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Mathematical Analysis of Stock Prices Prediction in a Financial Market Using Geometric Brownian Motion Model

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Abstract**—**The relevance of geometric Brownian motion (GBM) in modelling the behaviour of stock market prices (SMP) cannot be over emphasized taking into consideration the volatility of the SMP. Consequently, there is need to investigate how GBM models are being estimated and used in financial market to predict SMP. To achieve this, the GBM estimation and its application to the SMP of some selected companies are studied. The normal and log-normal distributions were used to determine the expected value, variance and co-variance. Furthermore, the GBM model was used to predict the SMP of some selected companies over a period of time and the mean absolute percentage error (MAPE) were calculated and used to determine the accuracy of the GBM model in predicting the SMP of the four companies under consideration. It was observed that for all the four companies, their MAPE values were within the region of acceptance. Also, the MAPE values of our data were compared to an existing literature to test the accuracy of our prediction with respect to time of investment. Finally, some numerical simulations of the graphs of the SMP, expectations and variance of the four companies over a period of time were presented using MATLAB programming software.

Keywords**—**Stock Market, Geometric Brownian Motion, normal and log-normal distribution, mean absolute percentage error.

I. Introduction

GENERALLY, financial market may be described as a market place where creation and trading of financial assets such as shares, debentures, bonds, derivatives, currencies and so on take place. It plays sensitive role in many countries' economy and represents a middle man between the savers and investors by mobilizing funds between them. It is classified into four different parts namely; nature of claims, maturity of claims, timing of delivery and organization structure. In the last few years, the financial market has gone through some tremendous changes which include but not limited to financial derivatives which have led to security of financial risks, low cost of transportations, high liquidity, investor protection and transparency in pricing information. One of the most interesting things about derivatives is the acquisition of insurance for every risky asset in the market thereby making derivatives the cheapest source of funds available for shareholders.

Brownian motion (BM) is often used in modelling the movement of time series variables in finance and also asset prices in a financial market. It was discovered by a biologist known as Robert Brown who examined pollen particles floating in water under a microscope, [1]; he observed that the pollen particles show a jumpy motion and concluded that the particles were 'alive'.

From the work of [2]-[5], the GBM process was used to model the SMP for investment in financial institutions such as pensions, banks and insurance. Unlike other volatility models such as constant elasticity of variance (CEV) model, Heston volatility (HV) model, Jump diffusion process etc., the instantaneous expected drift and instantaneous volatility is constant and usually assumes that the distribution of asset return is either normal or log-normal. However, financial data often have heavier tails that can be captured by the standard GBM. Hence there is need to use non-normal distributions to deal with the heavy tails [6]-[9]. The GBM is also the model behind the Black-Scholes (BS) European options pricing formula as well as far more complex derivatives. Some authors such as [10]-[12] used the GBM in real options analysis while [13] used it in representing future demand in capacity studies. The GBM is quite useful in studying stock price index because it assumes that the percentage changes are independent and identically distributed over equal and non-overlapping time length [14]. In [15], they showed that in the BS model; only the volatility parameter is present, but not the drift parameter since the model is derived on the bases of arbitrage-free pricing. Also, [15] showed that for GBM simulations, the drift parameter and volatility parameter are necessary; they observed that the higher the drift value, the higher the simulated prices over the period under consideration.

In as much the GBM process is well appreciated by many in the financial market, there are increasing numbers of literature interested in testing how valid the GBM model is and even its accuracy in forecasting the SMP. In [16], GBM model was used to predict the future closing prices of some small-sized firms in Bursa Malaysia; they concentrated on small-sized firms because the asset prices are lower and inexpensive for individual investors. Finally, they studied the accuracy of the predictions made with the GBM model over different periods, and also at the period required for data inputs into the model. In [17] and [18], the process of checking if a given time series follows the GBM model was studied; they proceed to study different methods in which seasonal variation from a time series can be removed. Furthermore, it was discovered in their

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study of four different industries, the time series for usage of established services satisfied the condition for a GBM process, whereas data from growth of emergent services did not satisfy the condition. Also, [19] studied the validity of the GBM assumptions. In a financial market, a GBM process is used in modelling the price process of the risky asset evolving from continuous time setting. Some notable scholars have tired extending this model with the use of t-distribution [20].

Although there are a lot of literatures on the pricing of securities in corporate finance, there are still debates on which method is most reliable. Hence most financial investors are very much interested in modelling the SMP, options and derivatives for an informed investment decisions for optimal productivity in investment [21]. In this paper, we consider four companies' SMP for a shorter period of time different from the one in [22] and compare our MAPE values with that of [22] to test for the accuracy of our prediction. Also, we examine the GBM model by using normal and lognormal distribution to determine the relationship between the expectation and the risk involve in the stock prices of each investment by the four companies.

