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Channel Estimation for Orthogonal Frequency Division Multiplexing Systems over Doubly Selective Channels Based on the DCS-DCSOMP Algorithm

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***Abstract*—**The Doppler shift generated by high-speed movement and multipath effects in the channel are the main reasons for the generation of a time-frequency doubly-selective (DS) channel. There is severe inter-carrier interference (ICI) in the DS channel. Channel estimation for an orthogonal frequency division multiplexing (OFDM) system over a DS channel is very difficult. The simultaneous orthogonal matching pursuit (SOMP) algorithm under distributed compressive sensing theory (DCS-SOMP) has been used in channel estimation for OFDM systems over DS channels. However, the reconstruction accuracy of the DCS-SOMP algorithm is not high enough in the low Signal-to-Noise Ratio (SNR) stage. To solve this problem, in this paper, we propose an improved DCS-SOMP algorithm based on the inner product difference comparison operation (DCS-DCSOMP). The reconstruction accuracy is improved by increasing the number of candidate indexes and designing the comparison conditions of inner product difference. We combine the DCS-DCSOMP algorithm with the basis expansion model (BEM) to reduce the complexity of channel estimation. Simulation results show the effectiveness of the proposed algorithm and its advantages over other algorithms.

***Keywords*—**OFDM, doubly selective, channel estimation, compressed sensing.

I. Introduction

OFDM technology is used in many communication systems because of its strong anti-interference capability, high frequency utilization and flexible resource allocation [1]. In fast time-varying scenarios, OFDM systems suffer from both frequency-selective (FS) fading due to multipath effects and time-selective fading due to Doppler shift. This results in a time-frequency DS channel [2]. The DS channel exhibits sparse characteristics, and the impulse response of each propagation path changes rapidly with time, which destroys the orthogonality among subcarriers and generates severe ICI [3]-[5]. To achieve effective equalization of the ICI of the DS channel, the channel estimation requires a large number of pilot frequency subcarriers. The spectrum utilization is thus reduced. CS theory has been shown to solve this problem [6]. It is able to sample and reconstruct the signal at a frequency much lower than the Nyquist sampling rate and obtain good channel estimation performance using only a small number of pilot subcarriers.

The estimation accuracy of the channel response depends on the reconfiguration performance of the CS algorithm. Among various CS reconstruction algorithms, the greedy algorithm is widely used in channel estimation due to its simple structure and good reconstruction performance [7]-[10]. In [7], the location and magnitude of the non-zero components of the channel impulse response are estimated by the orthogonal matching pursuit (OMP) algorithm, and all the selected atoms are orthogonalized at each step of the reconstruction process. In [8], an improved OMP algorithm is proposed to solve the OFDM system channel estimation problem by introducing intersect operation onto two different partial sparse delay path supports. In [9], channel estimation for a hydroacoustic communication system is accomplished by the multipath matching pursuit (MMP) algorithm, which searches for multiple candidate desired paths that are most relevant to the residual vector in each iteration and selects the best candidate path. In [10], the SOMP algorithm in the distributed compressive sensing (DCS) framework is used to achieve accurate estimation of multipath channels. The DCS-SOMP algorithm uses a multidimensional measurement matrix to jointly reconstruct the sparse signal, resulting in an effective improvement in channel estimation accuracy. At low SNR, severe noise and complex interference in the DS channel can cause a large impact on the received signal, resulting in a large deviation of the index set selected by the algorithm from the set of non-zero element positions of the channel vector. The DCS-SOMP algorithm selects indexes by inner product sum, which improves the probability of correct index selection and estimation accuracy at low SNR. However, its index selection is based on a single judgment condition. This can easily lead to misclassification situations. If more valid judgment conditions are added, then the channel estimation accuracy in the low SNR stage will be improved even more.

