Aircraft Selection Process Using Reference Linear Combination in Multiple Criteria Decision Making Analysis

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***Abstract*—**This paper introduces a new method for multiple-criteria decision making (MCDM) that avoids order reversal and ensures consistency in decision-making. The proposed method involves range targeting of benefit and cost criteria vectors for range normalization of the initial decision matrix. The Reference Linear Combination (RLC) is used to avoid the rank reversal problem. The preference order generated from the target score matrix does not require relative comparisons between alternatives but relies on a chosen reference solution point after transforming the original decision matrix into an MCDM problem by specifying the minimum and maximum bounds of each criterion. The efficiency and applicability of the proposed RLC method were demonstrated in the selection of commercial passenger aircraft.

***Keywords*—**Aircraft selection, reference linear combination (RLC), multiple criteria decision-making, MCDM.

I. Introduction

The goal of multiple criteria decision-making (MCDM) analysis is to choose the best alternative from a set of options based on a combination of quantitative and qualitative evaluations. However, the normalization process used in MCDM methods can lead to the rank reversal problem (RRP), where introducing or removing an alternative changes the preference ordering of the alternatives. This problem has been extensively studied in the literature, and several MCDM approaches, including AHP, ELECTRE, PROMETHEE, ORESTE, TOPSIS, VIKOR and others, have been found to be susceptible to rank reversal [1-55].

The MCDM algorithms mentioned above have been a source of inspiration for the development of other techniques that aim to address the RRP issue with varying degrees of success. The current paper introduces a novel MCDM algorithm called Reference Linear Combination (RLC), which is free from rank reversal and offers a consistent preference ordering pattern. The proposed algorithm is simple to implement and can be easily integrated into the conventional MCDM problem framework since it uses the existing MCDM score matrix and criteria weighting factors.

The proposed RLC method is utilized to evaluate the selection of commercial passenger aircraft, which is a highly competitive industry due to economic globalization and technological advancements. The increase in fuel prices during times of economic crisis negatively affects profits and falling ticket prices have led to a rise in air travel over other modes of transportation. In this competitive environment, the selection of the best aircraft can significantly impact a company's profitability. Decision-makers in competitive airline market consider not only traditional cost-related factors but also the needs of both the company and its customers. As the selection criteria and their relative weights vary significantly in MCDM environment, this work aims to identify the evaluation criteria and rank the commercial passenger aircraft [26-55]. On the other hand, often enough information is not always available in structuring a traditional MCDM problem, in these cases fuzzy MDCM methods that can effectively handle uncertain and vague information can be applied [56-105].

The paper is structured as follows. Section 2 provides a brief overview of the basics of the traditional MCDM problem. The procedural steps of the proposed RLC methodology are then described. In Section 3, the RLC approach is applied to a multiple-criteria aircraft selection problem. Finally, Section 4 concludes the paper and offers perspectives and recommendations for future research.

II. Methodology

A. Multiple Criteria Decision-Making (MCDM) Analysis

In decision making theory, a multiple criteria decision-making analysis problem is characterized by a set of alternatives $A_i = \{A_1, ..., A_i\}$ $(i > 2)$ which the best decision must be made, according to a given set of criteria $C_j = \{C_1, ..., C_j\}$ $(j > 1)$ and the score $i\,x\,j$ $S = [S_{ij}]$ whose component S_{ij} is the score of the alternative A_i based on criterion C_j.Each criterion has an importance normalized weight $\omega_j \in [0,1]$ with $\sum_{j=1}^{J} \omega_j = 1$.

The MCDM problem is considered to be classical if all criteria C_j and all alternatives A_i are known as well as all their related scores values S_{ij} expressed quantitatively and the weighting factor ω_j of each criteria C_j. Unclassical MCDM problems refer to problems involving incomplete or qualitative information. The set of normalized weighting factors is denoted by $\omega_j = \{\omega_1, ..., \omega_j\}$. Depending on the context of the MCDMA problem, the score can be interpreted either as a cost or as a benefit. The score matrix $S = [S_{ij}]$ is sometimes also called benefit or payoff matrix in the

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literature. The classical MCDM problem aims to select the best alternative $A^* \in A$ given S and the weighting factors ω_j of criteria.