II. Methodology

A. The Itô's Lemma

In this section, we explore a very important mathematical tool known as the Itô's lemma to deal with the stochastic process that we have and obtain a solution for our model. The rules involved in the differentiation and integration of real valued functions are different in stochastic calculus; hence, the Itô's lemma is one of the main theorems in a stochastic environment. Itô's lemma is a way of expanding functions or approximating in a series in dt. In particular,$(d\mathcal{W}_t)^2 = dt$, but $dt^2 = dtd\mathcal{W}_t = 0$.

Theorem1. (Itô's Lemma). Let $\mathcal{Z}_t$ be a stochastic process, and suppose we have $\hbar(t,z) : \mathcal{R}_+ \times \mathcal{R} \to \mathcal{R}$. we have $\mathcal{Z}_t = \hbar(t, \mathcal{W}_t)$, where $\mathcal{W}_t$ is the standard BM. Then,

$$d\mathcal{Z}_t(\mathrm{t}) = d\hbar(t,\mathcal{W}_t) = (\frac{\partial \hbar}{\partial t} + \frac{1}{2}\frac{\partial^2 \hbar}{\partial z^2})dt + \frac{\partial \hbar}{\partial z}d\mathcal{W}_t \quad (1)$$

Proof. From Taylor series,

$$d\hbar(t,\mathcal{W}_t) = \frac{\partial \hbar}{\partial t}dt + \frac{\partial \hbar}{\partial z}d\mathcal{W}_t + \frac{1}{2}\frac{\partial^2 \hbar}{\partial z^2}\,(d\mathcal{W}_t)^2 + \cdots,$$

such that other terms are 0 by Itô process multiplication rules and so, we obtained

$$d\hbar(t,\mathcal{W}_t) = \begin{pmatrix} \frac{\partial \hbar}{\partial t}dt + \frac{\partial \hbar}{\partial z}d\mathcal{W}_t + \frac{1}{2}\frac{\partial^2 \hbar}{\partial z^2} \\ = (\frac{\partial \hbar}{\partial t} + \frac{1}{2}\frac{\partial^2 \hbar}{\partial z^2})\,dt + \frac{\partial \hbar}{\partial z}d\mathcal{W}_t \end{pmatrix}.$$

We can make use of theorem 1 to derive the stochastic differential equation using GBM model

Proposition1. If $\mathcal{Z}_t$ is model by a GBM, such that $dz_t = \mu z_t dt + \sigma z_t d\mathcal{W}_t$ then,

$$log(z_t) - log(z_0) = \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\mathcal{W}_t(t) \quad (2)$$

Proof. Firstly, let write down a formal differential equation analogue to Itô's lemma in theorem 1 as follows

$$d\hbar = \frac{\partial \hbar}{\partial z}dz_t + \frac{1}{2}\frac{\partial^2 \hbar}{\partial z^2}dz_t.dz_t.$$

Using $\hbar(z) = \log(z_t)$ and replacing the quadratic variation term with a formal "square" this becomes

$$dlog(\mathcal{Z}_t) = \frac{d\mathcal{Z}_t}{\mathcal{Z}_t} - \frac{(d\mathcal{Z}_t)^2}{2\mathcal{Z}_t^2} \quad (3)$$

Substituting the differential dynamic of $d\mathcal{Z}_t$, we have

$$\frac{d\mathcal{Z}_t}{\mathcal{Z}_t} = \frac{\mu\mathcal{Z}_t dt + \sigma\mathcal{Z}_t d\mathcal{W}_t}{\mathcal{Z}_t} = \mu dt + \sigma d\mathcal{W}_t \quad (4)$$

Turning to the second term, we must now deal with the differential "square" substituting in our SDE,

$$(d\mathcal{Z}_t)^2 = (\mu\mathcal{Z}_t dt + \sigma\mathcal{Z}_t d\mathcal{W}_t)^2 \quad (5)$$

Simplifying (5), we have

$$(d\mathcal{Z}_t)^2 = \sigma^2\mathcal{Z}_t^2\sqrt{dt^2} = \sigma^2 z_t^2 dt$$

where $(dt)^2 = 0, dtd\mathcal{W}_t = 0$ *and* $d\mathcal{W} = \sqrt{dt}$, it follows that

$$\frac{(dz_t)^2}{z_t^2} = \sigma^2 dt. \quad (6)$$

Substituting (5) and (6) into (3), we have

$$dlog(\mathcal{Z}_t) = \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\mathcal{W}_t(t) \quad (7)$$

B. GBM with Normal Distribution

Asset return distributions are frequently presumed to follow either a normal or a lognormal distribution. It can also follow GBM based on the Gaussian process. However, many empirical studies have shown that return distributions are usually not normal [23], [24].

