World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:17, No:4, 2023

Open Science Index, Computer and Information Engineering Vol:17, No:4, 2023 publications.waset.org/10013062.pdf

Standard Fuzzy Sets for Aircraft Selection using Multiple Criteria Decision Making Analysis

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***Abstract*—**This study uses two-dimensional standard fuzzy sets to enhance multiple criteria decision-making analysis for passenger aircraft selection, allowing decision-makers to express judgments with uncertain and vague information. Using two-dimensional fuzzy numbers, three decision makers evaluated three aircraft alternatives according to seven decision criteria. A validity analysis based on two-dimensional standard fuzzy weighted geometric (SFWG) and two-dimensional standard fuzzy weighted average (SFGA) operators is conducted to test the proposed approach's robustness and effectiveness in the fuzzy multiple criteria decision making (MCDM) evaluation process.

***Keywords*—**Standard fuzzy sets (SFSs), aircraft selection, multiple criteria decision making, intuitionistic fuzzy sets (IFSs), SFWG, SFGA, MCDM.

I. INTRODUCTION

The civil aviation industry delivers orders to customers with various configuration options such as engines, design types, and avionics in aircraft, in line with customers' demands and expectations. Customers must make accurate decisions and act prudently against potential negative situations, especially considering the high cost and investment aspect. Aircraft selection is a difficult decision-making process that requires multidimensional evaluations containing conflicting multiple criteria.

Therefore, aviation can be divided into three separate groups for aircraft selection: general aviation, military aviation, and commercial aviation. In addition to this classification, it is also possible to divide it into another classification according to motor type, range, and seating capacity. Choosing the wrong aircraft type can lead to critical loss or even bankruptcy. There are many different types and models of aircraft in the aviation sector, and as the aircraft size decreases, the number of manufacturers increases globally.

Aviation industry customers pay attention to certain decision criteria in order of importance when selecting an aircraft. These criteria can be evaluated as direct operating costs, aircraft purchase price, and performance, ease of maintenance of the aircraft, low maintenance, and operating costs, operational flexibility, comfort, and the aircraft's ability to maintain its future value.

Especially airlines determine their fleet plans and aircraft selection with numerous criteria. Along with these criteria, they aim to meet customer expectations at the highest level and increase market share and profitability. As the size and number of aircraft decrease, the selection criteria may differ.

As expectations, purposes, and functions vary, it is naturally inevitable that the selection criteria for aircraft also differs.Therefore, in this situation where there are numerous conflicting criteria and alternatives in individual aircraft selection, multiple criteria decision making (MCDM) approach is proposed to select the passenger aircraft. In the capital-intensive aviation industry, where investment costs are high, selecting the right aircraft enables companies or individuals to make informed decisions.

The AHP and TOPSIS methods were used in the selection of combat aircraft. Based on 21 sub-criteria among 5 aircraft alternatives, an MCDM ranking order was reached. Maximum speed, altitude, range, takeoff weight, usability in all types of air operations, durability, ability to fly in all types of weather conditions and day and night conditions, maximum flight sortie, all types of air firing ability, maximum ammunition carrying capacity, electronic warfare capability, radar system capability, compatibility with all types of weapons firing and communication systems, auxiliary flight indicator systems, emergency rescue systems, pilot support systems in all types of operations, purchase cost, maintenance and sustainment costs, economic life and equipment continuity criteria were used [1].

MCDM methods were used in the aircraft selection problem based on 10 criteria among 5 aircraft alternatives. Direct operating cost, price, performance, technology level, maintenance ease, operational flexibility, comfort and value preservation, external noise, technical support, and cargo capacity criteria were studied [2].

Fuzzy MCDM method was used in the selection of training aircraft. The study was conducted based on 16 criteria among 7 aircraft alternatives used for training purposes. Fuel capacity, power, service ceiling, maximum and minimum g-limits, maximum speed, cruise speed, the maximum speed with landing gear down, usage speed with flaps open, stall speed with engine off, maximum cruise speed, maximum climb rate at sea level, landing distance, takeoff and 50ft reach distance, and full stop distance for landing were considered [3].

