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Inventing a Method of Problem Solving: The Natural Movement of the Mind to Solve a Problem

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Abstract—The major objective of this study was to devise a method for solving mathematical problems. Three concepts including faculty of understanding, faculty of guess, and free mind or beginner's mind provided the foundation for this method. An explanatory approach along with a hermeneutic process was taken in this study to support the assumption that mathematical knowledge is constantly developing and it seems essential for students to solve math problems on their own using their faculty of understanding (interpretive dialogue) and faculty of guess. For doing so, a kind of movement from the mathematical problem to mathematical knowledge should be adopted for teaching students a new math topic. The research method of this paper is review, descriptive and conception development. This paper first reviews the research findings on the NRICH'S project (NRICH is part of the family of activities in the Millennium Mathematics Project) with the aim that these findings form the theoretical basis of the problem-solving method. Then, the curriculum, the conceptual structure of the new method, how to design the problem and an example of it are discussed. In this way, students are immersed in the story of discovering and understanding the problem formula, and interpretive dialogue with the text continues by following the questions posed by the problem and constantly reconstructing the answer to find a formula or solution to solve the problem.

***Keywords*—**Interpretive dialogue, NRICH, inventing, a method of problem solving.

I. Introduction

TYPICALLY, to teach a new math topic, students first learn how to use mathematical knowledge, rules and formulas for solving the given math problem, then a math exercise on the subject is given to the students and they are asked to solve it using their mathematical knowledge acquired through the learning process. In this way, objectives such as thinking and problem-solving as well as using mathematical knowledge and formulas for solving math problems are met.

However, teaching math in schools based on the innovative method represented in this paper is as follows: first, students are provided with some problems on the new math topic along with their answers, then they make their efforts to discover or create mathematical knowledge, rules and formulas for solving that math problem using the innovative method explained in this paper based on guessing and interpretive dialogue, and finally they are expected to utilize acquired knowledge and formulas to solve similar math problems. Skills in thinking and problem-solving, as well as discovering mathematical knowledge and formulas are the main objectives which are fulfilled in this method. It should be noted that the NRICH assumptions have been used as a theoretical basis. This method is derived from a hermeneutic approach and uses inductive structures and processes and supports the Beginner mind and seeks to discover, construct and understand meaning (mathematical knowledge and rules and formulas for problem solving). To put it simply, this method is a kind of mind game in which the faculties of understanding and guess are used to discover and create meaning for solving math problems. The main hypothesis of this individualism approach is that mathematical knowledge is consistently developing and students need to discover, create and renovate their basic mathematical knowledge with the help of the natural and mental faculties of understanding and guess. The natural movement of the mind mentioned in the research title refers to the two mental activities existing in this method for solving problems including faculty of understanding and faculty of guess. The concept points out to the first encounter of a beginner's mind (basic knowledge) with math problems and their answers for discovering mathematical knowledge, formulas or rules through their dialogue on the problem and its answer by the use of their natural faculties of understanding and guess. The assumption underlying this method is that when a beginner's mind faces a math problem, they can naturally (without having learned a method or formula in advance) discover mathematical knowledge or formulas for solving math problems through guessing as well as interpretive dialogue on the problem and its answers. As a result, it is important to allow students with beginner's mind to be first faced with a math problem and its answer, then mental activities such as guessing and interpretative dialogue must be practiced in order to develop the faculty of understanding and guess. The research method of this paper is review, descriptive analysis and conception development. Conception development is the research that is designed to invent and defend a concept or conceptual structure [22]. This research seeks to address the following questions:

- Instead of directly teaching mathematics, how should math be taught in schools to motivate learners to discover or create math knowledge and formulas for solving math problems?
- Are students able to find out problem solving rules and formulas and mathematic knowledge simply by using such skills as beginner's mind, guessing, and dialogue on the problem and its answer without having learned any method or formula about the given subject in advance?
- What is an explanatory method in teaching mathematics?

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II. NRICH Mathematics

One of the innovations available at The Cambridge University is NRICH Mathematics [3]. The NRICH website was launched in 1996 as a math club with the mission of using information and communication technology to conduct research into the impact of technology on teaching math, supporting and promoting interest in math [1], [23]. Its mathematical activities focus on the development of problem solving. Rich mathematical exercises increase students' persistence, develop the ability to create mathematical reasoning, increase their confidence to solve new problems, and develop the ability to creatively apply mathematical knowledge in new contexts [4].