A random variable $\mathcal{Z}$ has a normal distribution with mean μ and varianceσ^2, where $\mu \in, \mathbb{R}$ and$\sigma > 0$, if its density is $g(\mathcal{Z}) = \frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(z-\mu)^2}{2\sigma^2}}$.

If g is a nonnegative function, then

$$\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(z-\mu)^2}{2\sigma^2}}dz = 1 \quad (8)$$

Note that by the changes of variables $\frac{(z-\mu)^2}{2} = t$

$$\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(z-\mu)^2}{2\sigma^2}}dx = \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt \quad (9)$$

Theorem2. If X has a normal distribution with parameters μ and σ^2, then $E[\mathcal{Z}] = \mu$ and $Var(\mathcal{Z}) = \sigma^2$

C. Evaluation Methods of GBM

According to [23], we have three tools for predicting models that have to do with time period t. These include;

number of period forecasts, n; actual value in time period at time t, R_t; and forecast value at time period t, W_t. MAPE seems to be the most widely used to evaluate the forecasting method that considers the effect of the magnitude of the actual values. It is a measure of prediction accuracy of a forecasting method in statistics. It usually expresses accuracy as a percentage and is defined as follows:

$$MAPE = \frac{1}{n}\sum\left|\frac{R_t - W_t}{R_t}\right| \times 100 \quad (10)$$

III. Main Result

In this section, we shall derive the expectation and variance of $\mathcal{Z}_t$ given as $E[\mathcal{Z}_t]$ and $V[\mathcal{Z}_t]$ and try to extend it to the covariance $C\,[\mathcal{Z}_t.\mathcal{Z}_s]$ using normal distribution and log-normal distribution to obtain the Estimation of GBM for a financial market. Furthermore, we will apply it to four different Nigeria Stocks Exchange (NSE) firms and discuss the result.

We consider a random variable $\mathcal{Z}$ which is modeled by the GMB as follows

$$d\mathcal{Z}_t = \mu\mathcal{Z}_t dt + \sigma\mathcal{Z}_t d\mathcal{W}_t \quad (11)$$

and its solution is given as

$$\mathcal{Z}_t(t) = e^{ln\mathcal{Z}_t(0)+\left(\mu-\frac{\sigma^2}{2}\right)T+\sigma\mathcal{W}_t(t)}. \quad (12)$$

For probability distribution normal distribution is $Z\sim[\mu,\sigma^2]$ such that

$$\mathcal{Z}_t = e^{Z} \sim LN[\mu,\sigma^2] \quad (13)$$

The exponential of (13) is known as log-normal distribution while,

$$\mathcal{Z} = \begin{pmatrix} ln\mathcal{Z}_0 + + \left(\mu - \frac{\sigma^2}{2}\right)T \\ +\sigma\mathcal{W}_t(t)\sim N\left[ln\mathcal{Z}_t(0) + \left(\mu - \frac{\sigma^2}{2}\right)T, T\sigma^2\right] \end{pmatrix} \quad (14)$$

is called normal distribution.

By substituting (13) into (14) we have,

$$\mathcal{Z}(T = e^{z}\sim LN) = \left(e^{z}\sim LN\begin{bmatrix} ln\mathcal{Z}_t(0) \\ + \left(\mu - \frac{\sigma^2}{2}\right)T, T\sigma^2 \end{bmatrix}\right). \quad (15)$$

Hence, (15) is called log-normal distribution.

A. Estimation of GMB Using Standard Normal Distribution

$$E[e^{tz}] = \begin{pmatrix} \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{tz}e^{-\frac{t^2}{2}}dz \\ = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{2tz+z^2}{2}}dz \\ = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{(z-t)^2}{2}+\frac{t^2}{2}}dz \\ = \frac{e^{t^2}}{2}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{(z-t)}{2}} = \frac{e^{t^2}}{2} \end{pmatrix} \quad (16)$$

B. Estimation of GMB Using Log Normal Distribution

Using the log normal distribution, the moment generating function is;

$$E\left[e^{t(a+bz)}\right] = e^{at}E[e^{tbz}] = e^{at}e^{\frac{(tb)^2}{2}} = e^{at+\frac{(tb)^2}{2}}$$

such that, for Y, we have

$$Y = t\,(a + b\mathcal{Z})$$

Simplifying, (17), we have

$$Y = at + bt\mathcal{Z} \sim N[at, b^2t^2]$$

such that,

$$E[e^Y] = e^{E[Y]+\frac{1}{2}V[Y]} \quad (17)$$

Therefore,

$$Y \sim N\left[ln\mathcal{Z}(0) + \left(\mu - \frac{\sigma^2}{2}\right)T + \sigma^2 T\right]$$

implies that,

$$Y \sim N\big[E[Y], V[Y]\big]$$

Also, we can use (18) to determine the expected value and the variance of a GBM.