The Analytic Network Process (ANP) was used for aircraft selection for Turkish Airlines. The study was conducted based on 10 criteria among 3 aircraft alternatives. The sub-criteria included purchase cost, operating and spare parts cost, maintenance cost, reliability, depreciation cost, delivery time, useful life, dimensions, conformity to safety, reliability, and service quality [4].

The Novel Approach to Imprecise Assessment and Decision Environments (NAIADE) method, which is one of the MCDM methods, was used to select an aircraft for a local

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Open Science Index, Computer and Information Engineering Vol:17, No:4, 2023 publications.waset.org/10013062.pdf

company. The study was conducted based on 11 criteria among 8 aircraft alternatives. Purchase cost, operating costs, flexibility, cruising speed, availability of spare parts, landing and takeoff distance, comfort, and avionics were the criteria studied [5].

The Analytic Hierarchy Process (AHP) method was used for aircraft selection for regional flights in Europe. The study was conducted based on 6 criteria among 7 aircraft alternatives, including seat capacity, price, total baggage capacity, and payment conditions [6]. Also, fuzzy AHP approach to passenger aircraft type selection was applied to evaluate the alternatives [7].

Also, four aircraft alternatives were evaluated using the AHP and TOPSIS methods based on 9 criteria, including airport capacity, take-off distance, fuel tank capacity, engine power, length to wingspan ratio, climbing rate, wing dihedral angle, wingspan, and fuel consumption [8].

The AHP method was used on aircraft selection problem for airlines. The study was conducted with 3 aircraft alternatives based on 4 main criteria and 8 sub-criteria, including economic and technical performance, internal quality, and environmental impact [9].

In the business jet selection process, the MCDM methods were used and evaluated 5 aircraft alternatives based on 9 criteria, including initial purchase cost, fuel consumption, maintenance cost, range, speed, passenger capacity, flight safety, CO2 emissions, and comfort [10].

The aircraft selection process was evaluated using MCDM methods, and four aircraft alternatives were assessed based on five criteria, including range, price, speed, passenger capacity, and fuel consumption [11].

The aerobatic aircraft selection process was structured using the Stochastic Multiple Criteria Acceptability Analysis (SMAA) method. The study was conducted based on 5 criteria among 5 aircraft alternatives, including aircraft performance, international prestige, pilot adaptation, logistics performance, and economy [12].

In their studies, the authors evaluated military training aircraft using various methods such as combining multi-criteria decision making processes with fuzzy logic [13], applying Fuzzy Reference Ideal Method (FRIM) for military advanced training aircraft selection [14], and using a double fuzzy multi-criteria analysis to evaluate international high-performance aircrafts for defense purposes [15].

In the aircraft selection process in airlines, the Analytic Hierarchy Process (AHP) method supported by the Social Choice Theory was used. The study was conducted based on 9 criteria among 3 aircraft alternatives, including fuel consumption, environmental impact, maintenance and repair costs, passenger capacity, and operating costs [16].

MCDM methods for aircraft selection in the education fleet were used. They conducted their study based on 25 criteria among 3 aircraft alternatives. Criteria such as aircraft suitability for training, maintenance and operational sustainability, school budget adequacy, number of students to be trained, similarity to existing aircraft, status of schools in the country, suitability of physical features for aircraft, region's meteorological conditions, aircraft stability and durability, climb capability and maximum climb height, stopping speed, minimum take-off distance, cockpit ergonomics, aircraft power in case of an accident, qualifications and licenses of teacher pilots, operational characteristics, aircraft procurement cost, spare parts supply time and convenience, location and ease of operation, operation and insurance costs, oil and fuel expenses, suitability of flight personnel and related equipment, technical support, maintenance and repair costs, ease of maintenance and repair, and technical and service life of the aircraft were evaluated in their study [17].

The cargo aircraft selection decision was analyzed using fuzzy MCDM methods. They conducted their study based on 16 criteria among 4 aircraft alternatives. Criteria such as initial cost, maintenance cost, scrap value, spare parts cost, financing options, unit fuel cost, range, noise class, type compatibility, compatibility with the airports where the operation will be carried out, loading capacity, maintenance times, aircraft door size, flight speed, delivery time, and economic life were considered [18].

The integration of spherical fuzzy AHP-TOPSIS methods has been applied to the aircraft type selection process for regional airlines in Türkiye [19]. Also, the aircraft selection problem was investigated using both classical MCDM and non-classical MCDM methods such as fuzzy sets, determinate fuzzy sets, intuitionistic fuzzy sets, and neutrosophic sets [20-50].