In this paper, we propose a DCS-based inner product difference comparison SOMP algorithm (DCS-DCSOMP), which improves the index selection scheme of the DCS-SOMP algorithm. The inner product difference is defined as the evaluation index of index selection reliability. The number of candidate indexes and the probability of correct index selection are increased. Firstly, the measurement matrices are projected into two-dimensional space. We arrange the inner products of each column vector and the residual vector in descending order for every two-dimensional matrix. Then, we calculate the

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difference between the first two, and set the first corresponding column index as the candidate index. Finally, the inner product differences corresponding to each candidate index are sorted in descending order. We compare the difference between the first two with a threshold value. If the result is greater than the threshold, the candidate index with the largest inner product difference is added to the index set. On the contrary, it means that the inner product difference cannot be used as the judgment condition for selecting indexes. We execute the DCS-SOMP algorithm and add its selected index to the index set.

The remainder of this paper is organized as follows: Section II describes the system model of the OFDM system. Section III presents the estimation target and methods. Section IV provides the simulation results. Finally, conclusions are drawn in Section V.

Notations: Vectors and matrices are denoted by bold lowercase and uppercase letters. $diag(\cdot)$ denotes the diagonal matrix. $(\cdot)^T$ and $(\cdot)^H$ denote transpose and conjugate transpose respectively. $\|\cdot\|_p$ denotes the p norm of a matrix or vector. $\mathbf{I}_L$ denotes an $L \times L$ identity matrix. $\mathbf{A}_L$ denotes an $L \times L$ matrix. $\mathbf{0}_{M\times N}$ denotes an $M \times N$ zero matrix. $c_{l,k}$ denotes the element at index position (l,k). $[\mathbf{A}]_{i,j}$ denotes the element at position (i,j) in matrix $\mathbf{A}$. $\mathbf{c}_i$ denotes the i-th element of the vector $\mathbf{c}$. $[\mathbf{A}]_i$ denotes the i-th row of the matrix $\mathbf{A}$. $[\mathbf{A}]_{:,j}$ denotes the j-th column of the matrix $\mathbf{A}$. $\mathbb{C}^{M\times N}$ represents the set of $M \times N$ matrices in complex field.

II. Channel Model

In a pilot-assisted OFDM system, the number of subcarriers is set to N. An OFDM symbol is expressed as:

$$\mathbf{X} = (X[0], \cdots, X[N-1])^T \tag{1}$$

We perform an N-order IFFT operation on the signal before it is sent to obtain a time-domain signal. Then a cyclic prefix of length L_{CP} is added to the signal to reduce ICI. The transmitted signal of length $L+N$ can be expressed as:

$$\mathbf{x} = \mathbf{T}_{\mathrm{CP}}\mathbf{F}_N^H\mathbf{X} \tag{2}$$

where $\mathbf{T}_{\mathrm{CP}}$ is the cyclic matrix with cyclic prefix (CP) added and $\mathbf{F}_N^H$ is the normalized IFFT matrix of order N. The elements of $\mathbf{F}_N^H$ are specifically expressed as:

$$[\mathbf{F}_N^H]_{k,n} = \frac{1}{\sqrt{N}} e^{j\frac{2\pi kn}{N}} \tag{3}$$

With a sampling period T_s, the impulse response of the l-th path at time m under the DS channel can be written as:

$$h[m,l] = \sum_{i=0}^{I-1} \eta_i e^{j2\pi v_i m T_s} sinc(l - \frac{\tau_i}{T_s}) \tag{4}$$

in which η_i, τ_i and v_i denote the complex attenuation factor, time delay, and Doppler frequency associated with the i-th path respectively. The transmit signal is transmitted serially in the DS channel. The time domain signal at the receiver $y \in C^{N+L_{CP}}$ can be expressed as:

$$y[m] = \sum_{l=0}^{L-1} h[m,l]x[m-l] + w[m] \tag{5}$$

where $w[m]$ denotes the noise component. After CP removal and FFT operation, the received signal can be expressed as:

$$\mathbf{Y} = \mathbf{H}_F\mathbf{X} + \mathbf{W} \tag{6}$$

in which $\mathbf{W}$ denotes an additive noise matrix. $\mathbf{H}_F = \mathbf{F}_N\mathbf{H}_T\mathbf{F}_N^H$. $\mathbf{H}_T$ represents the process of adding CP and removing CP to the original signal. In a FS channel, $\mathbf{H}_T$ is a circular matrix and $\mathbf{H}_F$ is a diagonal matrix. But in a DS channel, $\mathbf{H}_F$ is no longer a diagonal matrix due to ICI.