The reputation of the NRICH mathematics project is for creative thinking in the field of mathematics enrichment. The concept of enrichment in this project indicates that this project's approach to teaching mathematics is an open and flexible approach that encourages experimentation and communication (group work and mathematical communication) [2]. The aims of this project are: 1. Enriching the mathematics curriculum experience and enhancing this experience for all learners; 2. Developing mathematical problem solving and high level mathematical thinking skills, 3. Offering challenging, inspiring and engaging (to encourage mathematical thinking, improve students' attitudes, etc.) activities; 4. Showing rich mathematics in meaningful contexts; 5. Sharing expertise in teaching mathematics through work in partnership with teachers and educational settings [4]. According to this project, the purpose of teaching is enrichment and learning through enrichment drawing on developing skills in problem solving and mathematical thinking are viewed the foundations for enrichment. Piggott says: “One of the things at NRICH try to offer in an enrichment curriculum is the opportunity to experience: "The joy of confronting a novel situation and trying to make sense of it - the joy of banging your head against a mathematical wall, and then discovering that there may be ways of either going around or over that wall." [5]”.

For a problem to be effective: 1. It does not matter how good the problem is at the desired level, it is important that the student be able to engage with it to learn, 2. It must be readily available, meaning that the problem must be comprehensible to be solved by the student [5]. “Dewey [7], Polya [10], Mason et al. [8], Ernst [9], Mayer [6], have each proposed models with processes that are divided into different stages to solve the problem. Piggott explains the C.A.P.E model, which has a number of common features with existing models [2], [5]”.

A. The C.A.P.E. Model Presented by Piggott [2], [5]

1. Comprehension
- “Making sense of the problem/retelling/creating a mental image”,
- “Applying a model to the problem”;
2. Analysis and synthesis
- “Identifying and accessing required pre-requisite knowledge”,
- “Applying facts and skills, including those listed in mathematical thinking (above)”,
- “Conjecturing and hypothesising (what if)”;
3. Planning and execution
- “Considering novel approaches and/or solutions”
- “Identifying possible mathematical knowledge and skills gaps that may need addressing”,
- “Planning the solution/mental or diagrammatic model”,
- “Execute”;
4. Evaluation
- “Reflection and review of the solution”,
- “Self-assessment about one’s own learning and mathematical tools employed”,
- “Communicating results” [2], [5].

B. Mathematical Thinking

Mathematical thinking refers to specific mathematical strategies used for solving various problems [5]. Or, to put it differently, mathematical thinking is a series of math skills used to solve problems effectively [2].

1. Piggott lists some of “the mathematical thinking strategies:
- Conjecturing/theorising;
- Being systematic;
- Identifying common structures (isomorphisms);
- Introducing variables;
- Generalising;
- Specialising/clarifying/looking for specific examples;
- Considering a special case (the particular);
- Solving simpler related problems;
- Reflecting on experience - have you met something like this before?
- Multiple representations;
- Working backwards;
- Identifying and describing patterns;
- Representing information - diagram, table
- Testing ideas - guessing and testing (hypothesizing) [2].”
2. Piggott: “Special math skills for problem solving:
- Specialising (specific action that comes out of the problem doping a particular thing to help to simplify or trying special cases, e.g. paper folding)
- Generalising (as identifying patterns general or common patterns formula looking for an essential shape or form)
- Using analogy (examine problems with a structure similar to the one in question and get ideas from it)
- Visualising (using pictures to represent or explain mathematical problem situations or their solutions)
- Identifying the particular
- Modelling
- Decomposing [5].”

C. Enrichment (Content, Teaching Approach, Aims)

Piggott believes that problem solving in terms of content includes general skills that explain the basic elements in the problem-solving process. Mathematical thinking is skills for effective problem solving. Piggott analyzes the dimensions of the enrichment curriculum (content, teaching approach, aims) in three dimensions, that supports problem solving [5]:

1. Content: “Engaging problems which:
- help use and expand problem-solving strategies,
- encourage mathematical thinking [5];
2. A teaching approach that:
- encourages an open flexibly productive environment,
- encourages collaboration and teamwork,
– Exploration (creates openness and space for exploration and innovation, encourages learners to explore other dimensions),
– Mathematical communication (promoting dialogue and communication),
– “The valuing and utilization of difference as a teaching tool”
– “The acknowledgment that mathematics is often hard” [5], [2].
2. The aims of an enrichment curriculum:
- Using the four-element C.A.P.E model to solve the problem
- Improving learners' attitudes towards mathematics,
- Appreciation of mathematics
- Supporting mathematical comprehension and thinking by developing conceptual structures [5].

Examining the goals, approaches of teaching and content, it can be concluded that openness and flexibility are the two main features of enrichment.