C. GBM for Expected Value

Making use of (18), we can derive the expected value of GBM

$$E[\mathcal{Z}_T] = E[e^Y] = e^{E[Y]+\frac{1}{2}V[Y]} = \begin{pmatrix} ln\mathcal{Z}(0) + \\ \left(\mu - \frac{\sigma^2}{2}\right)T \\ +\sigma^2 T \end{pmatrix} \quad (18)$$

By simplifying (18), we have

$$E[\mathcal{Z}_T] = \mathcal{Z}_0 e^{\mu T} \quad (19)$$

D. GMB for Variance

$$V[\mathcal{Z}_T] = E[\mathcal{Z}_T^2] - (E\,[\mathcal{Z}_T])^2 \quad (20)$$

From $E[\mathcal{Z}_T^2]$ we have,

$$E[\mathcal{Z}_T^2] = \begin{pmatrix} e^{2E[Y]+\frac{1}{2}V[2Y]} = e^{2E[Y]+\,2V[Y]} \\ = e^{2\left(ln\mathcal{Z}(0)+\left(\mu-\frac{\sigma^2}{2}\right)T\right)+2\,\sigma^2 T} \\ = e^{2(ln\mathcal{Z}(0)+\,\mu T)+\sigma^2 T} \end{pmatrix} = \mathcal{Z}_0^2 e^{2\mu T} \quad (21)$$

Therefore,

$$V[\mathcal{Z}_T] = \begin{pmatrix} \mathcal{Z}_0^2 e^{2\mu T+\sigma^2 T} - \mathcal{Z}_0^2 e^{2\mu T} \\ = \mathcal{Z}_0^2 e^{2\mu T}\left(e^{\sigma^2 T} - 1\right) \end{pmatrix} \quad (22)$$

E. GMB Using for Co-variance

Making use of (18), we can also derive the co-variance of GBM

$$C[\mathcal{Z}_T.\mathcal{Z}_s] = E[\mathcal{Z}_T\mathcal{Z}_s] - E[\mathcal{Z}_T]E[\mathcal{Z}_s] \tag{23}$$

From $E[\mathcal{Z}_T]$ we have,

$$E[\mathcal{Z}_T] = \mathcal{Z}_0 e^{\mu T} \tag{24}$$

From $E[\mathcal{Z}_S]$ we have,

$$E[\mathcal{Z}_S] = \mathcal{Z}_0 e^{\mu S} \tag{25}$$

From (12), $E[\mathcal{Z}_T\mathcal{Z}_s]$ becomes

$$E[\mathcal{Z}_T\mathcal{Z}_s] = E\left[\mathcal{Z}_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)T+\sigma W_T}\mathcal{Z}_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)S+\sigma W_S}\right] \tag{26}$$

$$E[\mathcal{Z}_T\mathcal{Z}_s] = \begin{pmatrix} \mathcal{Z}_0^2 e^{\left(\mu-\frac{\sigma^2}{2}\right)(T+S)} E[e^{\sigma W_T+\sigma W_S}] \\ = \mathcal{Z}_0^2 e^{\mu(T+S)-\frac{\sigma^2}{2}\mathrm{T}-\frac{\sigma^2}{2}\mathrm{S}}\, E[e^{\sigma W_T+\sigma W_S}] \end{pmatrix} \tag{27}$$

but

$$E[e^{\sigma W_T+\sigma W_S}] = e^{2\sigma^2\mathrm{S}+\frac{\sigma^2}{2}\mathrm{T}+\frac{\sigma^2}{2}\mathrm{S}} \tag{28}$$

Substituting (28) into (27), we have

$$E[\mathcal{Z}_T\mathcal{Z}_s] = \mathcal{Z}_0^2 e^{\mu(T+S)-\frac{\sigma^2}{2}\mathrm{T}-\frac{\sigma^2}{2}\mathrm{S}}\, e^{2\sigma^2\mathrm{S}+\frac{\sigma^2}{2}\mathrm{T}-\frac{\sigma^2}{2}\mathrm{S}} \tag{29}$$

Substituting (24), (25) and (29) into (23), we have

$$C[\mathcal{Z}_T.\mathcal{Z}_s] = \begin{pmatrix} \mathcal{Z}_0^2 e^{\mu(T+S)-\frac{\sigma^2}{2}\mathrm{T}-\frac{\sigma^2}{2}\mathrm{S}}\, e^{2\sigma^2\mathrm{S}+\frac{\sigma^2}{2}\mathrm{T}+\frac{\sigma^2}{2}\mathrm{S}} \\ -\mathcal{Z}_0^2 e^{\mu(T+S)} \\ = \mathcal{Z}_0^2 e^{\mu(T+S)}(e^{\sigma^2 S} - 1) \end{pmatrix} \tag{30}$$

F. GBM Using for Probability Density

Making use of the standard normal distribution $Z \sim N\ [\mu, \sigma\text{^}2\ ]$, we can also derive the probability density of GBM. For $Y\ = e^z$,

$$f_Z(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \tag{31}$$

IV. Data Analysis and Numerical Simulations

Table I shows actual daily stock value of Company A, Company B, Company C and Company D for fifteen days.