In the literature review [51-75], it has been found that many studies apply a number of MCDM methods, including AHP, MAUT, TOPSIS, VIKOR, ELECTRE, ORESTE, and PROMETHEE, to the analysis of multiple criteria decision making problems. The MCDM methodology is adopted in these studies to address the challenges that arise when dealing with uncertain and vague information [76-125].

The paper is organized as follows: Section 2 provides a brief review of the standard fuzzy sets. In Section 3, the proposed decision-making approach is applied to solve a problem that involves the assessment of three passenger aircraft alternatives based on seven decision criteria. Finally, Section 4 presents a formal conclusion along with future suggestions.

II. METHODOLOGY

Standard fuzzy sets (SFSs) [76] can be categorized based on dimensionality, such as one-dimensional, two-dimensional, and multiple-dimensional sets. Two-dimensional intuitionistic fuzzy sets (IFSs) [80] represent a generalization of SFSs. In the present study, SFSs have been extended into two dimensions by defining two independent membership functions, namely membership degree and non-membership degree, for each element within the range of [0,1].

This extension permits the definition of independent membership functions with larger values, in contrast to IFSs that require the sum of the degree of membership and the degree of non-membership for each element to be at most 1.

Definition 1. Let X be a non-empty set. A standard fuzzy set A in X is given by

$$A = \{x, \mu_A(x) \mid x \in X\} \tag{1}$$

where the functions $\mu_A(x): X \to [0,1]$ and $\nu_A(x) = (1-\mu_A): X \to [0,1]$ define the degree of membership and the degree of non-membership of an element to the set *A*, respectively, with the condition that

$$\mu_A(x) + \nu_A(x) = 1, \forall x \in X \tag{2}$$

The degree of hesitancy is calculated as follows:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) = 0 \tag{3}$$

Definition 2. Let *X* be a non-empty set. An intuitionistic fuzzy set *I* in *X* is given by

$$I = \{x, \mu_I(x), \nu_I(x) \mid x \in X\} \tag{4}$$

where the functions $\mu_I(x): X \to [0,1]$ and $\nu_I(x): X \to [0,1]$ define the degree of membership and the degree of non-membership of an element to the set *I*, respectively, with the condition that

$$0 \le \mu_I(x) + \nu_I(x) \le 1, \forall x \in X \tag{5}$$

The degree of hesitancy is calculated as follows:

$$\pi_I(x) = 1 - \mu_I(x) - \nu_I(x) \tag{6}$$

Definition 3. Let *X* be a non-empty set. A two-dimensional standard fuzzy set *F* in *X* is given by

$$F = \{x, \mu_F(x), \nu_F(x) \mid x \in X\} \tag{7}$$

where the functions $\mu_F(x): X \to [0,1]$ and $\nu_F(x): X \to [0,1]$ define the degree of membership and the degree of non-membership of an element to the set *F*, respectively, with the condition that

$$0 \le \mu_F(x) + \nu_F(x) \le 2, \forall x \in X \tag{8}$$

In sequel, $A = (\mu_A, \nu_A)$ denotes a two-dimensional standard fuzzy number (SFN).

Definition 4. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two SFNs, then the addition and multiplication operations are defined as follows

$$A \oplus B = (\mu_A + \mu_B - \mu_A\mu_B, \nu_A\nu_B) \tag{9}$$

$$A \otimes B = (\mu_A\mu_B, \nu_A + \nu_B - \nu_A\nu_B) \tag{10}$$

$$A^C = (\nu_A, \mu_A) \tag{11}$$

Definition 5. Let $A = (\mu_A, \nu_A)$ be an SFN, then the score function $S(A)$ and accuracy function $H(A)$ of A can be respectively defined as follows

$$S(A) = \mu_A - \nu_A \tag{12}$$

$$H(A) = \mu_A + \nu_A \tag{13}$$

Definition 6. Let $A_i = (\mu_{A_i}, \nu_{A_i})$ $(i = 1, 2, ..., n)$ be a set of SFNs and $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be weight vector of A_i $\sum_{i=1}^{n} \omega_i = 1$, then a two-dimensional standard fuzzy weighted average (SFWA) operator is

$$SFWA(A_1, A_2, ..., A_n) = \left(\left(1 - \prod_{i=1}^{n} (1-\nu_{A_i})^{\omega_i}\right), \prod_{i=1}^{n} \mu_{A_i}^{\omega_i}\right) \tag{14}$$