D.Enrichment

The two basic concepts in enrichment are acceleration and extension. The concept of acceleration means that learners deliberately encounter advanced topics before the calendar age specified in the curriculum. “Extension is considered to be the exposure of learners to content not normally found in the standard curriculum and which might be considered appropriate to that chronological age or older [11].” Rich tasks and curriculum are not in conflict with each other. Rich tasks give learners the opportunity to ask questions about mathematical ideas and concepts, to broaden their understanding of mathematical concepts, to increase their self-confidence, to discover mathematical concepts. The goals that rich tasks for learners pursue are creativity in thinking, logical action, sharing ideas, combining results, analyzing perspectives, and evaluating findings. As a result, to meet these objectives, a classroom should have two main characteristics: it should be based on research assembly (exploration circles) and collaboration, and thus, it should promote imagination and establish communication [12].

1. Piggott describes the characteristics of a good problem as:
- Being apprehensible for a large group of learners (accessibility),
- Attracting the learner to mathematics, having an attractive starting point [12].
- Providing two opportunities for learners: to challenge learners to begin thinking by themselves, and to help learners have a sense of success (initial success),
- Leveling the problems: It is also possible for people in high demand to receive high-ceiling tasks,
- Having an opportunity to raise one’s issues,
- Providing various methods for problem-solving so that different responses are offered,
- Having an opportunity to discover various, more efficient, and more subtle solutions,
- Helping learners broaden their math skills, and deepen their mathematical knowledge,
- Providing creative treatment of the problem and use of imaginative knowledge for problem-solving,
- Being capable of revealing patterns previously recognized in the mathematical knowledge,
- Being capable of establishing communication between various areas of mathematics for problem-solving, or revealing pre-recognized basics principles,
- Having a space for cooperation and dialogue,
- Encouraging learners to achieve independence, increasing their self-confidence, and helping them critically deal with problems,

It is for the teacher to provide a rich experience for the learners [12], present a problem that has the potential to meet all or some of the above [13].

1. Features of a rich task [13]:
- being extendable and accessible for learners to understand,
- giving learners the opportunity to make decisions,
- involving learners in such activities as interpretation, testing, reflection, proving, and explanation,
- promoting and providing conditions for dialogue and communication,
- encouraging learners to originality and invention,
- encouraging learners to ask questions such as: 'what if' and 'what if not',
- amazing and enjoyable,
- “acceleration”

[11].

1. Criteria for identifying a rich mathematical activity:
- is accessible for all learners, i.e., available for the learner to understand,
- invites learners to make decisions,
- involves learners in such activities as interpretation, testing, reflection, proving, and explanation,
- encourages learners to explore other dimensions,
- promotes and provides conditions for dialogue and communication,
- encourages learners to ask questions such as: 'what if' and 'what if not',
- is surprising and astonishing,
- is enjoyable for learners [13].
2. Productive environments:
- focus on the learner rather than the content and the teacher,
- seek the independence of the learner,
- create openness and space for exploration and innovation,
- focus on the learner’s acceptance instead of judging the learner,
- Possibility of learning from the environment
- Possibility of variation in grouping

- have a flexible structure, not rigid,
- encourage learners to mental agility,
- focus on concepts, not procedures,
- use rich assignments for making higher-order thinking skills possible,
- foster creativity
- value constructive communication [14].

3. In preparation: Prepare items to support above classroom culture:

- Preparing, finding, and proposing open problems and rich assignments,
- Accessing to a wide range of online and paper resources
- Sharing good ideas with colleagues,
- Investigating how to use able pupils to support other children,
- Investigating how to use capable learners to support other learners,
- Preparing opportunities and encouragement, self-assessment, and the selection of materials,
- Making use of online communities
- Enjoying the unpredictable [14].

E. Creating a Space for the Development of Learners' Mathematical Thinking

- "Promoting a conjecturing atmosphere"
- "Careful use of questions and prompts"
- "Low threshold high ceiling tasks"
- "Modelling behavior"
- "Whole class discussion"
- "Highlighting behavior that you want to promote"
- "HOTS not MOTS" [15].

1) "Promoting a conjecturing atmosphere"

- "Accepting 'messy' work"
- "Valuing risk-taking and half-formed ideas"
- Promoting and encouraging conditions for dialogue and communication,
- Giving the opportunity to think [15].

2) "Careful use of questions and prompts"

- "What have you found out so far?"
- "Do you notice anything?"
- "Is that always true?"
- "Can you convince us?"
- "Can anyone think of a counter example?"
- "What if...?"
- "What might you try next?"
- "Is there a way you could organize your findings?" [15].

F. Various Aspects of Enrichment

Feng, in an article, has explained various aspects of enrichment and has shown these explanations in a figure, briefly [16]. The figure designed by Wai Yi Feng is shown in Fig. 1.