Table II shows the calculated values expected returns and instantaneous volatility of the SMP of the four companies. From (14), and Table II, we obtain Tables III and IV.

Table III shows a sample of predicting value for company A. It is a fifteen day forecasting data compared with the actual prices.

TABLE I
ACTUAL DAILY PRICES OF STOCK FOR FOUR COMPANIES [22]

Dates	Company A (Nestle Foods)	Company B (Dangote Cement)	Company C (First Bank)	Company D (Associated Bus Company)
01-May-19	3.00	2.85	3.40	3.80
02-May-19	3.00	3.00	3.55	3.80
03-May-19	3.00	2.94	3.55	3.80
06-May-19	3.09	2.93	3.83	3.79
07-May-19	3.10	2.80	4.12	3.79
08-May-19	3.07	2.90	4.02	3.78
09-May-19	3.07	2.90	3.77	3.74
10-May-19	3.09	2.94	3.90	3.78
13-May-19	3.00	2.80	3.82	3.78
14-May-19	3.25	2.80	3.79	3.78
15-May-19	3.35	2.75	3.54	3.78
16-May-19	3.35	2.80	3.58	3.78
17-May-19	3.31	2.75	3.60	3.75
20-May-19	3.40	2.79	3.67	3.74
21-May-19	3.40	2.65	3.63	3.78

TABLE II
ESTIMATED PARAMETER OF THE GBM NORMAL DISTRIBUTION

GBM Normal		
Index	μ	σ
Company A	3.17	0.15
Company B	2.84	0.09
Company C	3.72	0.19
Company D	3.78	0.02

TABLE III
PREDICTED PRICES AGAINST ACTUAL PRICES OF STOCK FOR COMPANY A

Date	Actual Values (N)	Predicted Values (N)
01-May-19	3.00	3.25
02-May-19	3.00	2.73
03-May-19	3.00	2.74
06-May-19	3.09	3.35
07-May-19	3.10	3.34
08-May-19	3.07	2.80
09-May-19	3.07	2.83
10-May-19	3.09	3.36
13-May-19	3.00	2.75
14-May-19	3.25	3.00
15-May-19	3.35	3.11
16-May-19	3.35	3.09
17-May-19	3.31	3.01
20-May-19	3.40	3.09
21-May-19	3.40	3.10

Table IV shows a sample of predicting value for company B. It is a fifteen day forecasting data compared with the actual prices.

From Table V, it is observed that all the MAPE values are less than 10%, it implies that the GBM model is a highly accurate model for forecasting these stock prices on the Nigeria stock market for fifteen days investment.

A. Forecast Measure

This measures the deviation between the forecasted values and the actual stock values. Table VI depicts a scale of judgment of forecast accuracy using MAPE.

TABLE IV
PREDICTED PRICES AGAINST ACTUAL PRICES OF STOCK FOR COMPANY B

Date	Actual Values (N)	Predicted Values (N)
01-May-19	2.85	2.92
02-May-19	3.00	3.08
03-May-19	2.94	3.01
06-May-19	2.93	3.00
07-May-19	2.80	2.87
08-May-19	2.90	2.97
09-May-19	2.90	2.97
10-May-19	2.94	3.01
13-May-19	2.80	2.86
14-May-19	2.80	2.86
15-May-19	2.75	2.71
16-May-19	2.80	2.86
17-May-19	2.75	2.81
20-May-19	2.79	2.85
21-May-19	2.65	2.70

TABLE V
MAPE VALUES FOR THE FOUR COMPANIES

Listed Company	MAPE Value (%)
Company A (Nestle Foods)	8.40
Company B (Dangote Cement)	2.25
Company C (First Bank)	0.51
Company D (Associated Bus Company)	0.52

TABLE VI
SCALE OF FORECAST ACCURACY [24]

MAPE (%)	Judgment of forecast accuracy
< 10%	Highly accurate
11% - 20%	Good forecast
21% - 50%	Reasonable forecast
>51%	Inaccurate forecast

We now measure the accuracy of the forecast model by calculating the MAPE values. Table V illustrates the MAPE values of the listed companies under consideration

