal{N}$'s negative production externality which we call *Consumers*. Each firm has an overall production capacity measured at $l_i : l_i > 0 \quad \forall\, i \in \hat{\mathcal{N}}$. This interaction forms an industry geographical network $G(\hat{\mathcal{N}}, \hat{g})$ with g representing the weighted spillover (waste disposal) links between firms. The network $G(\hat{\mathcal{N}}, \hat{g})$ is set at $t-1 \in \mathcal{T}$ so that it is exogenous to period t and $t+1$. At $t+1$ links are dissolved so that we study a single-period interaction. Given that if firm $j \in \mathcal{N}_i^{in}$, then $\hat{g}_{ji} > 0$ while $\hat{g}_{ji} = 0$ otherwise so that for each $j \in \mathcal{N}_i^{in}$, $\hat{g}_{ji} \in]0,1]$ represents the degree of exposure (link) to spillover from firm i to firm j. Each firm decides to produce a proportion of its capacity which it determines at $t \in \mathcal{T}$ denoted as r_i so that $\forall\, i \in \hat{\mathcal{N}}$, then $r_i \in\ [0,1[$. We call refer to this value as the *manufacturing rate* or simply the *rate*. This rate r_i remains fixed for the rest of $\mathcal{T}$ once set.

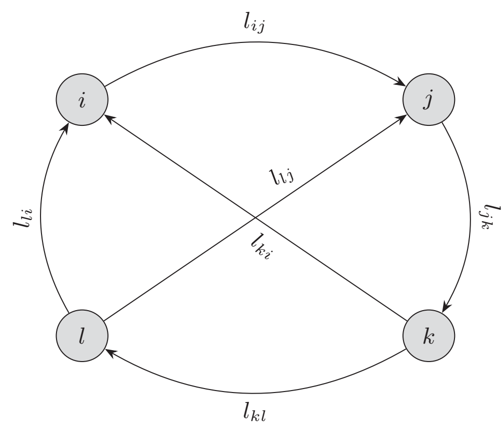
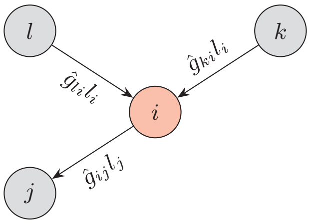


Fig. 1 A Bounded spillover Network (Industry): Arrows (edges) point to direction of third-parties and originate from producers

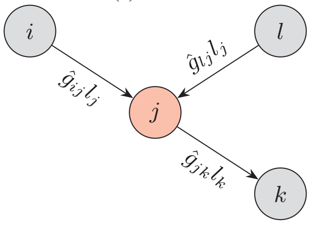
The firm $i \in \hat{\mathcal{N}}$ is primarily concerned about their producers and consumers as opposed to the entire network. It implies that regardless of the nature of the externality consumption network, each firm i can identify its position given a star that comprises of its producers and consumers.

[2]Whose discussion was based on the knowledge spillovers as they related citation patents for which being in the same geographical proximity increased chances of being cited.

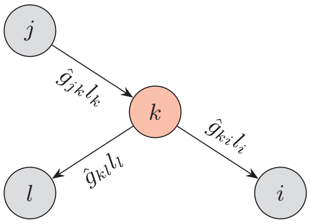
[3]As a criterion, agents whose endowments are taxed or subsidised need to be active.



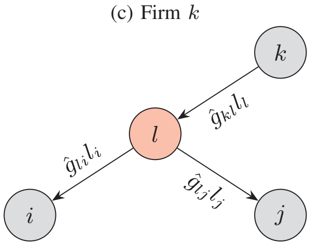
(a) Firm i



(b) Firm j



(c) Firm k



(d) Firm l

Fig. 2 Decomposed Network to capture pivotal links: We see that for decision-making purposes, it is direct incoming and outgoing links that are useful to a firm's decision

We take an example of a system $\hat{\mathcal{N}} = \{i, j, k, l\}$ as in fig. 1 where for each $\{i, j\} \in \hat{\mathcal{N}}$, $l_{ij} = \hat{g}_{ij}l_j$. We also observe the breakdown in fig. 2. It shows that from any directed network of externality consumption and externality production, for example, the network in fig. 1, relevant sub-networks can be derived. Such sub-networks as in fig. 2 capture each firm $i \in \hat{\mathcal{N}}$ externality production and consumption.

We describe a firm $i \in \hat{\mathcal{N}}$'s payoff where additionally, we assume that the firm incurs waste management expenses. This waste management expense increases directly with total waste produced and consumed by the firm through spillover from its 'producers'. As a determinant to the total administrative expenses, we include a feedback cost of waste spillover though the value given as $\kappa f(\mu_i(r_i)$. Here, we see the homogeneous constant κ which measures the level of efficiency in managing waste production generated by the $\mathcal{N}_i$ for each firm i. Our intuition is that a higher κ implies lesser efficiency in waste management while a lesser κ captures the extent to which firms internalise pollution/environmental hazards involving their manufacturing process due to the threat of increased regulation/penalty. It could also signal greater efficiency possibly arising from specialization, technical know-how, technological progress, and other factors that imply positive economies of scale for the firm in dealing with environmental hazards. The next part which we denote as μ_i for the given firm $i \in \hat{\mathcal{N}}$ captures the total firms production as well as a portion of the activities of its neighbours. More formally, we define $\mu_{i \in \hat{\mathcal{N}}}$ as follows;

$$\mu_i(r_i, r_{-i \in \mathcal{N}_i^{in}}) = l_i r_i + a \sum_{j \in \mathcal{N}_i^{in}} r_j \left(\hat{g}_{ji} l_j\right), \qquad (1)$$

where the parameter $a \in R_+$ captures recursive cost of waste spillover to a firm i which is a consumer. This is so that the greater κ is, the greater the perceived threat of regulation which forces firms to internalise more of pollution and other environmental hazards involved in their manufacturing process. Examples of this could include the purchase of protective equipment including face coverings, head protection, post-work health procedures for workers to reduce the risk of infections, etc. We shape the behaviour of this feedback section under the following assumption;

Assumption 1: $\forall$ firm $i \in \hat{\mathcal{N}}$, we hold that;

$$\frac{\partial f(\mu_i(r_i))}{\partial r_i} > 0, \quad \frac{\partial^2 f(\mu_i(r_i))}{\partial r_i^2} > 0.$$

We assume that the mapping $f(\mu_i) : R \rightarrow R_+$ for $R = (r_1 \times r_2 \times \ldots \times r_n)$ (the set of Strategy profile i.e., possible equilibrium rate of production by each firm) is convex for the proposed reason that cost rises at an increasing rate as total manufacturing activities (leading to waste production) to and from the firm rises. It should be said at this point that the parameter κ is normally expected to assume a very small value. As described earlier, the waste management/administrative cost includes resources devoted to the firm's waste production and consumption management.

A. *Payoffs and Strategic Substitution*

We then hold that the firm $i \in \hat{\mathcal{N}}$, $f(\mu_i) = (\mu_i)^2$ which fits well into assumption 1. Then the firm $i \in \hat{\mathcal{N}}$ has the following payoff function:

$$P_i(r_i | r_j, \ldots) = l_i r_i - \sum_{j \in \mathcal{N}_i^{in}} r_j \left(\hat{g}_{ji} l_j\right) - \kappa(\mu_i)^2. \qquad (2)$$

To elaborate, the payoff captures estimated profit of each firm $i \in \hat{\mathcal{N}}$ under the assumption that there is a fixed price for each product produced normalised to \$1. Hence, $l_i r_i$ is the direct revenue-other cost from production while $\sum_{j \in \mathcal{N}_j^{in}} r_j \left(\hat{g}_{ji} l_j\right)$ is the direct cost to the firm i arising from the consumption of negative spillover from its producers ($\mathcal{N}_i^{in}$). We thus define the firm $i \in \hat{\mathcal{N}}$ payoff as one that captures only the parts which are multiples of the action profile r. Let us have $\pi_i = 0.5 \left(\kappa l_i\right)^{-1}$ and $g_{ji} = \frac{\hat{g}_{ji} l_i}{l_i} = \frac{l_{ji}}{l_i}$ $\forall$

$j \in \mathcal{N}_i^{in}$, The linear reaction curve (best reply) for the firm i when $r_{i\in\hat{\mathcal{N}}} \in [0, R^{+}]$ is given as;

$$r_i^{br} = \max\left\{\pi_i - a\sum_{j\in\mathcal{N}_i^{in}} g_{ji}r_j, 0\right\}. \tag{3}$$

The value π_i reflects the autarkic amount production rate of the firm i. Typically, firm i desires a greater r_i as it wishes to use its capacity as much as possible. However, the magnitude of firm i's equilibrium rate of production depends on its best reply. Also, strategic substitution properties are captured in $\frac{\delta r_i}{\delta r_j} = -a g_{ji}$ for $j \in \mathcal{N}_i^{in}$.