It should be noted that the traditional MCDM problem, which relies only on a given score matrix S and a weighting vector ω_j for criteria, is incomplete. This is because the physical limits of score values for each criterion are not specified, making most MCDM problems ill-defined. MCDM problems are always likely to be ill-defined because it is challenging or impossible to collect all relevant technical parameters and validate them against observations.

To solve an MCDM problem fully, it is necessary to specify the absolute bounds of score values for each criterion. By doing so, the ill-defined MCDM problem becomes a well-defined one, where all score values for each criterion lie between its limits. A direct and straightforward approach is proposed to solve well-defined MCDM problems using the RLC method. However, to transform an ill-defined MCDM problem into a well-defined MCDM problem, additional information is needed, which may be available but not utilized in existing methods or may be introduced based on expert judgment or reasonable assumptions depending on the criteria involved. Once the ill-defined MCDM problem is transformed into a unique well-defined problem, the rank-reversal-free RLC method can provide the best multiple criteria decision-making solution with preference ordering of all alternatives.

B. Reference Linear Combination (RLC)

In RLC approach, the criteria are always assumed to be independent of each other, which ensures that no redundant information is used in the MCDM problem. This is done to avoid any potential bias in the results. The principle RLC method is based on the computation of normalized distance $d_{ij}(A_i, S_j^*)$ of each alternative A_i with respect to the reference solution point (RSP) S_j^* chosen for each criterion C_j, and their weighted average distance $d_{ij}(A_i, S_j^*) = \sum_{j=1}^{J} \omega_j d_{ij}(A_i, S_j^*)$ which is also a true distance metric.

Rank reversal is a phenomenon that can occur in MCDM when the ranking of alternatives changes as a result of adding or removing alternatives from the set being considered. This can happen if the reference solution point, against which the alternatives are compared, is not chosen appropriately.

To avoid rank reversal, it is important to select the reference solution S^* a priori, before considering any score values of alternatives. Additionally, S^* should be chosen independently of the score values of alternatives so that the distance $d(A_i, S_j^*)$ of any chosen alternative A_i to S^* is independent of the distance of $d(A_{i'}, S_j^*)$ for $i' \neq i$. In other words, the ranking of alternatives based on their distance from S^* should not be affected by the ranking of the distances between the alternatives themselves.

By selecting S^* in this way, the preference ordering based on $d(A_i, S_j^*)$ will be stable, and the rankings of alternatives will not change when new alternatives are added or removed from the set being considered. This is because the values of $d(A_i, S_j^*)$ for the modified MCDM problem will remain the same, regardless of which alternatives are added or removed.

It is important to note that the choice of reference solution S^* can have a significant impact on the final rankings of alternatives. Therefore, it is essential to carefully select S^* based on the problem context and criteria being considered.

Definition 1:Consider $(N > 2)$ metric spaces $E_j = \{E_1, E_2, ..., E_N\}$. One denotes $d_j(x_j, y_j)$ as a true metric chosen for measuring the distance between points x_j and y_j of E_j. One considers N-dimensional points defined as $x = [x_1\ x_2 ... x_N]^t$ and $y = [y_1\ y_2 ... y_N]^t$ belonging to $E_j = \{E_1, E_2, ..., E_N\}$. Then for any real factor $\omega_j \geq 0$, the weighted average distance $d(x, y)$ defined $d(x, y) = \sum_{j=1}^{J} \omega_j d_j(x_j, y_j)$.

Proof 1: To prove that $d(x, y)$ is a true distance, one must prove the four properties:

1) Positiveness: $\forall (x, y) \in E^2, d(x, y) \geq 0$
2) Symmetry: $\forall (x, y) \in E^2, d(x, y) = d(y, x)$
3) Separation: $\forall (x, y) \in E^2, d(x, y) = 0 \Leftrightarrow x = y$
4) Triangular inequality: $\forall (x, y) \in E^3, d(x, z) \leq d(x, y) + d(y, z)$.