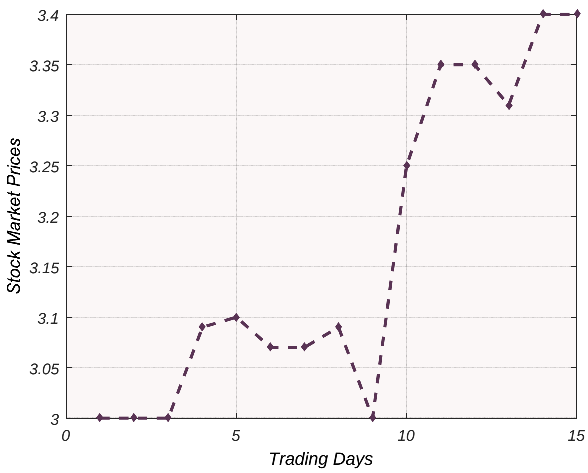

Fig. 1 SMP of company A

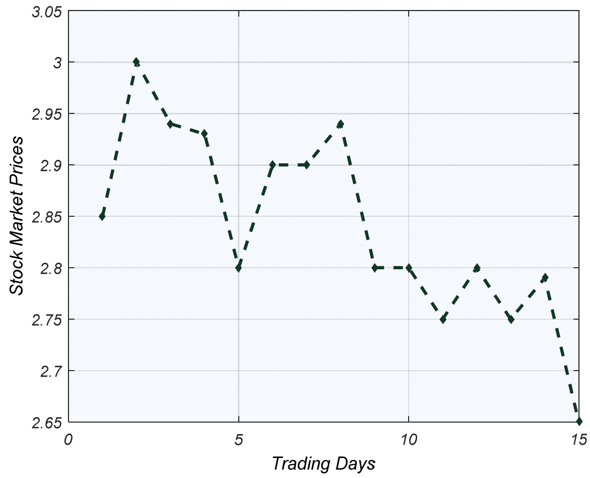

Fig. 2 SMP of company B

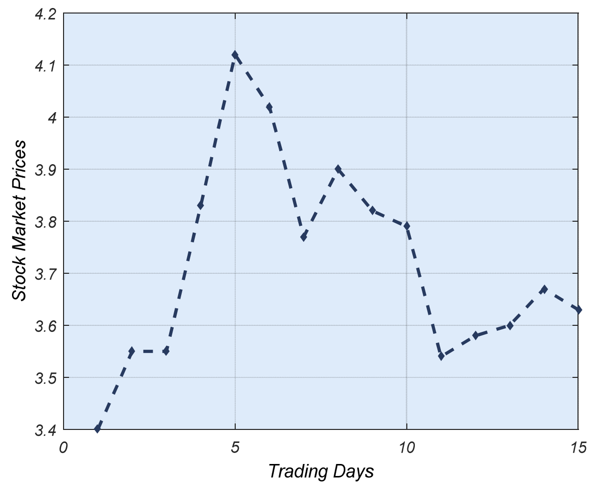

Fig. 3 SMP of company C
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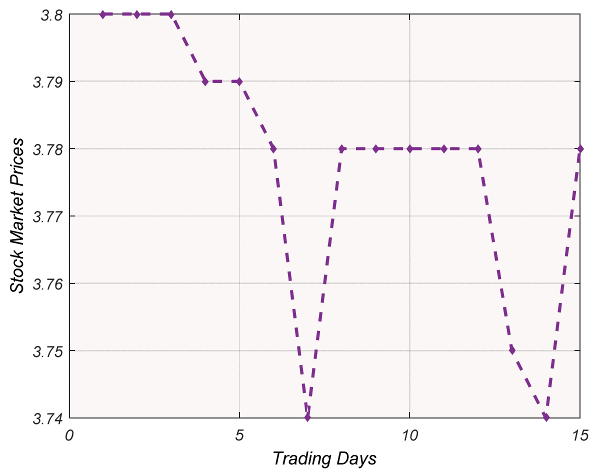