Let $G = [g_{ji}]$ be a zero-diagonal matrix and the game arising from (3) be denoted as $\Gamma(G, r, a)$. We make a distinction between participating firms and those who do not participate in $\Gamma(G, r, a)$. Assume a subset $\mathcal{S} \subset \hat{\mathcal{N}}$. We have the formal definition;

Definition 1: A firm $i \in \hat{\mathcal{N}}$ is a sink-node $\iff \mathcal{N}_i^{out} = \{\}$.

A Sink-node is a firm i for which $_i = 0$ and as such, has no capacity it is willing to utilise while at the same time, bears the externality cost from producers it is connected to. This may also mirror other non-business based entity like locals who suffer the pollution from productive activities. Let the set $\mathcal{N} = \{1, \ldots, n\}$ be so that $\mathcal{S} \cup \mathcal{N} \subseteq \hat{\mathcal{N}}$ and $\forall$ firm $i \in \mathcal{N}$, $\mathcal{N}_i^{out} \neq \{\}$ and also $\mathcal{N}_i^{in} \neq \{\}$. This distinction is important for example if we have a firm i such that $\mathcal{N}_i^{out} = \{\}$, then $g_{ji} = \infty$ as $l_i = 0$. It means we are unable to define firm i's best reply as it makes no decision. Furthermore, we could have also the firm i such that $\mathcal{N}_i^{in} = \{\}$. Let G_i represent the $i-th$ row of the matrix G, we would have $G_i = (0)_{i\in\mathcal{N}}$ leading to a pure strategy Nash Equilibrium $r_i = \pi_i$. This is described as *strategic dominance* as its externality production rate is made in isolation. To avoid these instances, we introduce another important but common concept to directed networks as follows;

Definition 2: A directed graph $G(\mathcal{N}, g)$ is *strongly connected* (SC) if and only if for every $\{0, n\} \in \mathcal{N}$, there exist a **closed directed walk** (the sequence $0, g_{01}, 1, g_{12}, \ldots, g_{n-1,n}, n, g_{0,n}, 0)$ from 0 to 0.

Then going further, we will rely on the assumption written below;

Assumption 2: The graph $G(\mathcal{N}, g)$ is strongly connected so that the set $\forall$ firm $i \in \mathcal{N}$, firm i is a strongly connected firm (SCF).

This as such ensures that we avoid dominant equilibrium outcomes or undefined best replies given sink nodes (for any firm $i \in \mathcal{S}$, $l_i = 0$ such that $r_i = \infty$).

B. Pure Strategy Solutions

We present the existence of the equilibrium and conditions for uniqueness. To support our next few results, we define a key attribute which is the positive definiteness of a directed network as follows;

Definition 3: Let M be a matrix and $\nu_1(M), \ldots, \nu_n(M)$ be the eigenvalues of the matrix (M). Then M is positive definite if and only if it holds that;

$$\nu_1\left(\frac{M + M^\top}{2}\right), \ldots, \nu_n\left(\frac{M + M^\top}{2}\right) > 0. \tag{4}$$

This definition is useful given the vast amount of public good in network literature emphasises symmetric matrix. Let the minimum eigenvalue of a matrix M be denoted as $\nu_{min}(M)$, we have the following lemma;

Lemma 1: The matrix $(I + aG)$ is positive definite in so far $a \in \left]0, \frac{1}{\left|\nu_{min}(\frac{G+G^\top}{2})\right|}\right[$.

Proof: See Appendix for proof. ■

This attribute represents the boundary condition for the attenuation parameter $'a'$ which is similar to the *Network Normality* condition introduced in [1] with key modifications based on directed network properties leading to an asymmetric network matrix G. This is so that each firm captures the amount pollution caused by producers to determine its own equilibrium rate of production to consumers without consideration for its implication to such consumers.[4]

We denote $\boldsymbol{\pi} = (\pi_i)_{i\in\mathcal{N}} \in R_+^n$ as the autarkic-rate column vector while $r^* = (r_i^*)_{i\in\mathcal{N}} \in R_+^n$ is the Nash equilibrium vector. Following the best reply in (3), draw a distinction between active and inactive firms in the definition below.

Definition 4: A firm $i \in \mathcal{N}$ is thus defined as *active* if and only if $r_i^*(\mathcal{N}, a) \in]0, R_+]$ and non-active if $r_i^*(\mathcal{N}, a) = 0$. Let the set of active firms be denoted with the set $\mathcal{A} \subseteq \mathcal{N}$ and hence non-active firms be $\mathcal{N} - \mathcal{A}\mathcal{N}$. Then using intuitions from [7], [11] and more closely, [1], we have the following;

Proposition 1: Let $r = (r_i)_{i\in\mathcal{N}} \in R_+^n$ be a externality production rate vector for firms. A set of production rates vector $r^*(\mathcal{A}, a)$ with active firms $\mathcal{A} \neq \{\}$ is a Nash equilibrium if and only if the following conditions hold;

1)
$$(I + aG)_{\mathcal{A}\times\mathcal{A}} r_{\mathcal{A}}^* = \boldsymbol{\pi}_{\mathcal{A}}$$

2)
$$aG_{\mathcal{N}-\mathcal{A}\times\mathcal{A}} r_{\mathcal{A}}^* \geq \boldsymbol{\pi}_{\mathcal{N}-\mathcal{A}}$$

Proof: See Appendix for proof. ■

The proposition above translates to the fact that firms become non-active when targets are achieved by simply charging a zero rate and thus, substitute for equilibrium rate of production of active firms in such a way that the outcome is the same or is greater than the outcome from the non-active firms' autarkic equilibrium rate of production to consumers. It also holds then that Nash equilibrium for the game has to include at least one active firm such that A cannot be a null set. A simple computational algorithm takes $2^{|\mathcal{N}|} - 1$ iteration representing possible combinations of active firms. It is noteworthy that even if we relax assumption 2 so that $\hat{\mathcal{N}} = \{\mathcal{N}, \mathcal{S}\}$, Equilibrium is simply obtainable by computing for $\mathcal{N}$. Hence, for each firm $i \in \mathcal{N}$ such that a firm $j \in \mathcal{S} \cap \mathcal{N}_i^{out}$, then $l_i = l_{ij} + \ldots$.

The property of sub-grouping firms at Nash Equilibrium into active and inactive components raises questions as to

[4]The magnitude to which a firm $i \in \mathcal{N}$ equilibrium rate of production r_i affects all other firms' outcomes.

uniqueness given that inactive firms are at corner solution in equilibrium. However, the properties of the equilibrium as it relates to its uniqueness is stated below;

Proposition 2: Given the parameter $'a'$ meets the boundary conditions as in lemma 1, there always exists a unique Nash equilibrium in pure strategies for the game $\Gamma(G, r, a)$ and the unique Nash equilibrium is always asymptotically stable.

Proof: From [19] concept of *diagonal strict concavity*, we understand that a sufficient condition for the payoff $P(r)$ to be diagonally strictly concave, then $H(r, 1) + H(r, 1)^\top$ must be negative definite where $H(r, 1)$ is the Jacobian with respect to r of $P^{'}(r)$. Since it holds that the Jacobian $H(r, 1) = -(I + aG)$, then the condition is achieve should $(I + aG)$ be positive definite which lemma 1 satisfies. It is then shown that Nash equilibrium is unique if and only if lemma 1 is satisfied. Additionally, the proof of *Theorem 1* in [1] addresses any concern as to the relevance of inactive firms to the uniqueness. ■

Furthermore, we draw the following statement from the proposition 1 as follows:

Corollary 1: Assume that $\forall\ i, j \in \mathcal{A},\ l_i = l_{-j}$ so that $\boldsymbol{\pi} = \pi 1_{\mathcal{A}}$. This means that $r^*(\mathcal{A}, -a) = \pi \boldsymbol{b}(\mathcal{A}, -a)$ so that $\forall$ firm $i \in \mathcal{A}$;

$$r_i^*(G, \mathcal{A}, a) = \pi \beta_i(\mathcal{A}, -a),$$

where $\beta_i(\mathcal{A}, -a)$ refer to the Bonacich independence index[5] or simply independence index of an active firm i implying $\boldsymbol{b}(\mathcal{A}, -a) = (\beta_i(\mathcal{A}, -a))_{i \in \mathcal{A}} \in R_+^n$.

Proof: Because we have the following;

$$\boldsymbol{b}(\mathcal{A}, -a) \stackrel{\text{def}}{=} (I + aG)^{-1}_{\mathcal{A}\times\mathcal{A}} 1_{\mathcal{A}}. \tag{5}$$

■

This implies that the Nash equilibrium rate is of each firm is directly proportional to their independence index. The independence index is so named because $G = [g_{ji}]$ accounts for the strength of incoming links. Also since in the series, $(I - aG)_{\mathcal{A}\times\mathcal{A}} 1_{\mathcal{A}}$ dominates $((I - aG)_{\mathcal{A}\times\mathcal{A}} \boldsymbol{\pi}_{\mathcal{A}}$ dominates the Nash equilibrium $r(\mathcal{A}, -a))$, then the greater the strength of g_{ji} for each firm i, the lower its $\beta_i(\mathcal{A}, -a)]$. This then hints as to which firm i charging less amount in externality production rate. We explore some special network properties in relation to this in the next section.