III. Channel Estimation

A. BEM in Channel Estimation

The BEM can represent the DS channel within a certain period of time through the linear combination of multiple basis functions, which can effectively reduce the estimation complexity [11]. Each $\mathbf{h}_l$ can be written as:

$$\mathbf{h}_l = \begin{pmatrix}\mathbf{b}_0 \, \mathbf{b}_1 \, \cdots \, \mathbf{b}_{Q-1}\end{pmatrix}\begin{pmatrix} c[0,l] \\ c[1,l] \\ \vdots \\ c[Q-1,l]\end{pmatrix} + \boldsymbol{\xi}_l \tag{7}$$

where Q is the BEM order, $\boldsymbol{\xi}_l = (\xi[0,l], \cdots, \xi[N-1,l])^T$ denotes the BEM modeling error, $\mathbf{b}_q = (b[0,q], \cdots, b[N-1,q])^T$ denotes the q-th ($q \in [0, Q-1]$) BEM basis function, $c[q,l]$ denotes the corresponding coefficient of $\mathbf{b}_q$. It can be known from (7) that the introduction of BEM reduces the number of coefficients to be obtained from NL to QL. We further define $\mathbf{h}'_n = (h(n+L_{CP}, 0), \cdots, h(n+L_{CP}, L-1))^T \in \mathbb{C}^L$, q-th BEM coefficient vector as $\mathbf{c}_q = (c[q,0], \cdots c[q,L-1])^T \in \mathbb{C}^L$, and error vector as $\boldsymbol{\xi}'_n = (h(n,0), \cdots, h(n,L-1))^T \in \mathbb{C}^L$. Then, (7) can be written as:

$$\mathbf{h}' = (\mathbf{B} \otimes \mathbf{I}_L)\mathbf{c}' + \boldsymbol{\xi}' \tag{8}$$

in which $\mathbf{h}' = (\mathbf{h}'^T_0, \cdots, \mathbf{h}'^T_{N-1})^T \in \mathbb{C}^{NL\times 1}$, $\mathbf{c}' = (\mathbf{c}_0{}^T, \cdots, \mathbf{c}_{Q-1}{}^T)^T \in \mathbb{C}^{QL\times 1}$, and $\boldsymbol{\xi}' = (\boldsymbol{\xi}'^T_0, \cdots, \boldsymbol{\xi}'^T_{N-1})^T$. In the next discussion, we temporarily ignore $\boldsymbol{\xi}'$. Now (6) can be expressed in BEM as:

$$\mathbf{Y} = (\sum_{q=0}^{Q-1} \mathbf{B}_q\mathbf{C}_q)\mathbf{X} + \mathbf{W} \tag{9}$$

where $\mathbf{H}_F = (\sum_{q=0}^{Q-1} \mathbf{B}_q\mathbf{C}_q)$, $\mathbf{B}_q = \mathbf{F}_N diag(\mathbf{b}_q)\mathbf{F}_N^H$, and $\mathbf{C}_q = diag\left(\sqrt{N}\mathbf{F}_N\left(\mathbf{c}_q^T, 0_{1\times(N-L)}\right)^T\right)$. Our estimation target is converted from $\mathbf{H}_F$ to $\{\mathbf{c}_q\}_{q=0}^{Q-1}$.