III. Explaining the Method

A. Curriculum

1. A type of planning in a mathematics curriculum is that, mathematical knowledge is taught, then the problem is given and the student is asked to solve the problem according to the mathematical knowledge. The goals of this type of curriculum are: 1. Thinking and problem solving, 2. and using mathematical knowledge to solve the problem.

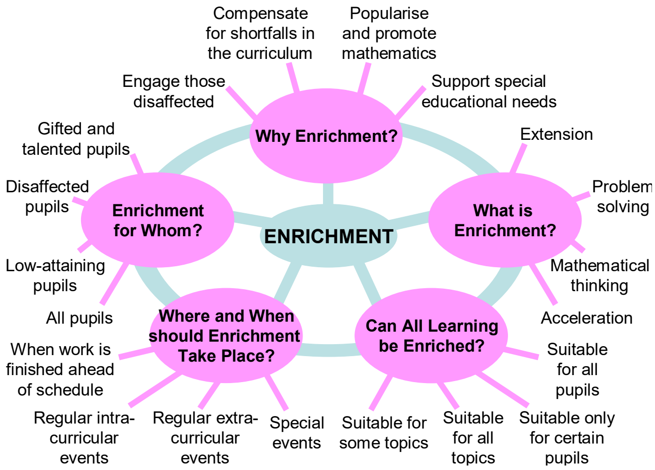

Fig. 1 Different aspects of enrichment

2. The curriculum of the method in this paper will be as follows that, to teach a new subject, both the problem and the answer to the problem are given first and then with the help of the innovative method in this paper, which is interpretive dialogue with the text, students will come up with a formula or problem-solving strategy and build math knowledge. The goals of this type of curriculum are: 1. Thinking and problem solving, 2. Building math knowledge.
3. *Curriculum aims:* This method (explained in Table I and Fig. 2) aims to evoke a sense of interpretation; an interpretation in the form of interpretative dialogue and a hermeneutic cycle to explore mathematical problem solution formulas. This method also seeks to strengthen the power of guessing; this method is characterized by creativity (novel, valuable, and explorative task [24]), the understanding of the mathematical knowledge, the obtaining of the pleasure of discovery, the sense of independence in learning, and the conversion of the child into a mathematician. In this method, children are given the opportunity to discover as much as they can alone. This method is an individualistic method and the student must solve the problem alone and build mathematical knowledge. It should be noted that the NRICH assumptions have been used as a theoretical basis. The natural movement of the mind means encountering with a beginner mind (basic knowledge) with mathematical problems and their answers and using the natural faculty of understanding and guess to discover meaning (knowledge, rules and problem-solving formulas). The natural movements of the mind also refer to the two mental faculties of guess and understanding that exist in our nature and have activities or movements. Therefore, three concepts are important in this method: 1. Faculty of

Understanding, 2. Faculty of Guess, 3. Free mind or beginner mind.

B. Faculty of Understanding (Hermeneutic Cycle and Interpretive Dialogue)

Understanding in this method is influenced by Gadamer's theories. And the method used to achieve understanding is interpretive dialogue with the text or the hermeneutic cycle of questions and answers. Hermeneutic experience has a defect, experience is imperfect, and the question arises from the defect of experience, and this question has the property of negation. The right question is the question that creates openness to build and reconstruct and create something, and it is the right question that provides knowledge by providing an opportunity to build and reconstruct and create something. In this interpretive dialogue, the question takes precedence, and this question arises from the defect of experience and imposes itself on us. In this interpretive dialogue, the question of negation and reconstruction continues in an internal dialogue until something is created or made. Socratic dialogue or question and answer is a clear example of hermeneutic understanding. This question-and-answer process is ongoing, and the openness of our new question-and-answer process is constantly being understood and reconstructed [19]. In questioning, we finally get to the wall of negation (negation means things were not what he thought they were). In other words, we conclude that "I still don't know". The adjective openness is a clear attribute of genuine questioning, asking originally means being outdoors because the answer is not yet definite. We seek to find the real answer step by step through questioning. And there is only one way to find the right question, and that is through immersion [18]. The condition of dialogue is also horizontal. Here, understanding is made possible in the dialogue. In this dialogue, the horizons get closer to each other, and through dialogue, one can enter each other's world and horizons. In order to enter each other's worlds, one must establish relations, linguistic relations. In order to be in agreement with another and horizontal thought, one must ask the question of establishing the same relation. Understanding is obtained by merging horizons. Understanding in one sense means understanding it as the horizon of the question and it is obtained by combining the horizon of the text and the horizon of the interpreter. The art of the interpreter is to keep the question open about the meaning of the phenomenon and to move towards the essence of what is being asked. Gadamer considers the beginning of the hermeneutic discussion to be Plato's dialectical dialogue. Plato's dialectic, or Socratic dialogue, takes place between dialogue partners who are open to the truth and understanding in order to approach the truth or give birth to the truth. Openness means listening to someone who listens well and asks good questions [19]. For Gadamer, hermeneutics is the art of understanding; hermeneutics means an artistic act in the first place. The arts in question are preaching, interpretation, explanation and interpretation, and of course the basic art is to understand that wherever the meaning of something is not clear and obvious, it is necessary [19]. For Heidegger and Gadamer, the hermeneutic cycle reflects the idea that any understanding is temporary, and that the process of realizing motion understanding is endless [20]. Gadamer adds that in interpreting a text we cannot separate ourselves from the meaning of a text [21]. The reader belongs to the text that he or she is reading, Gadamer also argues "Understanding is always an interpretation, and an interpretation is always specific, an application. For Gadamer the problem of understanding involves interpretive dialogue which includes taking up the tradition in which one finds oneself. Texts that come to us from different traditions or conversational relations may be read as possible answers to questions. To conduct a conversation, says Gadamer, means to allow oneself to be animated by the question or notion to which the partners in the conversational relation are directed [21]." Hirsch (1967) interprets the text as a reconstruction of the meaning or meanings desired by the author. Understanding the text is the result of a dialectical process between writer and reader. Knowing and realizing something about the person who wrote it adds to the validity of the interpretation [21].