C. Equilibrium and Inactive Firms

Our proposition 1 shows that Nash equilibrium could be such that $\mathcal{N} - \mathcal{A} \neq \{\}$. A firm $i \in \mathcal{N} - \mathcal{A}$ thus has an $r_i = 0$ as its equilibrium rate of production. We draw a swift distinction between inactive firms in our model and the concept of *free-riders* found in major public goods in networks papers such as [10], [11] as well as [1]. To understand this is to understand the best replies given in (3) as an outcome of the payoff. We observe that waste management is a main objective of the firm and as such, strategic substitution arises in a bid to reduce such management cost. So while a firm that linked to a producer cannot influence (directly) the rate of production for its producer, it can alter its corresponding production rate to balance and optimize waste management expenses. For this reason, producing nothing (a zero rate) arises from the fact the present waste management expenses is quite substantial that a positive rate would be even more harmful to the firm.

The idea here is that an inactive firm $i \in \mathcal{N} - \mathcal{A}$ is not necessarily *free-riding* the provision of other firms but on the other hand is simply avoiding any further cost as a result of its own decision since its producers have increased such overhead (waste management) cost to the maximum.

[5]So as not to confuse it with Bonacich Centrality which is $\beta_i(G^\top, a)$ for a firm i.

III. Intervention and Welfare Policies

In this section, we define outcomes based on Nash externality production rates and then observe the welfare properties of the model. More precisely, we highlight various possible policy initiatives to which a maximising Planner could adopt and its estimate the overall impact. To study welfare, we adopt the standard utilitarian approach. As such, we introduce the following definition;

Definition 5: The welfare from the game $\Gamma(G, r, a)$ is defined especially for firms who produce as;

$$W(r, \mathcal{A}, a) \stackrel{\text{def}}{=} \sum_{i \in \mathcal{A}} P_i, \tag{6}$$

This implies we use welfare as the aggregate payoff of all firms who produce. To define such payoff, we write the following lemma;

Lemma 2: Assume $\mathcal{N}$ and the game $\Gamma(G, r, a)$, $\forall$ firm $i \in \mathcal{A}$, payoff given Nash equilibrium is as follows;

$$P_{\mathcal{A}} = diag(B)\left((I + aG)^{-1}\boldsymbol{\pi}_{\mathcal{A}}\right) - K, \tag{7}$$

where $K = \left(\frac{2+a}{4\kappa a}\right)_{i\in\mathcal{A}}$ and $B = \left(\frac{(a+1)}{a} l_i\right)_{i\in\mathcal{A}}$ are both column vectors.

Proof: See Appendix for proof. ■

It is important to note the implication of (7). We see here that firms utility for charging is mainly dependent on their individual Nash equilibrium rate. This means that if we were to observe (7) and our best reply in (3) we then have an idea of kinds of policy implications for the model which we explore in the coming sections.

A. Capacity and Welfare Neutrality

In this part, we explore the possibility of intervention policies and their welfare impact. If we assume a system where each firm's production capacity is varied by a uniform rate which we denote as λ, the new capacity is given as λl_i for each $i \in \mathcal{N}$. We also assume such policies are applied specifically to the the active set $\mathcal{A}$ the same which means the network graph $G_{\mathcal{A}}$ should remain unchanged. Given λl_i for all firm $i \in \mathcal{N}$. The initial mark of the policy λ is such that payoff is written as;

$$P_i^\lambda(r_i) = \lambda\left(l_i r_i - \sum_{j\in\mathcal{N}_i^{in}} (\hat{g}_{ji} l_j)\, r_j\right) - \kappa\,(\lambda\mu_i)^2 \tag{8}$$

The diagram (fig. 3) shows a Planner P whose objective is to maximise $\sum_{i\in\mathcal{A}} P_i$. More specifically, the fig. 3a represents an instance where a Planner increases production capacity (possibly to reach its original maximum capacity) while fig. 3b shows a case where capacity is increased. The arrows show the policy action. In terms of equilibrium, we introduce the following lemma:

Lemma 3: The active set $\mathcal{A}$ remains fixed $\forall$ $\lambda 1$ even though $r^{\lambda} = \lambda^{-1} r$.

Proof: For the Nash equilibrium given such policies, we have for all active firms that;

$$r_i^{\lambda} = \frac{\pi_i}{\lambda} - a \sum_j \frac{\hat{g}_{ji} l_j}{\lambda l_i} r_j^{\lambda} = \frac{\pi_i}{\lambda} - a \sum_j \frac{\hat{g}_{ji} l_j}{l_i} r_j^{\lambda}. \tag{9}$$

This is so that rewriting in vector form, our Nash for active firms is given as;

$$r_{\mathcal{A}}^{\lambda} = (I + aG_{\mathcal{A}})^{-1} \frac{\boldsymbol{\pi}_{\mathcal{A}}}{\lambda} = \lambda^{-1} r_{\mathcal{A}}.$$

For this set combination $\mathcal{A}$ and $\mathcal{N} - \mathcal{A}$ to not be the Nash equilibrium set would mean that the following has to hold;

$$\frac{a}{\lambda} G_{\mathcal{N}-\mathcal{A}\times\mathcal{A}} r_{\mathcal{A}} < \frac{\boldsymbol{\pi}_{\mathcal{N}-\mathcal{A}}}{\lambda}.$$

However, multiplying the equation above by λ gives the condition as;

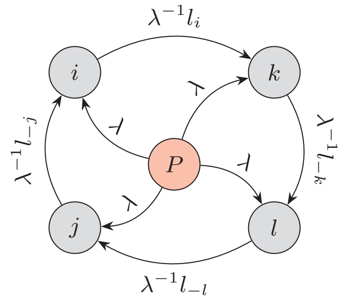
$$a G_{\mathcal{N}-\mathcal{A}\times\mathcal{A}} r_{\mathcal{A}} < \boldsymbol{\pi}_{\mathcal{N}-\mathcal{A}},$$

which is a contradiction to the original equilibrium of $a G_{\mathcal{N}-\mathcal{A}\times\mathcal{A}} r_{\mathcal{A}} \geq \boldsymbol{\pi}_{\mathcal{N}-\mathcal{A}}$. ■

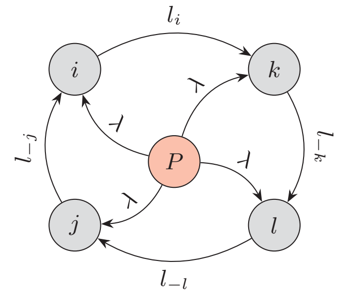
Some explanation of this lemma is that since uniform capacity change λ is homogeneous and since inactive firms are such that $a G_{\mathcal{N}-\mathcal{A}\times\mathcal{A}} r_{\mathcal{A}}^{\lambda} \geq \boldsymbol{\pi}_{\mathcal{N}-\mathcal{A}}^{\lambda}$, then it means that while it is that $r_{i\in\mathcal{A}}^{\lambda} = \lambda^{-1} r_i$, it is also the case that $\pi_{i\in\mathcal{N}-\mathcal{A}}^{\lambda} = \lambda \pi_i$. So if because π rises and falls at equal magnitude for each firm, set of active firms remains constant. As such the magnitude of uniform capacity change or intermediate intervention is not relevant in terms of what the composition of the active set would be at Nash equilibrium. In that light, we summarise the effect of such a homogeneous intervention policy as follows;

Proposition 3: Given the homogeneous policy λ, $\Delta W(r^{\lambda}, \mathcal{A}, a) = 0$, hence welfare is neutral.

Proof: See Appendix for proof. ■



(a) Intervention in a system with existing frictions



(b) Intervention in a system without existing frictions

Fig. 3 Directions of a Planner P's intervention to 4 firms

We then move to observe the impact of mutually exclusive policy $\lambda_i l_i$ $\forall$ $i \in \mathcal{A}$ such that $\lambda_i \lambda_j$ for all $i, j \in \mathcal{A}$. We observe here that policies are restricted to active set, we assume strictly that such policy intervention is such that leaves active set unchanged. For simplicity, one can initially assume the policy λ_i is applied to a single firm while holding others fixed as shown in fig. 4 where this time a regulator increases only one firm externality consumption. In practice, it could be through eliminating uniform capacity change for a single firm while leaving others constant as shown in Fig. 4.