1) Positiveness: Because $d_1(x_1, y_1)$ is a true distance defined in E_1, one has $d_1(x_1, y_1) \geq 0$ for all $(x_1, y_1) \in E_1 x E_1$, and because $\omega_1 \geq 0$, one has $\omega_1 d_1(x_1, y_1) \geq 0$. Similarly $d_2(x_2, y_2)$ being a true distance in E_2 and $\omega_2 \geq 0$, one has always $\omega_2 d_2(x_2, y_2) \geq 0$ for all $(x_2, y_2) \in E_2 x E_2$. Hence the quantity $\omega_1 d_1(x_1, y_1) + \omega_2 d_2(x_2, y_2) \geq 0$, which proves the positiveness of $d(x, y)$.

2) Symmetry: Because symmetry holds for d_1 and d_2, one has $\forall (x, y) \in E^2 d(x, y) = \omega_1 d_1(x_1, y_1) + \omega_2 d_2(x_2, y_2) = \omega_1 d_1(y_1, x_1) + \omega_2 d_2(y_2, x_2) = d(x, y)$ which proves the symmetry property of $d(x, y)$.

3) Separation: Because separation holds for d_1 and d_2 that is $d_1(x_1, x_1) = 0$ and $d_2(x_2, x_2) = 0$, one has $\forall (x, x) \in E^2$ the following equality $d(x, y) = \omega_1 d_1(x_1, y_1) + \omega_2 d_2(x_2, y_2) = \omega_1 .0 + \omega_2 .0 = 0$, which proves the separation property of $d(x, y)$.

4) Triangular inequality: Let's verify that the triangular inequality holds. Because d_1 and d_2 are considered as true

distances, they satisfy the triangular inequalities. That is, for all $(x_1, y_1, z_1) \in E_1 x E_1 x E_1$

$$d_1(x_1, z_1) \le d_1(x_1, y_1) + d_1(y_1, z_1)$$

and for any multiplicative factor $\omega_1 \ge 0$, one has also

$$\omega_1 d_1(x_1, z_1) \le \omega_1 d_1(x_1, y_1) + \omega_1 d_1(y_1, z_1)$$

Similarly, one has for any multiplicative factor $\omega_2 \ge 0$

$$\omega_2 d_2(x_2, z_2) \le \omega_2 d_2(x_2, y_2) + \omega_2 d_2(y_2, z_2)$$

The following inequality is always valid and can be obtained by adding the two positive (or zero) left-hand sides and the two positive (or zero) right-hand sides of the previous inequalities, and then rearranging terms.

$$\omega_1 d_1(x_1, z_1) + \omega_2 d_2(x_2, z_2) \le$$

$$\omega_1 d_1(x_1, z_1) + \omega_2 d_2(x_2, y_2) + \omega_1 d_1(y_1, z_1) + \omega_2 d_2(y_2, z_2)$$

This valid inequality can be expressed equivalently as

$d(x, z) \le d(x, y) + d(y, z)$, which proves that $d(x, y)$ satisfies the triangular inequality for all $(x, y, z) \in E^3$. This completes the proof.

By induction, this proof can be directly extended to the general case involving $(N > 2)$ metric spaces, proving that for any $\omega_j \ge 0$ and using any distance d_i chosen in E_1, $i = 1, 2, ..., I ...., n$, $d_i(x_i, y_i) = \sum_{i=1}^{n} \omega_i d_i(x_i, y_i)$ is also a true distance.

C. Reference Solution Point (RSP)

In MCDM problems, the Reference Solution Point (RSP) is the point used to define the ideal solution and determine the preference ordering of alternatives. The RSP can be chosen based on the designer's preference and the specific requirements of the MCDM problem at hand. Typically, the RSP is determined from the upper and lower bounds of the scores of each criterion, based on the preference ordering. However, in some cases, the RSP can be an expected or nominal reference point between these bounds.

The MCDM problem involves sorting or selecting alternatives based on their proximity to the defined RSP. The closer an alternative is to the RSP, the better it is according to the MCDM solution. The determination of the RSP and subsequent evaluation of alternatives is a critical step in the MCDM process and must be done carefully to ensure an accurate and effective solution.