The difference between this method and Gadamer's idea is that it is ultimately possible to reach final understandings. There is an ultimate meaning and we must discover it. In this method, the ultimate meaning is the same formula or principles of mathematical solution. This method approaches Hirsch in terms of ultimate meaning. Of course, this method also establishes a relationship with Dewey's view to reach a final understanding of the formula or mathematical knowledge.

John Dewey's problem-solution [17] stages are: 1. Becoming aware of difficulty, 2. Identifying the problem, 3. Assembling and classifying data and formulating hypotheses (a suggested solution), 4. Accepting or rejecting tentative hypotheses, 5. Formulating conclusions and evaluating them [7], [17]. In Dewey's view, we set a goal and constantly pursue it and evaluate our efforts based on that goal. Similar to Dewey's perspective, in this method a mathematical problem, along with its final answer, is given to the student, who does his best to explore the formulas to solve this problem and its relevant mathematical knowledge, thus constantly making evaluations based on the answer. Mathematical knowledge, or a formula, is correct when they can be used to arrive at this final answer. Of course, in the next step, there is a valid mathematical knowledge or formula that can answer other mathematical problems.

C. Faculty of Guess: Conscious Guessing

In this method, a mathematical problem solving strategy or a mathematical formula is guessed by examining the evidence, and then the formula is tested on other mathematical problems to see if the formula is correct or not. If the guessed strategy or formula is correct, a mathematical knowledge and an initial understanding are made. If the guessed math strategy or formula is wrong, one can gain a new understanding or learn something by examining why the guess is wrong, so even wrong guesses are instructive. Of course, testing is not just about checking the answer to other problems, but the question itself can have a negating property and negate the guessed answer. In this question-and-answer process, in which we are constantly understanding and reconstructing our understanding of mathematical knowledge, guess can be the answer to questions arising from the text that must be tested and analyzed, and the result of the test and analysis gives our initial understanding. This understanding develops as the process of interpretive dialogue with the text continues to achieve the final understanding (problem-solving formula, mathematical knowledge). Cohen (1974) says: What is guessed cannot be believed until it is tested, and in order for our guess to be conscious, we must use past successes and failures [25]. Conscious guessing is not a simple or superficial expression, but it can be considered a cognitive activity. In other words, Dorst et al. have referred to a cognitive image of guessing: "guessing is a cognitively basic activity one that we constantly engage in as we think, talk, and reason. Moreover, it's an activity that makes sense to engage in, for it's part of how computationally limited creatures like us cope with an intractably complex and uncertain world" [26]. Guess, then, is not separate from rational thinking, as Holguín puts it: "We can then say that rational thinking is thinking in terms of one's best guess" [28]. Conscious conjecture arising from the examination of evidence can therefore open the door to the discovery of mathematical formulas or knowledge, and acts as an exploratory method for constructing mathematical meaning or knowledge and discovering formulas and problem-solving rules. Built-in math gives us formulas and solutions to solve a problem, but the math being built requires guessing and testing guesses and expanding one's understanding to arrive at final formulas and final understandings. Pólya (1966) says: "First, guess; then prove...Finished mathematics consists of proofs, but mathematics in the making consists of guesses" [27].