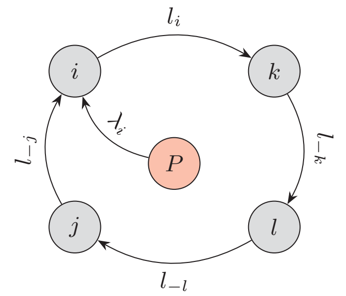


Fig. 4 Ring network with 4 firms where a regulator decides to increase total firm i's externality production to $\lambda_i l_i$

More broadly, the concept of the policy is that links of firms could be increased at a heterogeneous proportion. The impact of such policy on welfare goes as follows;

Theorem 1: Given a policy $(\lambda_i, \lambda_j, \ldots)$ so that $\lambda_i \lambda_j$ for all $i, j \in \mathcal{A}$ we have the following outcome:

$$\Delta W^{\lambda}(r^{\lambda}, \mathcal{A}, a) = 0 \tag{10}$$

Hence such policy is welfare neutral.

Proof: See Appendix for proof. ■

This means that a regulator cannot improve the welfare of active players by simply increasing/reducing one or more active firm network intensity even if it is by varying amounts. Welfare Neutral policies are also found in major public literature such as [7] and [26] whom both showed neutrality to the aggregate provision of public good and individual consumption of private good in so far as wealth redistribution does not change the set of active players involved. In an extension to this, [1] adds that small transfers that leave an active set the same are also neutral only when such transfers are made between the active set themselves. To contrast with our results yield neutrality without transfer policies. Because each firm's utility is based on their individual Nash equilibrium, payoffs are neutral which leaves overall welfare unchanged. Additionally, interventions are not be restricted to active firms and due to the homogeneous nature of the intervention, the magnitude of λ is pertinent in influencing the outcome in so far equilibrium rate of production by firms are limited to non-negative prodution rate.

B. Resource Allocation

To access a possible impact of theorem 1, we observe a policy change of $\Delta\ \kappa_i$ for $i \in \mathcal{A}$ (i.e, firms who produce at Nash equilibrium). Let us have the following definition;

Definition 6: For any firm $i \in \mathcal{N}$, we have that

- Subsidy $\Rightarrow \frac{\Delta\kappa_i}{\kappa_i} = \gamma_i^-$
- Tax burden $\Rightarrow \frac{\Delta\kappa_i}{\kappa_i} = \gamma_i^+$

Tax burden here imply burden of tax (indirect) borne by the manufacturing firm. An example may be the residual amount of excise duty not charged to final consumers[6] of a firms product. We assume then that $\gamma_i = \gamma\ \forall$ firm $i \in \mathcal{A}$ so that the policy is applied at a homogeneous proportion to all active firms. The payoff of each firm i is written as;

$$\forall\, i \in \mathcal{N}, \quad P_i^{\gamma}(r_i) = l_i r_i - \sum_{j \in \mathcal{N}_i^{in}} (\hat{g}_{ji} l_j)\, r_j - (1+\gamma)\kappa(\mu_i)^2 \tag{11}$$

We summarise the effect in the following results;

Lemma 4: Given γ, welfare differential is as follows;

$$\Delta W^{\gamma}(r^{\gamma}, \mathcal{A}, a) = 1^{\top} P_{\mathcal{A}} \frac{-\gamma}{(1+\gamma)}. \tag{12}$$

Remark 1: This implies that if $\gamma \in [-1, 0[$, then $\Delta W^{\gamma}(r^{\gamma}, \mathcal{A}, a) > 0$ while if $\gamma \in [0, 1[$ then $\Delta W^{\gamma}(r^{\gamma}, \mathcal{A}, a) < 0$ and its interpretation is simply that subsidies improve welfare while taxes reduce welfare.

Note that the active set $\mathcal{A}$ also remains fixed $\forall\ \gamma \in]0, 1[$. Results, in this case, are unsurprising as a lighter burden means firms are less sensitive to the volume of combined externality (arising from feedback and producers). Examples of such policies could be through providing outsourcing facilities to a portion of its waste management or maybe policies that improves personnel skill designed to manage waste more efficiently. When, however, this policy applies in a heterogeneous manner to firms, it then becomes isomorphic to resource transfers which we explore in detail subsequently.

In lemma 3 as well as theorem 1, it is noted that given a policy $\lambda_i 1$ such that externality production becomes $\lambda_i l_{-1}$ for any $i \in \mathcal{A}$, $\Delta W(G, a) = 0$ in so far as the active firms $\mathcal{A}$ remains fixed. Given our results above, we have the following results;

Proposition 4: Given the game $\Gamma(G, \mathcal{A}, a)$ there exists $\Delta W(r, \mathcal{A}, \gamma, \lambda) \in]0, R_{++}[$ (not necessarily Pareto) at zero cost to a Planner in so far as there exists $\sum_{i \in \mathcal{A}} \lambda_i l_i$ which is in monetary terms and the active set $\mathcal{A}$ remains fixed.

Proof: Strictly holding $\mathcal{A}$ fixed, let $\sum_{i \in \mathcal{A}}(1-\lambda_i)l_{-1}$ be the amount the regulator charges in order to reduce spillover links from each firm i (building from proposition 3), then this is the case so far $\sum_{i \in \mathcal{A}}(1-\lambda_i)l_{-1} = \gamma \sum_{i \in \mathcal{A}} \left(\kappa_i(\mu_i)^2\right)$ which then guarantees Pareto improvement among active firms. For non-Pareto improvement, subsidised administrative cost $\gamma_i \kappa_i()^2$ need not apply to all firms in $\mathcal{A}$. In this case, the criteria shown in theorem 2 become useful. ■

Our result above arises from the fact that so far as the active set remains fixed, the regulator can instead of eliminating uniform capacity change, create one at no cost to overall welfare. This also grants resources to subsidise one or more firms in a way that improves welfare. Pareto improvement is possible if $X = \sum_{i \in \mathcal{A}}(1-\lambda_i)l_{-1}$ is split such that $\gamma \sum_{i \in \mathcal{A}} \kappa_i(\mu_i)^2 \leq X$. We observe now that γ is constant so that its effect on welfare corresponds to lemma 4. This is a unique form of transfer compared to those found in the mainstream public good in networks literature such as [1], [3], etc. This is because in this case, transfers could be simply from one firm to another through different variables the firm faces.

IV. Intervention Targeting

We project in this section the relationship between Bonacich externality measures and firms' quality, especially in terms of marginal welfare given a resource-constrained Planner. We here generalise the Planner to one who wishes to grant waste management expenses subsidy to maximise overall welfare $(\Delta W^{\gamma}(r^{\gamma}, \mathcal{A}, a))_{max}$ of active firms $\mathcal{A}$. Then if the set $\Phi(\mathcal{A})$ represents the possible combinations of firms, the Planner has $|\Phi(\mathcal{A})| = 2^{|\mathcal{A}|} - 1$ amount of alternative actions as to the distribution of subsidy intervention to achieve $(\Delta W^{\gamma}(r^{\gamma}, \mathcal{A}, a))_{max}$. This is such that the earlier discussed "$\gamma\ \forall$ firm $i \in \mathcal{A}$" is a strategic element in $\Phi(\mathcal{A})$ arising from the $C(|\mathcal{A}|, |\mathcal{A}|)$ combination, where $C(a, b) = \frac{a!}{(a-b)!b!}$. On the other extreme, let $\phi \subset \Phi$ be the subset arising the combination $C(|\mathcal{A}|, 1)$, This then means that $|\phi| = |\mathcal{A}|$ such that the Planner calculates the total welfare from subsidising for a single firm $i \in \mathcal{A}$. We then wish to show the qualities of firm $i \in \mathcal{A}$ which yields the greatest payoff from the strategy subset ϕ. Literature in recent times have, within network spillover problems come up with various targeting criteria; The *Key-Player* concept introduced in [4], The highest threat index (which is the Bonacich centrality) introduced in [12] as well as the *top Principal Components* as another eigenvalue related measure used in [13].