Definition 7. Let $A_i = (\mu_{A_i}, \nu_{A_i})$ $(i = 1, 2, ..., n)$ be a set of SFNs and $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be weight vector of A_i $\sum_{i=1}^{n} \omega_i = 1$, then a two-dimensional standard fuzzy weighted geometric (SFWA) operator is

$$SFWG(A_1, A_2, ..., A_n) = \left(\prod_{i=1}^{n} \mu_{A_i}^{\omega_i}, \left(1 - \prod_{i=1}^{n} (1-\nu_{A_i})^{\omega_i}\right)\right) \tag{15}$$

Definition 8. Let $A_i = (\mu_{A_i}, \nu_{A_i})$ and $B_i = (\mu_{B_i}, \nu_{B_i})$ be two SFNs. The distance between these two SFNs is obtained by normalized Minkowski distance family as follows.

$$d(A,B) = \left(\frac{1}{2n}\sum_{i=1}^{n} (\mu_{A_i} - \mu_{B_i})^\delta + (\nu_{A_i} - \nu_{B_i})^\delta + (\pi_{A_i} - \pi_{B_i})^\delta\right)^{1/\delta} \tag{16}$$

where $\delta = (1, 2, 3, ..., \infty)$, $\delta = 1$ denotes Manhattan distance, $\delta = 2$ denotes Euclidean distance, $\delta = 3$ denotes Minkowski distance, and $\delta = \infty$ denotes Chebyshev distance.

A. Multiple Criteria Decision-Making (MCDM) Analysis

In decision making theory, a multiple criteria decision-making analysis problem is characterized by a set of alternatives $A_i = \{A_1, ..., A_i\}$ $(i > 2)$ which the best decision must be made, according to a given set of criteria $C_j = \{C_1, ..., C_j\}$ $(j > 1)$ and the score $i\,x\,j$ $X = [X_{ij}]$ whose component X_{ij} is the score of the alternative A_i based on criterion C_j. Each criterion has an importance normalized weight $\omega_j \in [0,1]$ with $\sum_{j=1}^{J} \omega_j = 1$.

The MCDM problem is considered by using all criteria C_j and all alternatives A_i as well as all their related score values X_{ij} expressed quantitatively and the weighting factor ω_j of each criteria C_j. The set of normalized weighting factors is denoted by $\omega_j = \left\{\omega_1, ..., \omega_j\right\}$. Depending on the context of the MCDMA problem, the score can be interpreted either as a cost or as a benefit. The score matrix $X = [X_{ij}]$ is sometimes also called benefit or payoff matrix in the literature. The classical MCDM problem aims to select the best alternative $A^* \in A$ given X and the weighting factors ω_j of criteria.

B.Aircraft Selection Problem

The multiple criteria decision-making (MCDM) methods can be effectively applied to determine the most suitable aircraft selection. Therefore, in this study, a suggested MCDM approach is used to evaluate and select the best aircraft among several alternatives. To evaluate the aircraft alternatives, a set of criteria is determined based on a comprehensive literature review and expert opinions.

The identified decision criteria include benefit type and cost type criteria for evaluation of the aircraft alternatives. Benefit criteria are flight range (C1), number of seats (C2), maximum takeoff weight (C3), luggage volume (C4), and payload (C5), cost criteria are fuel consumption (C6) and purchase cost (C7). The potential narrowbody aircraft considered are A1, A2, and A3, which are evaluated according to the determined criteria. The steps of the two-dimensional standard fuzzy MCDM the problem are presented as follows:

Step 1. The decision matrix is established.

The initial decision matrix $X = [x_{ij}]_{mxn}$ for the alternatives (A_i), the decision criteria (C_j), and the criteria weights (ω_j) is constructed.

Step 2. The decision matrix is normalized.

$$x_{ij} = \begin{cases} \{\mu_A - \nu_A \,, & \Omega_B \\ \nu_A - \mu_A \,, & \Omega_C \end{cases}$$

where Ω_B denotes benefit type criteria, and Ω_C denotes cost type criteria,

Step 3. The criteria weights are computed.