D. Beginner Mind

A Japanese mindfulness instructor has made a remarkable statement about the beginner mind, which is: "In the beginner's mind, there are many possibilities, but in the expert's, there are few." [29]. But the meaning of this concept here is not in accordance with the meaning of the beginner mind in the mindfulness. The beginner mind means that in this method for exploration, to make meaning and reach understanding, it does not require a lot of knowledge and an expert mind, but a basic and beginner knowledge is enough. For example, in this method, a beginner's elementary third-grader's knowledge of what and how the +, -, ×, =, and numbers are used to discover and construct knowledge, rules, and formulas for solving power problems in mathematics. This is an explorative and subjective play that can be done by a student with the primary knowledge of a mathematical issue. This student first faces a problem and its answer, then tries to discover the formula to solve this problem.

TABLE I
THE NATURAL MOVEMENT OF THE MIND AS AN INTERPRETIVE DIALOGUE TO SOLVE A PROBLEM (METHOD STRUCTURE)

Problem solving steps	Questions driving an interpretive dialogue
Facing the problem and astonishment (Step1)	What is this? How should it be solved? What is the formula for solving this problem? What principles does this issue have in its heart? The moment we face the problem and I say to myself: What is this? What to do with it now? In fact, such an astonishment starts a conversation with the problem.
Examine the evidence and guess the formula (Step2)	Do you notice anything? What have you ever realized? How did this answer come about? How does this evidence suggest such an answer? What has changed and what has remained the same? Why has it changed like this? Is this always the case? Which signs and symptoms mean to me and which one should we discover? Which meaning should we look for? (The mind moves towards a unification or discovery of the formula, that is, to gain an understanding of the formula by examining the evidence, which is the initial conjecture or understanding).
Guessed formula test (initial understanding of evidence review) (Step3)	Can the rest of the questions be solved with this formula? Is this strategy that was discovered visible in the answers to the other questions? For example, when a negative power is deducted from the numerator to Denominator, its sign changes. Has this happened in response to other questions? Analyze Why was your guess incorrect or correct?
Review the results and turn the result into an initial understanding, then develop understanding. (Step4)	Is such a formula and strategy valid? Can it solve the rest of the questions? Or can it be used to help solve other questions? Is this always true? What might you try next? Can we still accept this formula? Can other similar problems be solved with it? Can you convince us? Can you explain how you came up with such a formula? Can you think of a reciprocal example?
Organize findings and move on to a set of valid formulas and strategies (connected like puzzle pieces) (Step5)	Finally, a set of formulas and methods is made. In fact, mathematical knowledge is made by the student. Is there a way you can organize your findings?

E. Method Structure (Interpretive Dialogue with Text and Conscious Guessing)

Interpretive dialogue with the text continues by pursuing the questions that arise from the problem and constantly reconstructing the answer to arrive at a formula or strategy for solving the problem. In this way, we are in the story of discovering and understanding the formula of the problem. What makes a problem-solving model and classification is the nature of the questions. The steps are classified according to the type of questions. Questions that arise from the problem and are presented to the student will be followed up. In this method, we examine the evidence for the question arising from the text, then we test the answer and reconstruct the understanding and create a new understanding. In fact, we will have a dialectical hermeneutics, and that will be a way of discovering meaning. In fact, it means discovering the laws, principles and formulas that govern the subject. Therefore, there will be a two-way dialogue between a text horizon and an interpreter horizon, which if combined, could help construe the truth. In this method, no formula is presented, but the formula itself is discovered. Children should discover as much as they can. In this method of education, we go beyond the basic level and a nine-year-old child can learn the contents of a student thirteen years old and even older. In this way, the next level can be the story and the dialogue process can take on a social and interpersonal dimension. In this method, the question is given priority and the reasonableness of the answer to the question depends on whether the horizon of the question is the same as the horizon of the text or the problem, or in fact the question arose from the text or the problem.

The natural movement of the mind denotes using guessing and interpretive dialogue to solve a problem.

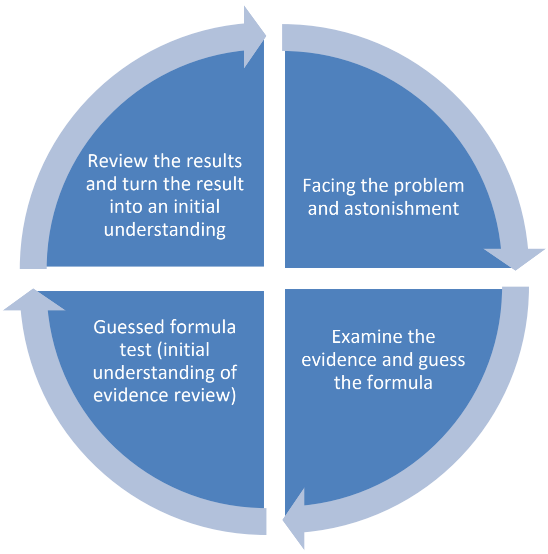

Fig. 2 The natural movement of the mind as an interpretive dialogue to solve a problem