It can be seen from [11] that the pilot signal at the receiver can be specifically expressed as:

$$\begin{cases} [\mathbf{Y}]_{P_0} = \left(diag(\mathbf{P}_e)[\mathbf{F}'_N]_{P_{\frac{Q-1}{2}}}\right)(\mathbf{\Lambda}_0\mathbf{c}_0) + \mathbf{W}_0 \\ [\mathbf{Y}]_{P_{\frac{Q-1}{2}}} = \left(diag(\mathbf{P}_e)[\mathbf{F}'_N]_{P_{\frac{Q-1}{2}}}\right)\left(\mathbf{\Lambda}_{\frac{Q-1}{2}}\mathbf{c}_{\frac{Q-1}{2}}\right) + \mathbf{W}_{\frac{Q-1}{2}} \\ [\mathbf{Y}]_{P_{Q-1}} = \left(diag(\mathbf{P}_e)[\mathbf{F}'_N]_{P_{\frac{Q-1}{2}}}\right)(\mathbf{\Lambda}_{Q-1}\mathbf{c}_{Q-1}) + \mathbf{W}_{Q-1} \end{cases} \quad (10)$$

where $\mathbf{P}_e$ is the effective pilot sequence. $[\mathbf{F}'_N]_{P_{\frac{Q-1}{2}}} \in \mathbb{C}^{M\times L}$ is the sub-matrix taken according to the row index set $P_{\frac{Q-1}{2}}$ in the FFT matrix, $P_{\frac{Q-1}{2}} = P_e$, $\{\mathbf{W}_q\}_{q=0}^{Q-1} \in \mathbb{C}^{M\times 1}$ includes noise and model error, $diag(\mathbf{P}_e)[\mathbf{F}'_N]_{P_{\frac{Q-1}{2}}}$ is the measurement matrix common to all equations. We set $diag(\mathbf{P}_e)[\mathbf{F}'_N]_{P_{\frac{Q-1}{2}}}$ to $\mathbf{\Phi}$. Then (10) can be written as:

$$[\mathbf{Y}]_{P_q} = \mathbf{\Phi}(\mathbf{\Lambda}_q\mathbf{c}_q) + \mathbf{W}_q, \forall q \in \{0,1,\cdots,Q-1\} \quad (11)$$

Equation (11) is of the same form as the expression of CS theory; we can get $\mathbf{c}_q$ by estimating $\mathbf{\Lambda}_q\mathbf{c}_q$.

B. DCS Reconstruction Algorithm

As shown in (10), the estimation of $\{\mathbf{c}_q\}_{q=0}^{Q-1}$ is actually a CS reconstruction problem. The BEM-based channel model exhibits joint sparsity in the time domain [12]. The channel estimation method based on DCS makes full use of the joint sparsity among multiple related signals to achieve better channel estimation effect. Scholars have applied the DCS-SOMP algorithm to OFDM system channel estimation [11]. The main idea of the DCS-SOMP algorithm is to use the inner product sum to evaluate the correlation between the columns of the measurement matrix and the residuals. We first project the measurement matrix to a two-dimensional space. In each two-dimensional matrix, we calculate the inner product of the column vector and the residual vector and add them according to the column index. The proportional relationship between the inner product and the correlation is used to find the column index with the strongest correlation with the residual, that is, the position of the non-zero element of the sparse vector. Then we sort the inner product sum in descending order. We add the column index corresponding to the first bit to the index set and repeat the operation until the number of elements in the index set is equal to the sparsity. Finally, the column vectors corresponding to the index set are taken out to form a new matrix, and the signal is reconstructed by QR decomposition. Obviously, finding as many correct non-zero element positions as possible is the key to improving the reconstruction effect.

However, the index selection method of the DCS-SOMP algorithm is not perfect. Due to the influence of noise, the probability of the non-zero element position being correctly selected is not high at low SNR. Therefore, we choose to make improvements in the index selection phase. We add reliable judgment conditions to improve the effectiveness of the index selection scheme at low SNR.

Our improvement idea is to design an index selection scheme based on the DCS-SOMP algorithm. Instead of taking the inner product sum as the only judgment condition, the results of independent reconstruction of each two-dimensional matrix and the influence of noise are used as the design basis. The purpose is to achieve a steady improvement in the reconstruction effect.