The importance weights ω_j of decision criteria are assessed by the DMs using two-dimensional standard fuzzy weighted geometric (SFWG) operator.

Step 4. Weighted normalized matrix is computed.

The two-dimensional standard fuzzy weighted geometric (SGWG) and (SFWA) operator are used to compute the weighted normalized matrix.

Step 5. The alternatives are ranked according to their score function $S(A) \in [-1,1]$ values in decreasing order. The bigger value $i^* = \arg\max S(A_i)$ corresponds to the best MCDM solution A^*, that is $A^* = A_i$.

C. Standard Fuzzy Set Application

The solutions of the defined problem through the proposed standard fuzzy method are presented in the following algorithm.

Step 1. The proposed approach is applied to the most appropriate aircraft selection among three alternatives in MCDM problem. These alternatives (A1, A2, and A3) are evaluated according to seven criteria determined based on comprehensive literature review and expert opinions.

A team of experts are formed to evaluate the suppliers using the proposed approach. Three decision-makers are selected, consisting of aircraft experts and expert academics on multiple criteria decision making in a fuzzy environment and are abbreviated as DM1, DM2, and DM3 respectively.

Step 2. The evaluations of the aircraft by the decision-makers in accordance with the defined objectives and criteria, using two-dimensional standard fuzzy set values are presented in Table 1.

Table 1. Two-dimensional standard fuzzy decision matrix for each decision maker

Criteria	DMs	A1		A2		A3	
C1	DM1	0,6	0,7	0,3	0,9	0,7	0,9
	DM2	0,8	0,9	0,5	0,6	0,4	0,7
	DM3	0,4	0,8	0,3	0,7	0,7	0,5
C2	DM1	0,2	0,6	0,8	0,9	0,5	0,8
	DM2	0,7	0,4	0,6	0,5	0,6	0,3
	DM3	0,5	0,4	0,7	0,9	0,7	0,8
C3	DM1	0,4	0,7	0,5	0,7	0,9	0,3
	DM2	0,5	0,3	0,8	0,4	0,6	0,7
	DM3	0,6	0,3	0,7	0,6	0,6	0,4
C4	DM1	0,5	0,4	0,6	0,3	0,5	0,3
	DM2	0,9	0,7	0,6	0,5	0,7	0,5
	DM3	0,4	0,5	0,7	0,4	0,5	0,3
C5	DM1	0,6	0,8	0,6	0,4	0,7	0,5
	DM2	0,3	0,9	0,5	0,8	0,6	0,3
	DM3	0,5	0,4	0,6	0,7	0,5	0,4
C6	DM1	0,6	0,7	0,9	0,4	0,7	0,4
	DM2	0,3	0,6	0,5	0,9	0,8	0,5
	DM3	0,4	0,7	0,8	0,6	0,4	0,7
C7	DM1	0,6	0,3	0,9	0,7	0,8	0,3
	DM2	0,9	0,4	0,8	0,5	0,7	0,8
	DM3	0,8	0,5	0,7	0,3	0,6	0,7

Steps 3. Original two-dimensional standard fuzzy set numbers are converted to their corresponding normalized numbers as shown in Table 2.

Table 2. Normalized two-dimensional standard fuzzy decision matrix for each decision maker

Criteria	DMs	A1		A2		A3	
C1	DM1	0,6	0,7	0,3	0,9	0,7	0,9
	DM2	0,8	0,9	0,5	0,6	0,4	0,7
	DM3	0,4	0,8	0,3	0,7	0,7	0,5
C2	DM1	0,2	0,6	0,8	0,9	0,5	0,8
	DM2	0,7	0,4	0,6	0,5	0,6	0,3
	DM3	0,5	0,4	0,7	0,9	0,7	0,8