F. Problem Design

- The problem must be unfamiliar to the student or beyond his or her age. For a third-grader, a problem can be posed several years above his or her academic age. Of course, not every problem can be raised.
- A beginner's knowledge is sufficient to explore and construct mathematical knowledge and mathematical rules and formulas (for example, to discover mathematical knowledge and mathematical problem-solving formulas in the thematic domain of power in math, one needs to recognize addition, subtraction, multiplication, division and their signs).
- It should be accessible for a wide range of students.
- They challenge learners to think for themselves.
- Mathematical problems should be designed in various levels.
- When math problems are designed, students should be encouraged to utilize their creativity and imagination to create a mathematical body of knowledge in guessing, hypothesizing, and testing the hypotheses.
- In each case, there is no limit to the number of questions and answers available. Questions and answers can be posed as far as the student can reach the formula or strategy.
- By designing a "Rich Problem is Not Enough", the teacher must help the students to finally build a complete set of problem-solving formulas and math knowledge about a math topic.

1. Examples of training problems are solving power problems, solving negative power fraction problems (both the problem and the answer to the problem are given in general to discover mathematical knowledge and formula by examining the answers)

$5^6 \times 5^7 =$ $3^4 \times 3^5 =$ $6^2 \times 6^5 =$ $9^1 \times 9^3 =$ $2^5 \times 2^7 =$,.....	$5^6 \times 5^7 = 5^{13}$ $3^4 \times 3^5 = 3^9$ $6^2 \times 6^5 = 6^7$ $9^1 \times 9^3 = 9^4$ $2^5 \times 2^7 = 2^{12}$,......

Fig. 3 Examples of training problems

$$\frac{9 \times 10^{-6} \times 12 \times 10^{10} \times 3^{-2}}{8 \times 10^4} =$$

,.....

$$\frac{10^{-7} \times 15 \times 10^8}{3 \times 10^4 \times 10^{-7}} =$$

,....

$$\frac{10^{-3} \times 9 \times 10^6}{4 \times 10^3 \times 10^{-2} \times 25} =$$

$$\frac{9 \times 10^{-6} \times 12 \times 10^{10} \times 3^{-2}}{8 \times 10^4} = \frac{9 \times 12 \times 10^{10}}{8 \times 10^4 \times 10^6 \times 3^{+2}} = \cdots$$

,.....

$$\frac{10^{-7} \times 15 \times 10^8}{3 \times 10^4 \times 10^{-7}} = \frac{10^8 \times 15}{10^4 \times 3} = \frac{10^4 \times 5}{1} = 10^4 \times 5$$

,......

$$\frac{10^{-3} \times 9 \times 10^6}{4 \times 10^3 \times 10^{-2} \times 25} = \frac{10^{+2} \times 9 \times 10^6}{4 \times 10^3 \times 10^{+3} \times 25} = \cdots$$

Fig. 4 Examples of training problems

2. Example of method implementation (Check Fig. 4):

- Initial encounter with this problem and its answer: $\frac{9\times10^{-6}\times12\times10^{10}\times3^{-2}}{8\times10^4} = \frac{9\times12\times10^{10}}{8\times10^4\times10^6\times3^{+2}} = \cdots$

Step1. Raising a question from an unknown and unfamiliar problem: What is this problem? How should this problem be resolved? What is the formula for solving this problem? What is the use of this mathematical expression?

Step2. Examine the evidence to make a guess (dialogue with the problem and its answer to discover the rules, problem solving formulas and understanding mathematical knowledge). Observing the answer to the problem: What has changed: when 10^{-6} is transferred from the numerator to the denominator, its sign is reversed, and becomes 10^{+6}.

Guessing: Every 10^{-x} is transferred from the numerator of the fraction form to the denominator of the fraction, only its sign is reversed and becomes 10^{+x}.

Step3. Guess test: Is this guess valid? Is this always the case? If 10^{-6} is always transferred from the fraction to the denominator, does it become 10^{+6}? To test the initial guessing, the answer to a mathematical problem in which 10^{-6} exists in the numerator of the fraction is examined. For example, by examining a mathematical problem $\frac{10^{-3}\times 9\times 10^{6}}{4\times 10^{3}\times 10^{-2}\times 25}$, we can see that the initial guess is correct. When this guess is observed in other answers to other problems, then this guess is inductively proved and an initial understanding is obtained.

Step4. Initial understanding: Every 10^{-x} is transferred from the numerator of the fraction form to the denominator of the fraction, only its sign is reversed and becomes 10^{+x}. This understanding can be developed in a hermeneutic cycle. The question that arises from this initial understanding can be: Can this understanding be correct for 3^{-x} or 5^{-x} or not?. Or the student may ask himself again: What has changed? How has it changed?