We begin with a naive scenario. We assume a Planner with unlimited finance but one who wishes to subsidise the

[6]Consumers here do not indicate other firms who suffer spillover but households purchasing actual manufactured products.

administrative cost by a γ amount a single selected firm to maximise overall network welfare. Formally, we define the Planner's problem within the strategy $\phi \subset \Phi$ stated as:

$$\max_{\gamma} \{P_i^{\gamma} - P_i | i = 1, \dots, n\} \qquad s.t \quad \gamma^{-} = \gamma_i | i \in \{\mathcal{A}\}. \tag{13}$$

The choice firm $i \in \mathcal{A}$ then has a payoff written as:

$$P_i^{\gamma}(r_i) = \left(l_i r_i - \sum_{j \in \mathcal{N}_i^{in}} (\hat{g}_{ji} l_j)\, r_j \right) - (1+\gamma)\kappa(\mu_i)^2 \tag{14}$$

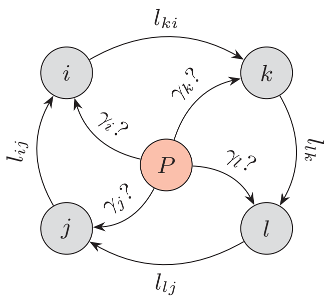


Fig. 5 Ring network with 4 firms to which the Planner makes a decision which to subsidise

Hence the question is which firm should the Planner subsidise. We observe the following equation of the measure of a firm $i \in \mathcal{N}$;[7]

$$\beta_i(G^{\top}, -a) \stackrel{\text{def}}{=} \sum_{k=0}^{+\infty} (-a)^k \sum_{j=1}^{n} \left(\left(G^{\top} \right)^k \right)_{ij} \tag{15}$$

This is such that $\boldsymbol{b}(G^{\top}, -a) = (I + aG^{\top})^{-1}1 = \left(\beta_i(G^{\top}, -a)\right)_{i \in \mathcal{N}} \in R_+^n$. The measure above is related to the Bonacich centrality used to capture prestige and network influence as proposed by [9]. However, it measures the weakness of firms link to their consumers. This means that the greater $\beta_i(G^{\top}, -a)$ is for firm i, the smaller the weight of the direct link to $\mathcal{N}_i^{out}$. Going further, $\beta_i(G^{\top}, -a)$ is referred to as the *externality index* for firm i. We as such present the following results.

Theorem 2: We assume that $l_i = l_{-j}\ \forall\ i, j \in \mathcal{A}$. The welfare differential $\Delta W(r^{\gamma}, \mathcal{A})$ is at maximum if and only if subsidy γ_i such that for firm i;

$$\beta_i(G_{\mathcal{A}}^{\top}, -a) \geq \beta_{j \neq i}(G_{\mathcal{A}}^{\top}, -a),$$

Hence firm i has the largest externality index.

Proof: See Appendix for proof. ■

This result shows the relationship between externalities on outgoing links based on weighted interconnections and a welfare improving intervention[8]. To summarise this point, recall that we can also write firm i's centrality measure as below,

$$\beta_i(G_{\mathcal{A}}^{\top}, -a) = 1 - a \sum_{j \in \mathcal{N}_i^{out}} g_{ij} \beta_j(G_{\mathcal{A}}^{\top}, -a). \tag{16}$$

[7]We still hold in this part that $\mathcal{N} = \mathcal{A}$.
[8]In this case, a subsidy.

This means that for every unit increase in π_i, it negatively impacts each $r_{j \in \{\mathcal{N}_i^{out} \cap \mathcal{A}\}}$, thus a negative externality. Then given that the equilibrium rate of production by an active firm serves as a form of negative externality, the subsidy should be given to the firm that generates the least externality in the network. This is as subsidy here increases strategic substitution since it increases the potential r_i for any firm whose $\kappa(\mu_i)^2$ is reduced. This serves as an identifier for *pressure points* of our model in contrast to other network targeting works.

A more practical and justifiable scenario would be where the Planner has limited resources. In this instance, the Planner wishes to maximise total welfare and as such, measures the impact of channeling subsidy to a single firm versus splitting proportionally across all active firms. To select the firm to consider allocating resources to, let us rewrite the problem of the Planner from (13) as follows;

$$\max_{\gamma_i | i \in \mathcal{A}} \{P_i^{\gamma_i} - P_i | i = 1, \dots, n\}, \tag{17}$$

$$s.t \quad \gamma_i = \gamma_i | i \in \{\mathcal{A}\} \quad and,$$
$$\gamma_i \kappa_i(\mu_i)^2 \leq X$$

It follows then that $\gamma_i \leq -\frac{X}{\kappa_i(\mu_i)^2}$ where X represents the cash endowment of the regulator. In this case, we then derive another corollary from theorem 2 as,

Corollary 2: Assuming a regulator who is cash-constrained and $l_i = l_{-j}\ \forall\ i, j \in \mathcal{A}$, the welfare differential $\Delta W^{\gamma_i}(r^{\gamma_i}, \mathcal{A}, a) | i \in \mathcal{A}$ is at maximum if and only if subsidy γ is applied to firm i which meets the following criteria,

$$\beta_i(G_{\mathcal{A}}^{\top}, -a) \frac{-\gamma_i}{1+\gamma_i} \geq \beta_{j \neq i}(G_{\mathcal{A}}^{\top}, -a) \frac{-\gamma_j}{1+\gamma_j}.$$

Proof: Let $\eta = \frac{2+a}{4a}$ and $\omega = \frac{a+1}{a}$. Since γ_i is not necessarily homogeneous across firms, then $\forall$ firm i such that $P_i = \ldots + (1+\gamma_i)\kappa(\mu_i)^2$, $\Delta W(r^{\gamma}, \mathcal{A}, a) = \frac{-\gamma_i \omega}{2\kappa(1+\gamma_i)} \beta_i(G_{\mathcal{A}}^{\top}, -a) + \frac{\eta \gamma_i}{1+\gamma_i} = \frac{\gamma_i}{1+\gamma_i} \left(\eta - \frac{\omega}{2\kappa} \beta_i(G_{\mathcal{A}}^{\top}, -a) \right)$ and we also hold that $\frac{\gamma_i}{1+\gamma_i} \to +\infty$ as $\gamma_i \to -1$ while keeping active set $\mathcal{A}$ strictly fixed. ■

The intuition then from our results is that welfare due to individual subsidy especially when the regulator has limited funds are best allocated to firms with a combination of greater proportional reduction in waste management expenses as well as lower negative spillover effects. An example of the Planner making this decision can be observed below;

Example 1 (Individual vs Group Targeting): Assuming the following manufacturing network below;

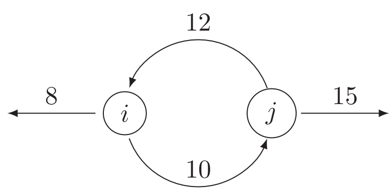


Fig. 6 Network with 3 firms and 4 spillover links (edges)

Other parameters are as follows, $a = 0.8$, $\kappa = 0.04$. This means we have $\pi = (0.699, 0.46)^\top$ and

$$G = \begin{bmatrix} 0 & 0.67 \\ 0.37 & 0 \end{bmatrix}.$$

So that $r^* = (0.54, 0.3)^\top$, $b(G^\top, -a) = (0.8368, 0.5515)^\top$ and $P = (19.198, 15.553)^\top$ which leaves the initial welfare $1^\top P = 34.751$.

We assume then that a Planner has \$2 to distribute. First, we have the waste management expenses as;

$$\kappa(\mu_i(r^*))^2 = 6.35 \text{ and,}$$
$$\kappa(\mu_j(r^*))^2 = 6.17.$$

We have $\Phi = \{\phi_1, \phi_2, \phi_3\}$ where $\phi_1 = \{i, j\}$, $\phi_2 = \{i\}$ and $\phi_3 = \{j\}$.

For the strategy ϕ_1, $\gamma_i = \gamma_j = \gamma$. This gives the value as $\gamma = -0.1587$. Strategy ϕ_2 gives $\gamma_i = -0.3149$ while Strategy ϕ_3 gives $\gamma_j = -0.324$.

Strategy 1(ϕ_1): Where $\gamma = -0.1587$.

We have the welfare improvement as;

$$\begin{aligned}\Delta W^\gamma(r^\gamma, \mathcal{A}, a) &= 1^\top P_{\mathcal{A}} \frac{-\gamma}{(1+\gamma)}, \\ &= 34.751 \frac{0.1587}{0.8413}, \\ &= 6.56.\end{aligned}$$

Strategy 2(ϕ_2): Where $\gamma_i = -0.3149$.

The welfare improvement is;

$$\begin{aligned}\Delta W^\gamma(\gamma_i, \mathcal{A}, a) &= \frac{-\gamma_i \omega}{2\kappa(1+\gamma_i)} \beta_i(G_{\mathcal{A}}^\top, -a) + \frac{\eta\gamma_i}{1+\gamma_i}, \\ &= \frac{0.7085}{0.0548}(0.8368) - \frac{0.2755}{0.6851}, \\ &= 10.41.\end{aligned}$$

Strategy 3(ϕ_3): Where $\gamma_j = -0.324$.

The welfare improvement is;

$$\begin{aligned}\Delta W^\gamma(\gamma_j, \mathcal{A}, a) &= \frac{-\gamma_j \omega}{2\kappa(1+\gamma_j)} \beta_j(G_{\mathcal{A}}^\top, -a) + \frac{\eta\gamma_j}{1+\gamma_j}, \\ &= \frac{0.729}{0.0508}(0.5515) - \frac{0.2835}{0.6760}, \\ &= 7.4928.\end{aligned}$$

Here, we see that the optimal intervention would be to spend the \$2 on subsidising firm i's waste management expenses which in itself, gives a total welfare improvement that supersedes splitting proportionately among both active firms. Also noticeable is the fact that firm i has a greater externality index $\beta_i(G_{\mathcal{A}}^\top, -a)$ in comparison to firm j which corresponds to our results. On a final note, it is worth pointing out that the sub-strategy combination $C(|\mathcal{A}|, b)$, where $1 < b < |\mathcal{A}|$, strategies are known as *group strategy*. This is even more distinct when the number of active firms exceeds 2 ($|\mathcal{A}| > 2$). Our analysis still implies the Planner weighs these strategies and indeed, the optimal could be found within such strategy. However, we have focused primarily on individual firms' quality which makes it a suitable target. Group-based interventions remain unexplored but relevant.