For each criterion $C_j\,(j = 1,.., J)$ the min and max bounds of this criterion are denoted respectively by $S_j^{\min}$ and $S_j^{\max}$. If for a criterion C_j the preference is larger score value is better, then the best reference solution for criterion C_j is $S_j^* = S_j^{\max}$, but if for criterion C_j the preference is smaller score value is better, then the reference solution point for criterion C_j is $S_j^* = S_j^{\min}$. The reference multiple criteria best solution S^* is defined as the point of coordinates $S_j^* = \{S_1^*, ..., S_j^*\}$ in the *N*-dimensional space.

Once the MCDMA is well-defined by using the specification of the bounds values of each criterion, the RSP method does not suffer from rank reversal because the evaluation of each alternative is done independently of the others. Therefore, removing an alternative or including a new alternative in the new well-defined MCDM problem does not change the preference order of alternatives. The RSP method must be adapted: The only condition is that each coordinate S_j^* of the RSP must be between the bounds $[S_j^{\min}, S_j^{\max}]$ of each criterion $C_j\,(j = 1,.., J)$ in the well-formulated MCDM problem. The application of the RLC method working with RSP is presented in the MCDM aircraft selection problem for convenience.

D. Distance Metric

To measure the proximity of an alternative $A_i\,(i = 1, ..., I)$ with respect to the reference solution point, one can use the weighted average distance $d_{ij}(A_i, S_j^*) = \sum_{j=1}^{J} \omega_j d_{ij}(A_i, S_j^*)$ which is a true distance metric. All distances $d_{ij}(A_i, S_j^*)$ for $j = 1, ..., J$ involved in the weighted average must be of the same kind. For instance, one may chose a Manhattan (L_1) distance for measuring the distance in E_1 metric space, and one may chose an Euclidean (L_2) distance for measuring the distance in E_2 metric space, and another possible Minkowski's distance related with E_3 , or Hausdorff distance etc.

Here, it is proposed to use the same distance metric for each criterion: the classical Manhattan (L_1) distance or Euclidean (L_2) distance for calculating $d_{ij}(A_i, S_j^*)$, but any other choice of distances is possible, and is theoretically allowed in RLC method including the hybrid weighted averaged distance.

E. Data Normalization

The need to normalize score values or distance values $d_{ij}(A_i, S_j^*)$ before calculating the weighted average distance $d_{ij}(A_i, S_j^*) = \sum_{j=1}^{J} \omega_j d_{ij}(A_i, S_j^*)$ to rank alternatives with respect to the reference solution point is important. The normalization is step required because criteria often have different natures, with varying physical units. It is challenging to assign a clear meaning to a weighted average distance that combines distances between objects of different natures. To address this issue, it is preferable to use unitless distances $d_{ij}(A_i, S_j^*)$ obtained through normalization in the calculation of the weighted average distance $d_{ij}(A_i, S_j^*) = \sum_{j=1}^{J} \omega_j d_{ij}(A_i, S_j^*)$. Normalization procedure is given as follows:

Normalization converts score values related to a criterion C_j into unitless values. This is achieved by dividing each score value S_{ij} by the range of possible score values for that criterion.

$$s_{ij} = \frac{S_{ij} - S_j^{\min}}{S_j^{\max} - S_j^{\min}} \tag{1}$$

where the normalized score value $s_{ij} \in [0,1]$, and $s_{ij} = 0$ if $S_{ij} = S_j^{\min}$, and $s_{ij} = 1$ if $S_{ij} = S_j^{\max}$.