Step5. Examine the evidence to make a guess.

Observing the answer to the problem: When 3^{-2} is transferred from the numerator form to the denominator, its symbol is reversed and becomes 3^{+2}.

Guessing: Every 3^{-x} is transferred from the numerator of the fraction form to the denominator of the fraction, only its sign is reversed and becomes 3^{+x}. By generalizing this guess, we can say: Every 5^{-x} or $6^{-x}, \ldots$ is transferred from the numerator of the fraction form to the denominator of the fraction, only its sign is reversed and becomes 5^{+x} or $6^{+x}, \ldots$.

Step6. Guess test: Is this guess valid? Is this always the case? To test the initial guessing, the answer to a mathematical problem in which 3^{-x} or 5^{-x} or $6^{-x}, \ldots$ exists in the numerator of the fraction is examined. For example, by examining a mathematical problem, $\frac{9\times 10^{-6}\times 12\times 10^{10}\times 3^{-2}}{8\times 10^{4}}$, we can see that the initial guess is correct. When this guess is observed in other answers to other problems, then this guess is inductively proved and an initial understanding is obtained.

Step7. Initial understanding: Any power number with negative power, from the numerator of the fraction form to the denominator of the fraction, becomes the positive power sign. This understanding can be developed in a hermeneutic cycle. The question that arises from the initial understanding may be: Is this understanding valid or not? Or if a power number with a negative power is transferred from the denominator of the fraction to the numerator of the fraction, is the same initial understanding true again or not? To answer this question, we first examine the answer to the problems that the negative power number is transferred from the denominator of the fraction to the numerator of the fraction.

Step8. Examine the evidence to make a guess.

Observing the answer to the problem: What has changed in this problem '$\frac{10^{-3}\times 9\times 10^{6}}{4\times 10^{3}\times 10^{-2}\times 25}$'?. When 10^{-2} is transferred from the denominator to the numerator, its symbol is reversed and becomes 10^{+2}.

Guessing: Every 10^{-x} or 3^{-x} or $4^{-x}, \ldots$ is transferred from the denominator of the fraction to the numerator of the fraction, only its sign is reversed and becomes 10^{+x} or 3^{+x} or $4^{+x}, \ldots$.

Step9. Guess test: Is this guess valid? Is this always the case? If 10^{-x} *or* 3^{-x} *or* $4^{-x}, \ldots$ is the denominator of the fraction to the numerator of the fraction, is the sign inverted?. To test the initial guessing, there are answers to a mathematical problem in which 10^{-x} *or* 3^{-x} *or* $4^{-x}, \ldots$ exist in from of the denominator. When this guess is observed in other answers to other problems, then this guess is inductively proved and an initial understanding is obtained.

Step10. Initial understanding: Each power number with a negative power that is transferred from the denominator to the numerator is the sign of that power number becomes positive.

Step11. Organizing understandings (higher understanding)

- Understanding1: For any power number with a negative power, if it is transferred from the numerator to the denominator of the fraction, its sign becomes positive.
- Understanding 2: If transferred from the denominator to the numerator, each power number with a negative power has its sign reversed and becomes positive.
- Understanding 3: This is derived from Understanding 1 and 2: Each power number with a negative power, if transferred from the numerator to the denominator and vice versa, has its negative sign changed into a positive one.

This understanding can be developed in a hermeneutic cycle with a question: Any power with a positive power, if transferred from the numerator to the denominator and vice versa, will the positive power sign change into negative?

IV. Conclusion

The main purpose of this paper is to present a method to solve the problem. The research method of this paper is review, descriptive and conception development. The planned type of curriculum this method is that, to teach a new subject, both the problem and the answer to the problem are given first. And then with the help of this method, which is interpretive dialogue with the text and guessing, students will come up with a formula or problem-solving strategy and build math knowledge. The goals of this type of curriculum are: 1. Thinking and problem solving 2. Building math knowledge. It should be noted that the NRICH assumptions have been used as the theoretical basis of this method. In this paper, we first review the research findings about the NRICH project with the aim that these findings form the theoretical basis of the new problem-solving method. Then, the curriculum, the conceptual structure of the new method, how to design the problem and an example of it are discussed. In this way, we are in the story of discovering and understanding the problem-solving formula and mathematical knowledge. In fact, we will have a dialectical hermeneutics, and that will be a way of discovering meaning. A pure two-way dialogue where two horizons face each other, the dialogue is established and the facts are identified, and then a fusion of horizons or understanding takes place. In this method, no formula is presented, but the formula itself and mathematical knowledge is discovered.

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