In the DCS-SOMP algorithm, the dimension of the measurement matrix A is equal to the BEM order. Index selection can be expressed as:

$$\underset{\omega\in[1,L]}{\operatorname{argmax}} \sum_{q=1}^{Q} \frac{|\langle \mathrm{r}_q, \mathbf{\Phi}_{\omega,q}\rangle|}{\|\mathbf{\Phi}_{\omega,q}\|_2} \quad (12)$$

where ω is the column index of $\mathbf{\Phi}$, q is the dimension index, and $\mathbf{r}$ is the residual. In each two-dimensional projective measurement matrix, we compute the inner product of each column and the residual. The index of the column with the largest inner product is set to a_q. We found that the non-zero element position of the signal has a high probability of being an element in $\{a_1, a_2, \cdots, a_Q\}$, which can be used as a new judgment basis for index selection.

In the low SNR stage, the noise has too much influence on the received signal. It is unreliable to use the inner product to evaluate the correlation between two column indexes whose inner product difference is small. On the contrary, the inner product difference can be used as an evaluation indicator for the reliability of the scheme. We can use the size of the inner product difference to select the element in x that is most likely to be a non-zero element position. The process is as follows:

Step1. In each dimension, sort the inner products of each column and the residual from large to small and calculate the difference between the first two digits to obtain inner product differences. If the difference between the inner product differences is small, it is unreliable to use the column index corresponding to the largest inner product difference as the non-zero element position.

Step2. Sort the inner product differences from large to small. Calculate the difference between the first two digits and compare the result $\Delta\mathbf{d}$ with the preset threshold ζ, which is usually set at 0.1~0.2. If $\Delta\mathbf{d} \geq \zeta$, the inner product difference can be used as the judgment basis for index selection.

Step3. Take the column index corresponding to the largest inner product difference as the final selection result. If $\Delta\mathbf{d} < \zeta$, the inner product difference can not be used as the judgment basis for index selection.

Step4. Execute the index selection scheme of the DCS-SOMP algorithm, and use the selected column index a_s as the final selection result.

According to the above analysis, we propose a BEM coefficient estimation scheme based on the DCS-DCSOMP algorithm, which is given in Algorithm 1.

IV. Simulation Results and Discussion

In this section, we simulate the proposed channel estimation scheme to verify the effectiveness of the proposed algorithm. The used parameters can be summarized as follows.

Algorithm 1 The DCS-DCSOMP based BEM Coefficients Estimation

Input: $\mathbf{Y} = ([\mathbf{Y}]_{P_0}, \cdots, [\mathbf{Y}]_{P_{Q-1}})$, $\mathbf{\Phi}$, ζ and the sparsity K

Output: $\{\hat{\mathbf{c}}_q\}_{q=0}^{Q-1}$

Procedures:

1) **Initialization** set the index set $\mathbf{\Omega}_I = \emptyset$ and the residual $\mathbf{r} = \mathbf{Y}$

2) **Index selection and residual estimation**

for $t = 1{:}K$

a) Execute **Step1**、**Step2**、**Step3** and **Step4** to obtain the inner product difference$\{\mathbf{d}_1, \cdots, \mathbf{d}_Q\}$、$\Delta\mathbf{d}$ and a_s.

b) **If** $\Delta\mathbf{d} \geq \zeta$, get the index m corresponding to the largest item in $\{\mathbf{d}_1, \cdots, \mathbf{d}_Q\}$.
$a_m \in \{a_1, a_2, \cdots, a_Q\}, \mathbf{\Omega}_I = \mathbf{\Omega}_I \cup a_m$. **end if**

c) **If** $\Delta\mathbf{d} < \zeta, \mathbf{\Omega}_I = \mathbf{\Omega}_I \cup a_s$. **end if**

d) Set the new element in $\mathbf{\Omega}_I$ after b) and c) is a_t,$\mathbf{\Phi}_t = (\mathbf{\Phi}_{t-1}, \lambda_{a_t})$, λ_{a_t} is the a_t-th column of $\mathbf{\Phi}$.