Fig. 4 SMP of company D

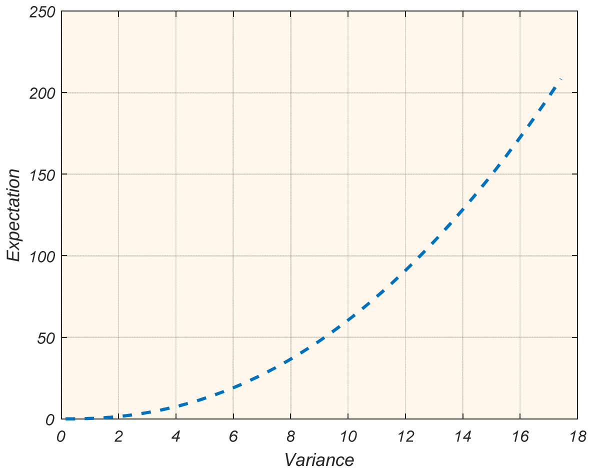

Fig. 5 Relationship between the expectation and variance of company A

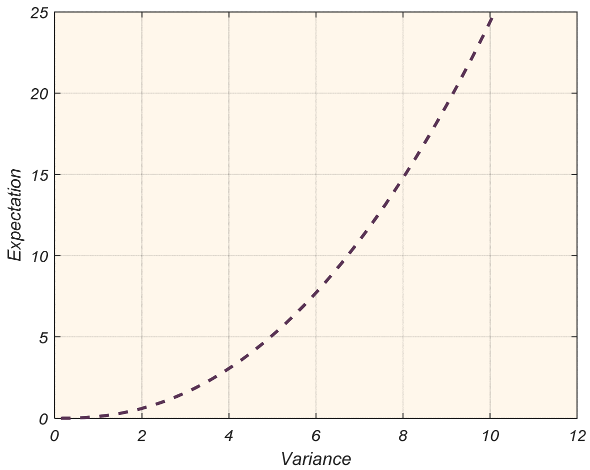

Fig. 6 Relationship between the expectation and variance of company B

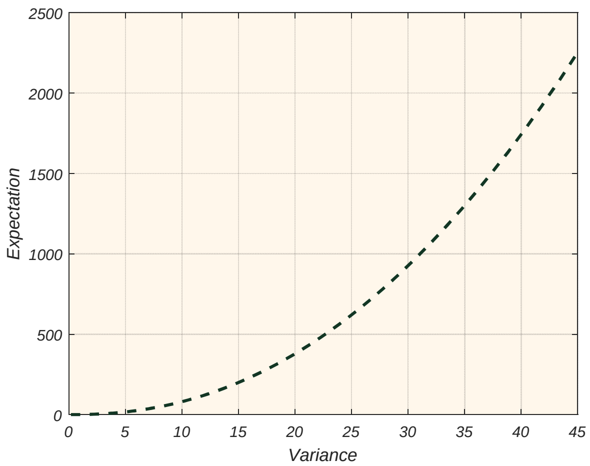

Fig. 7 Relationship between the expectation and variance of company C

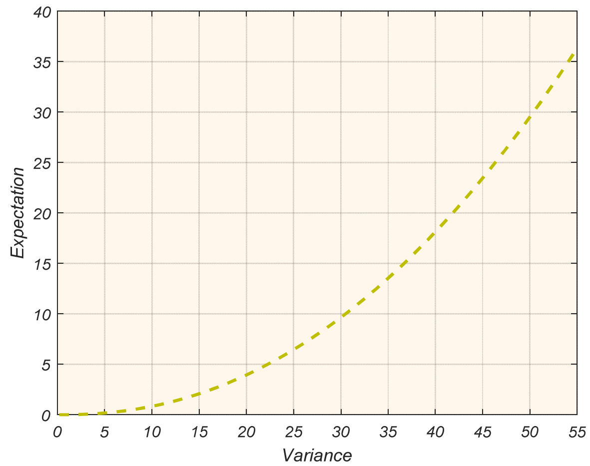

Fig. 8 Relationship between the expectation and variance of company D

V. Discussion and Conclusion

Fig. 1 represents the price of a stock for company A over time. It shows a relationship between the time in days and the closing price of company A's stock during the investment period. In Fig. 2, the price of a stock for company B over time is presented. It is a graph of time in days against the closing price of company B's stock during the investment period. Fig. 3 represents the effect of the price process of the variation in stock over time. It is a graph of the time against the closing price of company C's stock during the investment period. Fig. 4 represents the price of a stock over time. It shows the relationship between the time in days and the closing price of a company D's stock during the investment time. In all the graphs above, we observed the stochastic nature of the SMP whose price cannot be predicted based on past or future occurrences hence, the need for stochastic volatilities of which the GBM is one.

Also, Figs. 5-8 show a relationship between the respective expectations of the four companies A-D and their respective variances. It was observed that the expectation of the individual companies is directly proportional to their individual variances; the implication as shown on the graph indicates that the higher the risk involved in each company's assets, the higher the expectation.

In conclusion, the GBM model is one of the known models used in modelling SMP in a financial market. As at today, it is the most widely used model in practice due to its simplicity compared to other stochastic models. However, the study of GBM models has paved way for further improvement. Furthermore, the application of GBM model to financial market shows the model is appropriate for forecasting SMP on the floor of Nigeria stock exchange at least for the period of one month as stated in [22]. Also, we observed that reducing the investment days did not significantly affect the value of MAPE which was still within the region of acceptance. Finally, GBM gives a good opportunity to decide new and gain profit for the period of investment.

REFERENCES

[1] A. Ermogeneous. Brownian motion and its Applications in the Stock Market, Undergraduate Mathematics Day, Electronic Proceeding, Department of Mathematics, University of Dayton Commons (2006).

[2] C. R. Harvey, A. Siddique. Conditional skewness in asset pricing tests, Journal of Finance 55 (2000), 1263–1295. https://doi.org/10.1111/0022-1082.00247

[3] E. E. Akpanibah, U. O. Ini. Portfolio strategy for an investor with logarithm utility and stochastic interest rate under constant elasticity of variance model, Journal of the Nigerian Society of Physical Sciences, 2 (3) (2020), 186-196.