V. Concluding Remarks

We have shown strategic substituting behaviour of firms arising from firms making an inter-temporal externality production rate decision to make maximum profit in the face of waste management expenses. Such waste management expenses depend on the level of firms efficiency in managing overall waste arising from producers as well as the firm through feedback mechanisms. The outcome of this is a substitute game with mostly a unique equilibrium. Our best replies are very likened to notable works such as [8], [1] as well as [11] without boundaries and [2] with boundaries but with slightly different weight and directional properties. We identify neutrality and welfare-improving policies given various types of intervention. One main intuition from our model is that resources can be redirected from within and to the same firm such that the Planner improves welfare while suffering little to no additional cost. Lastly, we established that interventions targeted at firms that have a relatively higher degree of network centrality based on a weak link to consumers yield the most efficient welfare-based outcomes. This is because then raising such firms' externality production rate yields lower negative spillover to linked firms.

This work primarily pays more attention to cost coming from waste management and as such gives intuition towards strategic substitute under the assumption that the firm incurs an additional cost based on the additional volume of productive activities. As with regards to decisions on externality production rates, given that there are a host of other factors that might influence an equilibrium rate of production (externality production), then it is easily predicted that other forms of interaction including the possibility of games of complements could arise when such other factors are taken into account. Also, because we assume one-shot decision-making, we ignore instances where firms could work to increase administrative efficiency especially as it pertains to minimising waste management costs. This in itself could lead to new problems including moral hazard (for example, personnel might not reveal his/her true efficiency as it might alter remuneration). We believe this would make for a vital extension to the model. Another line of extension is linked to welfare whereby the Planner weights firms by order of importance such that payoffs are given weights. This could also shed a more realistic light on the impacts of policies on firms.

Appendix A
Proofs

A. *Proof of Lemma 1*

Intuitions on this concept are briefly discussed in [11]. Additionally, it should be noted that because G is a directed graph, then $(I + aG)$ being positive definite implies

$$1 + a\,\nu_{min}\left(\frac{G + G^\top}{2}\right) > 0, \tag{18}$$

hence the condition.

B. Proof of Proposition 1

Given (3), then for the active set $\mathcal{A}$, we would have for firm $i \in \mathcal{A}$ the following;

$$r_{i\in\mathcal{A}} = \pi_i - a \sum_{j\in\mathcal{N}_i^{in}, j\in\mathcal{A}} \frac{l_{ji}}{l_i} r_j. \tag{19}$$

Intuitively, any firm $l \in \mathcal{N} - \mathcal{A}$ would be such that the following holds;

$$r_{l\in\mathcal{N}-\mathcal{A}} = \pi_l - a \sum_{j\in\mathcal{N}_l^{in}, j\in\mathcal{A}} \frac{l_{jl}}{l_{-l}} r_j \leq 0,$$

which then translates to;

$$a \sum_{j\in\mathcal{N}_l^{in}, j\in\mathcal{A}} \frac{l_{jl}}{l_{-l}} r_j \geq \pi_l. \tag{20}$$

Writing (19) and (20) in vector form for the full set $\mathcal{N}$ completes the proof.

C. Proof of Lemma 2

Recall that $\hat{g}_{ji} = \frac{l_{ji}}{l_i}$.

We assume $\mathcal{N} = \mathcal{A}$. This means we can rewrite (2) as follows

$$P_i = l_i r_i - \sum_{j\in\mathcal{N}_i^{in}} l_{ji} r_j - \kappa(\mu_i)^2 \tag{21}$$

Also, from (3),

$$r_i = \pi_i - a \sum_{j\in\mathcal{N}_i^{in}} \frac{l_{ji}}{l_i} r_j$$

yielding;

$$l_i \pi_i = l_i r_i + a \sum_{j\in\mathcal{N}_i^{in}} l_{ji} r_j \tag{22}$$

Also from (22),

$$\sum_{j\in\mathcal{N}_i^{in}} l_{ji} r_j = \frac{l_i\pi_i - l_i r_i}{a} \tag{23}$$

then substituting (22) and (23) in (21) yields;

$$P_i = l_i r_i - \frac{l_i\pi_i + l_i r_i}{a} - \kappa(\mu_i)^2$$

which is also;

$$P_i = l_i r_i \frac{(a+1)}{a} - \frac{l_i \pi_i}{a} - \kappa\left(l_i\pi_i\right)^2$$

Given that we have $\pi_i = \frac{1}{2\kappa l_i}$, we then have our payoff as:

$$P_{i\in\mathcal{A}} = \frac{l_i(a+1)}{a} r_i - \frac{2+a}{4\kappa a}. \tag{24}$$

Let $\omega = \frac{(a+1)}{a}$ and $\eta = \frac{2+a}{4\kappa a}$, given (4), we have the expression with respect to firm $i \in \mathcal{A}$ Bonacich centrality as;

$$P_{i\in\mathcal{A}} = \omega l_i \left((I + aG)^{-1}\boldsymbol{\pi}_{\mathcal{A}}\right)_i - \eta \tag{25}$$

In vector for, this becomes;

$$P_{\mathcal{A}} = diag(B)\left((I + aG)^{-1}\boldsymbol{\pi}_{\mathcal{A}}\right) - K$$

such that $K = [\eta]^{\mathcal{A}\times 1}$ and $B = [\omega l_i]^{\mathcal{A}\times 1}$.

D. Proof of Proposition 3

So we have that given $\lambda = (1+\varepsilon)$, we have $P_i^{\varepsilon}(r_i) = \lambda\left(l_i r_i - \sum_{j\in\mathcal{N}_i^{in}}\left(\hat{g}_{ji} l_j\right) r_j\right) - \kappa\left(\lambda\mu_i\right)^2 + \xi r_i$. If we were to take the differential with respect to r_i; we end up with the best reply as follows;

$$r_i(\lambda) = \frac{\pi_i}{\lambda} - a\sum_j \frac{\lambda l_{ji}}{\lambda l_i} r_j = \frac{\pi_i}{\lambda} - a\sum_j \frac{l_{ji}}{l_i} r_j.$$

This is so that rewriting in vector form, our Nash for active firms is given as;

$$r(\lambda) = (I + aG_{\mathcal{A}})^{-1}\frac{\boldsymbol{\pi}_{\mathcal{A}}}{\lambda}.$$

We can simply deduce from (8) that the vector payoff for active firms is as follows;

$$P_{\mathcal{A}}^{\lambda} = \lambda diag(B)\left((I + aG_{\mathcal{A}})^{-1}\frac{\boldsymbol{\pi}_{\mathcal{A}}}{\lambda}\right) - K = P_{\mathcal{A}}$$

This is because granting εl_i to each firm $i \in \mathcal{N}$ yields (1) and (6). Hence payoff is homogeneous of degree zero, i.e $P_{\mathcal{A}}^{\lambda}(\lambda l_i) = P_{\mathcal{A}}(l_i)$. As such, welfare differential

$$W(r^*, \mathcal{A}) - W^{\lambda}(r^{\lambda}, \mathcal{A}) = 1^{\top}(P_{\mathcal{A}} - P_{\mathcal{A}}^{\lambda}) = 0.$$

E. Proof of Theorem 1

We assume that the Planner decides to change a firm i's total externality production by a parameter λ and let us have it that the policy intervention λ_i such that payoffs is written as;

$$\begin{aligned} P_i^{\lambda}(r_i) = \lambda_i l_i r_i - \sum_{j\in\mathcal{N}_i^{in}} \left(\hat{g}_{ji} l_j\right) r_j \\ - k\left(\lambda_i l_i r_i + a \sum_{j\in\mathcal{N}_i^{in}} \left(\hat{g}_{ji} l_j\right) r_j\right)^2 \end{aligned} \tag{26}$$

The addition of λl_i to firm i is strictly conditional on the following;

1) $\mathcal{A}(\lambda) = \mathcal{A}$, and
2) $a \in \left]0, \frac{1}{\left|\frac{G(\lambda)+G(\lambda)^{\top}}{2}\right|}\right[$.