e) Calculate the new residual
$\mathbf{r}_t = \mathbf{Y} - \mathbf{\Phi}_t(\mathbf{\Phi}_t^H\mathbf{\Phi}_t)^{-1}\mathbf{\Phi}_t^H\mathbf{Y}$

f) $t = t + 1$

end for

3) Calculate $\{\hat{\mathbf{\Lambda}}_q\hat{\mathbf{c}}_q\}_{q=0}^{Q-1}$ by QR decomposition

4) Calculate $\{\hat{\mathbf{c}}_q\}_{q=0}^{Q-1}$ from $\{\hat{\mathbf{\Lambda}}_q\hat{\mathbf{c}}_q\}_{q=0}^{Q-1}$ by inverse operation

The number of subcarriers is $N = 512$. The CP is $L_{CP} = 64$. The subcarrier spacing is $\Delta f = 15$ kHz. The number of channel taps is $L = 20$, in which the number of non-zero taps is $K = 6$. The data mapping method is 16 QAM.The BEM order is $Q = 3$. The simulation is carried out in a high-speed scene with a speed of 300 km/h.

The input parameters of Algorithm 1 are as follows. $\zeta = 0.12$, $K = 6$. In Fig. 1, we describe the NMSE performance comparison of each reconstruction algorithm. The advantages of the DCS algorithm are obvious, and the DCS-DCSOMP algorithm is 0.6dB lower than the DCS-SOMP algorithm. In Fig. 2, we describe the BER performance comparison of each reconstruction algorithm. The DCS-DCSOMP algorithm is 0.001 lower than DCS-SOMP. However, due to the influence of the channel, there is still a certain gap between the result and the optimal BER.

V. Conclusion

This paper proposed an improved DCS-SOMP algorithm that introduces the difference comparison operation in the index selection stage to improve the reconstruction accuracy at low SNR. We used BEM to transform the channel model and completed the channel estimation for an OFDM system over DS channels by the accurate estimation of BEM coefficient vector. Simulation results demonstrated that compared with the DCS-SOMP algorithm, the proposed algorithm could obtain better NMSE and BER performance. In the follow-up work, we will study the application of the proposed algorithm in other wireless systems.

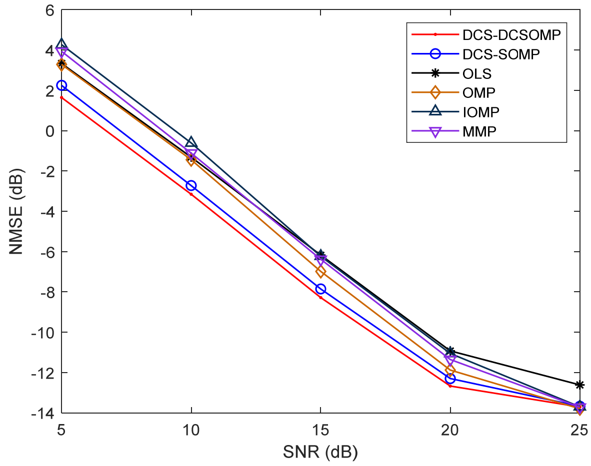

Fig. 1 Comparison of the NMSE performance

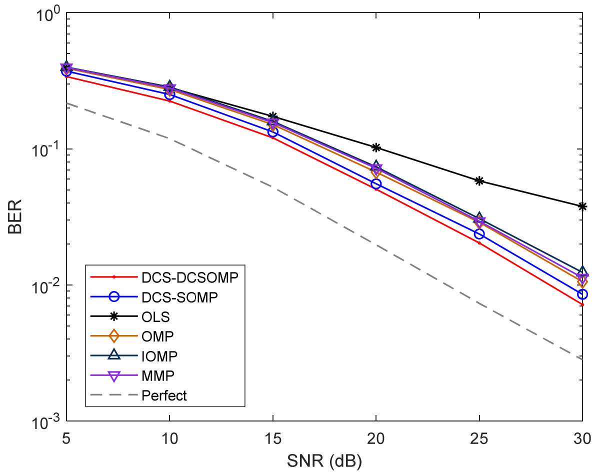

Fig. 2 Comparison of the BER performance

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