[4] B. O. Osu., C. U. Okonkwo, P. U. Uzoma, E. E. Akpanibah. Wavelent analysis of the international markets: a look at the next eleven (N11), Scientific African 7 (2020). 1-16.

[5] E. E. Akpanibah, B. O. Osu. Optimal Portfolio Selection for a Defined Contribution Pension Fund with Return Clauses of Premium with Predetermined Interest Rate under Mean variance Utility. Asian Journal of Mathematical Sciences 2(2), (2018), 19–29.

[6] K. Tan. Modeling returns distribution based on radical normal distributions', Journal of the Society for Studies on Industrial Economies 46(3), (2005), 449–467.

[7] A. Carol. Normal mixture diffusion with uncertain volatility: Modeling short- and long-term smile effects. Journal of Banking & Finance 28 (12), (2004), 2957–2980. https://doi.org/10.1016/j.jbankfin.2003.10.017

[8] K. Tan, S. Tokinaga. An approximation of returns distribution based upon GA optimized mixture distribution and its applications', in Proceedings of the 4th International Conference on Computational Intelligence, Robotics and Autonomous Systems, (2007), 307–312.

[9] K. Tan, M. Chu. Estimation of portfolio return and value at risk using a class of Guassian mixture distributions. The International Journal of Business and Finance Research 6(1), (2012), 97–107.

[10] H.B. Nembhard, L. Shi, M. Aktan. A real options design for quality control charts', The Engineering Economist 47(1), (2002), 28–50. https://doi.org/10.1080/ 00137910208965022

[11] S. Benninga, E. Tolkowsky. Real options – An introduction and application to R&D evaluation. The Engineering Economist 47(2), (2002), 151–168. https://doi.org/ 10.1080/00137910208965030.

[12] E. E.Akpanibah, U. O. Ini. An investor's investment plan with stochastic interest rate under the CEV model and the Ornstein-Uhlenbeck process, Journal of the Nigerian Society of Physical Sciences, 3 (3), (2021), 186-196.

[13] S. M. Ryan. Capacity expansion for random exponential demand growth with lead times', Management Science 50(6), (2006) 740–748. https://doi.org/10.1287/mnsc. 1030.0187

[14] S. Ross. Introduction to probability models, 7th edn., Harcourt Academic Press, New York. (2000).

[15] K.D. Brewer, Y. Feng, C. C. Kwan. Geometric Brownian motion, Option Pricing and Simulation: Some Spreadsheets Based Exercises in Financial Modelling, Spreadsheet in Education, (eJSiE), 5, Issue 3(2012).

[16] S.N. Abiding, M.M. Jaffar. Forecasting Share prices of Small Size Companies in Bursa, Malaysia, Applied Mathematics and Information Sciences, 8, (2014), 107-112.

[17] R. Krishna, C. Vaughan. Simulating Stock Prices Using Geometric Brownian motion: Evidence from Australian Companies, Australian Accounting, Business and Finance Journal, 10(3), (2016), 23-32.

[18] B. O. Osu, E. E. Akpanibah, K.N.C. Njoku. On the Effect of Stochastic Extra Contribution on Optimal Investment Strategies for Stochastic Salary under the Affine Interest Rate Model in a DC Pension Fund, General Letters in Mathematics, 2(3), (2017), 138-149.

[19] R.R. Marathe, S. M. Ryan. On the validity of the geometric Brownian motion assumption, The Engineering Economist 50(2), (2005) 159–192. https://doi.org/ 10.1080/00137910590949904.

[20] B. Nkemnole, O. Abass. Estimation of geometric Brownian motion model with a t-distribution– based particle filter. Journal of Economic and Financial Sciences 12(1), https://doi.org/10.4102/jef.v12i1.159. (2019).

[21] E. E. Akpanibah, B. O. Osu, B. I. Oruh, C. N. Obi. Strategic optimal portfolio management for a DC pension scheme with return of premium clauses. Transaction of the Nigerian association of mathematical physics, 8(1), (2019), 121-130.

[22] O. Sunday and M. S. Sulaiman. On the Application of Geometric Brownian motion Model to Stock Price Process on the Nigerian Stock Market. International Journal of Science for Global Sustainability 6(1), (2020), 80-87.

[23] K. D. Lawrence, R.K. Klimberg, S. M. Lawrence. Fundamentals of Forecasting Using Excel, Industrial Press Inc. America (2009).

[24] S.N. Abiding, M.M. Jaffar. A Review on Geometric Brownian motion in Forecasting the Share prices in Bursa Malaysia, World Applied Science Journal, 17 (Special Issue of Applied Math), (2012), 87-93.