The Nash equilibrium for firm i given λl_i is ;

$$r_i^{\lambda} = \frac{\pi_i}{\lambda_i} - a \sum_{j\in\mathcal{N}_i^{in}, j\in\mathcal{A}} \frac{g_{ji}}{\lambda_i} r_j^{\lambda}$$

While the equilibrium for all firm $j | j \in \mathcal{N}_i^{out} \cap \mathcal{A}$ is

$$r_j^{\lambda} = \pi_j - a \sum_{k\in\left(\mathcal{N}_k^{in} - \{i\}\right)\cap\mathcal{A}} g_{kj} r_k^{\lambda} - a\lambda_i g_{ij} r_i^{\lambda}$$

The vector payoff for active firms is then;

$$P_{\mathcal{A}}^{\lambda} = diag(B^{\lambda})\left((I + aG_{\mathcal{A}}^{\lambda})^{-1}\boldsymbol{\pi}_{\mathcal{A}}^{\lambda}\right) - K$$

where $B^\lambda = (\omega\lambda_i l_i, \omega l_j, \omega l_k, \ldots)^\top$, $\boldsymbol{\pi}^\lambda = \left(\lambda_i^{-1}\pi_i, \pi_j, \pi_k \ldots\right)^\top$ and lastly,

$$G^\lambda_{\mathcal{A}} = \begin{bmatrix} 0 & \frac{g_{ji}}{\lambda_i} & \ldots & \frac{g_{ni}}{\lambda_i} \\ \lambda_i g_{ij} & \ldots & \ldots & g_{ji} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_i g_{in} & g_{nj} & \ldots & 0 \end{bmatrix}$$

We then show that $diag(B^\lambda)\left((I + aG^\lambda_{\mathcal{A}})^{-1}\boldsymbol{\pi}^\lambda_{\mathcal{A}}\right) = diag(B)\left((I + aG_{\mathcal{A}})^{-1}\boldsymbol{\pi}_{\mathcal{A}}\right)$. First we have that

$$diag(B^\lambda)\left((I + aG^\lambda_{\mathcal{A}})^{-1}\boldsymbol{\pi}^\lambda_{\mathcal{A}}\right) = \begin{bmatrix} \omega\lambda_i l_j & 0 & \ldots & 0 \\ 0 & \ldots & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \omega l_n \end{bmatrix} \times \begin{bmatrix} m_{ii} & \frac{m_{ji}}{\lambda_i} & \ldots & \frac{m_{ni}}{\lambda_i} \\ m_{ij} * \lambda_i & \ldots & \ldots & m_{ni} \\ \vdots & \vdots & \vdots & \vdots \\ m_{in} * \lambda_i & m_{nj} & \ldots & m_{nn} \end{bmatrix} \times \begin{bmatrix} \frac{\pi_i}{\lambda_i} \\ \pi_j \\ \vdots \\ \pi_n \end{bmatrix}$$

This is then the same as;

$$\begin{aligned} diag(B^\lambda)\left((I + aG^\lambda_{\mathcal{A}})^{-1}\boldsymbol{\pi}^\lambda_{\mathcal{A}}\right) &= \begin{bmatrix} \omega\lambda_i l_i & 0 & \ldots & 0 \\ 0 & \ldots & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \omega l_{-n} \end{bmatrix} \\ &\times \begin{bmatrix} \frac{1}{\lambda_i}\left(m_{ii}\pi_i + m_{ji}\pi_j + \ldots + m_{ni}\pi_n\right) \\ m_{ij}\pi_i + \ldots + \ldots + m_{ni}\pi_n \\ \vdots \\ m_{in}\pi_i + m_{nj}\pi_j + \ldots + m_{nn}\pi_n \end{bmatrix} \\ &= \begin{bmatrix} \omega l_j\left(m_{ii}\pi_i + m_{ji}\pi_j + \ldots + m_{ni}\pi_n\right) \\ \omega l_j\left(m_{ij}\pi_i + \ldots + \ldots + m_{ni}\pi_n\right) \\ \vdots \\ \omega l_n\left(m_{in}\pi_i + m_{nj}\pi_j + \ldots + m_{nn}\pi_n\right) \end{bmatrix} \\ &= diag(B)\left((I + aG_{\mathcal{A}})^{-1}\boldsymbol{\pi}_{\mathcal{A}}\right) \end{aligned}$$

Say then we have $\lambda_i \neq \lambda_j \neq \ldots \neq \lambda_n$, we have our Nash equilibrium as;

$$\begin{aligned} (I + aG^\lambda_{\mathcal{A}})^{-1}\boldsymbol{\pi}^\lambda_{\mathcal{A}} &= \begin{bmatrix} m_{ii} & \frac{m_{ji}*\lambda_j}{\lambda_i} & \ldots & \frac{m_{ni}*\lambda_n}{\lambda_i} \\ \frac{m_{ij}*\lambda_i}{\lambda_j} & \ldots & \ldots & \frac{m_{ni}*\lambda_n}{\lambda_j} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{m_{in}*\lambda_i}{\lambda_n} & \frac{m_{nj}*\lambda_j}{\lambda_n} & \ldots & m_{nn} \end{bmatrix} \times \begin{bmatrix} \frac{\pi_i}{\lambda_i} \\ \frac{\pi_j}{\lambda_j} \\ \vdots \\ \frac{\pi_n}{\lambda_n} \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{\lambda_i}\left(m_{ii}\pi_i + m_{ji}\pi_j + \ldots + m_{ni}\pi_n\right) \\ \frac{1}{\lambda_j}\left(m_{ij}\pi_i + \ldots + \ldots + m_{ni}\pi_n\right) \\ \vdots \\ \frac{1}{\lambda_j}\left(m_{in}\pi_i + m_{nj}\pi_j + \ldots + m_{nn}\pi_n\right) \end{bmatrix} \end{aligned}$$

Which when multiplied by $diag(B^\xi)$ still yields the same expression that

$$diag(B^\lambda)\left((I + aG^\lambda_{\mathcal{A}})^{-1}\boldsymbol{\pi}^\lambda_{\mathcal{A}}\right) = diag(B)\left((I + aG_{\mathcal{A}})^{-1}\boldsymbol{\pi}_{\mathcal{A}}\right).$$

Proof of Lemma 4

then best replies are:

$$r^\gamma_i = \frac{\pi_i}{(1+\gamma)} - a \sum_{j \in \mathcal{N}^{in}_i, j \in \mathcal{A}} g_{ji} r^\gamma_j$$

While the vector payoff for active firms is then;

$$\begin{aligned} P^\gamma_{\mathcal{A}} &= diag(B)\left((I + aG_{\mathcal{A}})^{-1}\frac{\boldsymbol{\pi}_{\mathcal{A}}}{(1+\gamma)}\right) - \frac{K}{(1+\gamma)} \\ &= \frac{1}{(1+\gamma)} P_{\mathcal{A}} \end{aligned}$$

as such, welfare differential

$$W^\gamma(r^\gamma, \mathcal{A}) - W(r^*, \mathcal{A}) = 1^\top P_{\mathcal{A}} \frac{\gamma}{(1+\gamma)}$$

F. Proof of Theorem 2

The best replies for the firm i which is subsidised for is;

$$r^\gamma_i = \frac{\pi_i}{(1+\gamma)} - a \sum_{j \in \mathcal{N}^{in}_i, j \in \mathcal{A}} g_{ji} r^\gamma_j$$

While the vector payoff for active firms is then;

$$P^\gamma_{\mathcal{A}} = diag(B)\left((I + aG_{\mathcal{A}})^{-1}\boldsymbol{\pi}^\gamma_{\mathcal{A}}\right) - K^\gamma$$

where $\boldsymbol{\pi}^\gamma_{\mathcal{A}} = \left(\frac{\pi_i}{1+\gamma}, \pi_j, \ldots\right)^\top$, while $K^\gamma = \left(\frac{\eta}{1+\gamma}, \eta, \ldots\right)^\top$. As such, payoff vector differential;

$$\begin{aligned} P^\gamma(r^\gamma, \mathcal{A}) - P(r^*, \mathcal{A}) = diag(B)\left((I + aG_{\mathcal{A}})^{-1}(\boldsymbol{\pi}^\gamma_{\mathcal{A}} - \boldsymbol{\pi}_{\mathcal{A}})\right) \\ - (K^\gamma + K) \end{aligned} \tag{27}$$