C3	DM1	0,4	0,7	0,5	0,7	0,9	0,3
	DM2	0,5	0,3	0,8	0,4	0,6	0,7
	DM3	0,6	0,3	0,7	0,6	0,6	0,4
C4	DM1	0,5	0,4	0,6	0,3	0,5	0,3
	DM2	0,9	0,7	0,6	0,5	0,7	0,5
	DM3	0,4	0,5	0,7	0,4	0,5	0,3
C5	DM1	0,6	0,8	0,6	0,4	0,7	0,5
	DM2	0,3	0,9	0,5	0,8	0,6	0,3
	DM3	0,5	0,4	0,6	0,7	0,5	0,4
C6	DM1	0,7	0,6	0,4	0,9	0,4	0,7
	DM2	0,6	0,3	0,9	0,5	0,5	0,8
	DM3	0,7	0,4	0,6	0,8	0,7	0,4
C7	DM1	0,3	0,6	0,7	0,9	0,3	0,8
	DM2	0,4	0,9	0,5	0,8	0,8	0,7
	DM3	0,5	0,8	0,3	0,7	0,7	0,6

D.IFWG operator was applied to the aircraft selection problem

Step 4. The aggregated two-dimensional standard fuzzy decision matrix, which was obtained by combining the individual decision matrices consisting of two-dimensional standard fuzzy numbers are displayed in Table 3.

Table 3. Aggregated two-dimensional standard fuzzy decision matrix

Criteria	A1		A2		A3	
C1	0,577	0,818	0,356	0,771	0,581	0,753
C2	0,412	0,476	0,695	0,829	0,594	0,696
C3	0,493	0,472	0,654	0,584	0,687	0,499
C4	0,565	0,552	0,632	0,406	0,559	0,374
C5	0,448	0,771	0,565	0,670	0,594	0,406
C6	0,665	0,448	0,600	0,785	0,519	0,670
C7	0,391	0,800	0,472	0,818	0,552	0,712

Steps 5. The standard fuzzy evaluations of the criteria, which were assigned by the decision makers using two-dimensional standard fuzzy numbers are displayed in Table 4. Meanwhile, the aggregated two-dimensional standard fuzzy criteria weights that were obtained by using their equal weights, and the importance rankings (R_i) of the criteria are shown in Table 5.

Table 4. The two-dimensional standard fuzzy evaluations of the criteria for each decision maker

Criteria	DM1		DM2		DM3	
C1	0,7	0,1	0,8	0,3	0,7	0,6
C2	0,8	0,2	0,7	0,3	0,5	0,3
C3	0,9	0,3	0,7	0,5	0,8	0,5
C4	0,6	0,3	0,7	0,4	0,6	0,2
C5	0,8	0,4	0,5	0,4	0,9	0,3
C6	0,7	0,3	0,6	0,3	0,8	0,3
C7	0,6	0,3	0,7	0,1	0,9	0,4

Table 5. Aggregated criterion weights

Criteria	μ_A	ν_A	$S(\omega_j)$	R_i
C1	0,732	0,368	0,363	4
C2	0,654	0,268	0,386	3
C3	0,796	0,441	0,355	5
C4	0,632	0,305	0,327	7
C5	0,711	0,368	0,343	6
C6	0,695	0,300	0,395	2
C7	0,723	0,277	0,446	1

Step 6. The weighted decision matrix is created using the weight vectors obtained in Table 5. The weighted decision matrix is as in Table 6.

Table 6. Weighted decision matrix

Criteria	A1		A2		A3	
C1	0,669	0,534	0,469	0,581	0,672	0,597
C2	0,560	0,841	0,788	0,005	0,712	0,726
C3	0,570	0,755	0,713	0,012	0,742	0,738
C4	0,697	0,783	0,748	0,026	0,693	0,867
C5	0,565	0,581	0,666	0,001	0,691	0,826
C6	0,753	0,837	0,701	0,046	0,634	0,717
C7	0,508	0,640	0,581	0,182	0,651	0,709

Step 7. The rankings (R_i) of the alternatives, which were obtained after aggregating the score values for each two-dimensional standard fuzzy number in the weighted decision matrix are presented in Table 7.

Table 7. The rankings of the alternatives

Criteria	C1		C2	
A_i	μ_A	ν_A	$S(A_i)$	R_i
A1	0,032	0,917	-0,885	2
A2	0,054	1,000	-0,946	3
A3	0,070	0,884	-0,814	1

E. IFWA operator was applied to the aircraft selection problem

Step 8. The aggregated two-dimensional standard fuzzy decision matrix, which was obtained by combining the individual decision matrices consisting of two-dimensional standard fuzzy numbers are displayed in Table 8.