Where $\boldsymbol{\pi}^{\gamma}_{\mathcal{A}} - \boldsymbol{\pi}_{\mathcal{A}} = \left(\frac{\pi_i \gamma}{1+\gamma}, 0, \ldots, 0\right)^{\top}$, while $K^{\gamma} - K = \left(\frac{\eta\gamma}{1+\gamma}, 0, \ldots, 0\right)^{\top}$ We can then expand (27) as such;

$$
\begin{aligned}
P^{\gamma}(r^{\gamma}, \mathcal{A}) - P(r, \mathcal{A}) &= \begin{bmatrix} \omega l_i & 0 & \ldots & 0 \\ 0 & \ldots & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \omega l_{-n} \end{bmatrix} (I + aG_{\mathcal{A}})^{-1} \\
&\times \begin{bmatrix} -\frac{\pi_i \gamma}{1+\gamma} \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \begin{bmatrix} -\frac{\eta\gamma}{1+\gamma} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} \omega l_i & 0 & \ldots & 0 \\ 0 & \ldots & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \omega l_{-n} \end{bmatrix} \begin{bmatrix} -m_{ii} \frac{\pi_i \gamma}{1+\gamma} \\ -m_{ij} \frac{\pi_i \gamma}{1+\gamma} \\ \vdots \\ -m_{ik} \frac{\pi_i \gamma}{1+\gamma} \end{bmatrix} \\
&- \begin{bmatrix} -\frac{\eta\gamma}{1+\gamma} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} -m_{ii}\omega \frac{l_i \pi_i \gamma}{1+\gamma} \\ -m_{ij}\omega \frac{l_{-j} \pi_i \gamma}{1+\gamma} \\ \vdots \\ -m_{ik}\omega \frac{l_{-n} \pi_i \gamma}{1+\gamma} \end{bmatrix} - \begin{bmatrix} -\frac{\eta\gamma}{1+\gamma} \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\end{aligned}
$$

This then means that since $\pi_i = \frac{1}{2\kappa l_i}$, we have;

$$
\begin{aligned}
\Delta W(r^{\gamma}, \mathcal{A}|i) &= \frac{-\gamma\omega}{2\kappa(1+\gamma)} \left(m_{ii} + m_{ij} \frac{l_{-j}}{l_i} + \ldots + m_{in} \frac{l_{-n}}{l_i} \right) \\
&+ \frac{\eta\gamma}{1+\gamma}
\end{aligned} \tag{28}
$$

This means that if $l_i = l_{-j} \ \forall \ i, j \in \mathcal{A}$, then we have that the equation above becomes;

$$
\begin{aligned}
\Delta W(r^{\gamma}, \mathcal{A}|i) &= \frac{-\gamma\omega}{2\kappa(1+\gamma)} (m_{ii} + m_{ij} + \ldots + m_{in}) + \frac{\eta\gamma}{1+\gamma} \\
&= \frac{-\gamma\omega}{2\kappa(1+\gamma)} \beta_i(G^{\top}_{\mathcal{A}}, -a) + \frac{\eta\gamma}{1+\gamma} \\
&> 0 \qquad \text{in so far } \gamma < 0.
\end{aligned} \tag{29}
$$

We also observe that $\frac{-\gamma\omega}{2\kappa(1+\gamma)}$ as well as $\frac{\eta\gamma}{1+\gamma}$ is common to every active firm. This means that the firm i such that $\beta_i(G^{\top}_{\mathcal{A}}, -a)$ is greatest achieves the highest value of $\Delta W(r^{\gamma}, \mathcal{A}|i)$.

APPENDIX B
PSEUDO-CODE FOR COMPUTATION

Algorithm 1 Nash Equilibrium externality production Rate Algorithm

```
procedure DEFINE PARAMETERS
    A(k) ⊂ N, N − A(k) ⊂ N, N − A(k) ∩ A(k) = ∅,
  N − A(k) ∪ A(k) = N.
    max_k = 2^|N| − 1 (loop)
    loop:
    if k = 1 : 1 : max_k then
        r*_{A(k)} = (I + aG_{A(k),A(k)})^{-1} π_{A(k)}.
        r*_{N−A(k)} = 0.
    End If:
    r*_{A(k)} ≥ 0 and,
    aG_{N−A(k),A(k)} r*_{A(k)} ≥ π_{N−A(k)}.
    Else:
    goto loop.
```

ACKNOWLEDGMENT

Special gratitude to Prof Allouch Nizar for his rigorous support as well as Dr Sylvan Barde, Prof Miltadis Makris and Prof Frédéric Deroïan for their helpful comments.

REFERENCES

[1] Nizar Allouch. On the private provision of public goods on networks. *Journal of Economic Theory*, 157:527–552, 2015.

[2] Nizar Allouch and Maia King. Constrained public goods in networks. Technical report, School of Economics Discussion Papers, 2018.

[3] Nizar Allouch and Maia King. A network approach to welfare. 2018.

[4] Coralio Ballester, Antoni Calvó-Armengol, and Yves Zenou. Who's who in networks. wanted: The key player. *Econometrica*, 74(5):1403–1417, 2006.

[5] Mohamed Belhaj and Frédéric Deroïan. Group targeting under networked synergies. *Games and Economic Behavior*, 118:29–46, 2019.

[6] Mohamed Belhaj, Sebastian Bervoets, and Frédéric Deroïan. Efficient networks in games with local complementarities. *Theoretical Economics*, 11(1): 357–380, 2016.

[7] Theodore Bergstrom, Lawrence Blume, and Hal Varian. On the private provision of public goods. *Journal of public economics*, 29(1):25–49, 1986.

[8] Larry Blume, David Easley, Jon Kleinberg, Robert Kleinberg, and Éva Tardos. Network formation in the presence of contagious risk. In *Proceedings of the 12th ACM conference on Electronic commerce*, pages 1–10. ACM, 2011.

[9] Phillip Bonacich. Power and centrality: A family of measures. *American journal of sociology*, 92(5): 1170–1182, 1987.

[10] Yann Bramoullé and Rachel Kranton. Public goods in networks. *Journal of Economic Theory*, 135(1):478–494, 2007.

[11] Yann Bramoullé, Rachel Kranton, and Martin D'amours. Strategic interaction and networks. *American Economic Review*, 104(3):898–930, 2014.

[12] Gabrielle Demange. Contagion in financial networks: a threat index. *Management Science*, 64(2):955–970, 2016.

[13] Andrea Galeotti, Benjamin Golub, and Sanjeev Goyal. Targeting interventions in networks. *Econometrica*, 88 (6):2445–2471, 2020.

[14] Diego García-Gusano, I Robert Istrate, and Diego Iribarren. Life-cycle consequences of internalising socio-environmental externalities of power generation. *Science of the Total Environment*, 612:386–391, 2018.

[15] Adam B Jaffe, Manuel Trajtenberg, and Rebecca Henderson. Geographic localization of knowledge spillovers as evidenced by patent citations. *the Quarterly journal of Economics*, 108(3):577–598, 1993.

[16] Andrew Johnston, Kenneth Amaeshi, Emmanuel Adegbite, and Onyeka Osuji. Corporate social responsibility as obligated internalisation of social costs. *Journal of Business Ethics*, 170(1):39–52, 2021.

[17] Tatiana Moşteanu and Mihaela Iacob. Principles for private and public internalisation of externalities. a synoptic view. *Theoretical & Applied Economics*, 16(10), 2009.

[18] Kunju Vaikarar Soundararajan Rajmohan, Chandrasekaran Ramya, Manakkal Raja Viswanathan, and Sunita Varjani. Plastic pollutants: effective waste management for pollution control and abatement. *Current Opinion in Environmental Science & Health*, 12:72–84, 2019.

[19] J Ben Rosen. Existence and uniqueness of equilibrium points for concave n-person games. *Econometrica: Journal of the Econometric Society*, pages 520–534, 1965.

[20] Lesley Rushton. Health hazards and waste management. *British medical bulletin*, 68(1):183–197, 2003.

[21] Ana-Maria Şchiopu, Ion Apostol, Monica Hodoreanu, and Maria Gavrilescu. Solid waste in romania: Management, treatment and pollution prevention practices. *Environmental Engineering & Management Journal (EEMJ)*, 6(5), 2007.

[22] Bhavisha Sharma, Barkha Vaish, Vaibhav Srivastava, Sonu Singh, Pooja Singh, and Rajeev Pratap Singh. An insight to atmospheric pollution-improper waste management and climate change nexus. In *Modern age environmental problems and their remediation*, pages 23–47. Springer, 2018.

[23] Ibro Skenderovic, Becir Kalac, and Suad Becirovic. Environmental pollution and waste management. *Balkan journal of health science*, 3(1):2–10, 2015.

[24] Hezhong Tian, Jiajia Gao, Jiming Hao, Long Lu, Chuanyong Zhu, and Peipei Qiu. Atmospheric pollution problems and control proposals associated with solid waste management in china: a review. *Journal of Hazardous Materials*, 252:142–154, 2013.

[25] Hugh Ward. International linkages and environmental sustainability: The effectiveness of the regime network. *Journal of Peace Research*, 43(2):149–166, 2006.

[26] Peter G Warr. The private provision of a public good is independent of the distribution of income. *Economics letters*, 13(2-3):207–211, 1983.

[27] Simon J Wells. Financial interlinkages in the united kingdom's interbank market and the risk of contagion. 2004.

[28] Hong Yang, Mingguo Ma, Julian R Thompson, and Roger J Flower. Waste management, informal recycling, environmental pollution and public health. *J Epidemiol Community Health*, 72(3):237–243, 2018.

[29] K Zhichkin, V Nosov, and L Zhichkina. Waste management system in the brewing industry. In *IOP Conference Series: Earth and Environmental Science*, volume 337, page 012009. IOP Publishing, 2019.