Table 8. Aggregated two-dimensional standard fuzzy decision matrix

Criteria	A1		A2		A3	
C1	0,840	0,796	0,738	0,723	0,661	0,680
C2	0,726	0,756	0,745	0,740	0,654	0,577
C3	0,650	0,577	0,700	0,552	0,824	0,438
C4	0,853	0,458	0,679	0,391	0,766	0,356
C5	0,655	0,482	0,694	0,607	0,779	0,391
C6	0,709	0,438	0,757	0,711	0,605	0,607
C7	0,631	0,398	0,802	0,796	0,664	0,695

Step 9. The weighted decision matrix is created using the weight vectors obtained in Table 5. The weighted decision matrix is as in Table 9.

Table 9. Weighted decision matrix

Criteria	A1		A2		A3	
C1	0,880	0,557	0,801	0,623	0,738	0,657
C2	0,811	0,685	0,825	0,697	0,758	0,794
C3	0,710	0,685	0,752	0,702	0,857	0,776
C4	0,904	0,830	0,783	0,860	0,845	0,875
C5	0,740	0,785	0,771	0,709	0,837	0,833
C6	0,787	0,841	0,824	0,689	0,705	0,755
C7	0,717	0,869	0,853	0,644	0,744	0,720

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Step 10. The rankings (R_i) of the alternatives, which were obtained after aggregating the score values for each two-dimensional standard fuzzy number in the weighted decision matrix are presented in Table 10.

Table 10. The rankings of the alternatives

A_i	μ_A	ν_A	$S(A_i)$	R_i
A1	0,191	0,876	-0,684	2
A2	0,211	0,918	-0,707	3
A3	0,178	0,840	-0,662	1

The comparison between two-dimensional standard fuzzy sets based on SFWA and SFWG operators demonstrates that they produce identical ranking order patterns. This finding provides validation for the proposed MCDM methodology in ranking the alternatives for the aircraft selection problem.

The alternatives have been ranked based on a two-dimensional standard fuzzy sets approach using the SFWA and SFWG operators, and the resulting order is as follows:

$$S(A_2) \leq S(A_1) \leq S(A_3)$$

$$A_3 \geq A_1 \geq A_2$$

Finally, according to the fuzzy multiple criteria decision-making (MCDM) analysis conducted for the aircraft selection problem, alternative A_3 has been identified as the optimal choice.

The proposed fuzzy multiple criteria decision-making (MCDM) methodology and the steps involved in the two-dimensional standard fuzzy MCDM problem can be summarized as follows:

Step 1. Identify the decision-making problem. Identify the criteria and alternatives for the decision-making problem. The decision-making problem pertains to the selection of an aircraft and involves identifying the criteria that are used to evaluate the available alternatives.

Step 2. Construct a fuzzy group decision matrix. Determine the fuzzy values of each alternative for each criterion.

Step 3. Normalize the fuzzy decision matrix. This involves transforming the cost type criteria values into the benefit type criteria values in the matrix.

Step 4. Determine the criteria weights. Construct a fuzzy group decision matrix for criteria evaluation. The criteria weights are computed by using the SFWG or SFWA operator.

Step 5. Calculate the fuzzy weighted values of each alternative. This is done by multiplying the value of each alternative for each criterion by the weight of that criterion by using the SFWG and SFWA operator.

Step 6. Defuzzify the fuzzy weighted values. This involves converting the fuzzy weighted values into crisp values. This can be done using the score function.

Step 7. Select the alternative with the highest defuzzified fuzzy weighted value. This is the alternative that is the best overall, according to the criteria that have been defined.

III. CONCLUSION

This study introduces the concept of two-dimensional standard fuzzy sets, which share similarities with intuitionistic fuzzy sets in incorporating fuzziness in membership functions. The independent membership functions of two-dimensional standard fuzzy sets are also in the range of [0,1].

These two-dimensional standard fuzzy sets were utilized to solve an aircraft selection problem, and a comparative analysis was conducted with SFWA and SFWG operators to obtain ranking results. The comparison results validate the effectiveness of the proposed method. Further research can explore the development of different MCDM methods based on two-dimensional standard fuzzy sets to compare with the proposed approach.

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