It is essential to normalize the coordinates of the reference solution point as well to obtain the reference solution point. $S_j^* = \left\{S_1^*, ..., S_j^*\right\}$, where $s_j^* = \frac{S_j^* - S_j^{\min}}{S_j^{\max} - S_j^{\min}}$ for $j = 1, ..., J$. Therefore, the original Euclidean distance $d_{ij}(A_i, S_j^*) \in [0,1]$ defined by $d_{ij}(A_i, S_j^*) = | S_{ij} - S_j^* |$ is replaced by the unitless normalized Euclidean distance $d_{ij}(A_i, s_j^*) \in [0,1]$ defined by

$$d_{ij}(A_i, s_j^*) = | s_{ij} - s_j^* | \tag{2}$$

In this context, it is worth noting that each criterion is a one-dimensional problem. This means that the Euclidean distance $d(x, y) = \sqrt{(x - y)^2} = | x - y |$ between the alternative and the reference solution point is calculated using the difference between the normalized score value of the alternative and the normalized score value of the reference solution point for that criterion. It is worth noting that one gets

$$d_{ij}(A_i, s_j^*) = \left| \frac{S_{ij} - S_j^{\min}}{S_j^{\max} - S_j^{\min}} - \frac{S_j^* - S_j^{\min}}{S_j^{\max} - S_j^{\min}} \right| = \frac{| S_{ij} - S_j^* |}{| S_j^{\max} - S_j^{\min} |} \tag{3}$$

Once the normalized distances $d_{ij}(A_i, S_j^*)$ are calculated, the normalized weighted average distance $d_{ij}(A_i, s_j^*) \in [0,1]$ is defined by

$$d_{ij}(A_i, s_j^*) = \sum_{j=1}^{J} \omega_j d_{ij}(A_i, s_j^*) \tag{4}$$

F. Criteria Bounds

To use the RLC method, additional information regarding the bounds of the criteria is required to transform the original ill-defined MCDM problem into a well-defined one and find its solution. At present, it is unclear whether there is a general principle for automatic bound selection for the RLC method, making this a challenging open question.

Also, it is difficult to establish general principles because bound selection depends heavily on the nature of the criteria involved in the specific MCDM problem at hand. As a guideline, experts should be consulted to provide the necessary bounds for the RLC method. Sensitivity analysis can then be performed on the RLC result by varying the bounds to determine an acceptable margin of bound values and assess the robustness of the RLC solution.

G.Determining the Criteria Weights

For each criterion $C_j\,(j = 1,.., J)$, the criteria weighs can be determined using the subjective or objective methods. One can also use composite criteria weights ω_j. Here, the equal criteria $C_j\,(j = 1,.., J)$ weights ω_j are assigned by $\omega_j = 1/J$, where $\sum_{j=1}^{J} \omega_j = 1$.

H.Steps of RLC Method

To make it more convenient, the main steps of the RLC method are summarized below:

- Specify the criteria, alternatives, and their performance scores in a score matrix,
- Determine the importance weights for each criterion,
- Determine the reference solution point; Determine the upper and lower bounds for each criterion to transform the ill-defined MCDM problem into a well-defined MCDM problem,
- Normalize the score matrix,
- Calculate the weighted Euclidean / Manhattan distance between each alternative and the normalized reference solution point,
- Rank the alternatives based on their weighted Euclidean / Manhattan distances in ascending order,
- Check the sensitivity of the results with respect to the changes in the criteria weights as well as using an augmented or reduced decision matrix.

Step 1: Define the min and max bounds of classical (ill-defined / incomplete) original MCDM problem in order to transform it into a well-defined MCDM problem.

Step 2: Define the reference solution point of MCDM depending on preference order of each criterion (larger is better, or smaller is better).

Step 3: For each alternative $A_i\,(i = 1,.., I)$, compute its normalized distance with respect to reference solution for each criteria $C_j\,(j = 1,.., J)$.

Step 4: For each criterion $C_j\,(j = 1,.., J)$, define subjective or compute objective criteria weights ω_j.

Step 5: For each alternative $A_i\,(i = 1,.., I)$, compute its normalized averaged distance with respect to multiple criteria reference solution by

$$d_{ij}(A_i, s_j^*) = \sum_{j=1}^{J} \omega_j d_{ij}(A_i, s_j^*)$$

Step 6: Sort alternatives in increasing order using $d_{ij}(A_i, s_j^*) \in [0,1]$ values. The least value corresponds to the best MCDM solution A^*, that is $A^* = A_{i_*}$, where $i_* = \arg\min d_{ij}(A_i, s_j^*)$.

Step 7: Evaluate the impact of changes in the decision matrix and preference ordering on the results to determine the sensitivity of the RLC Method. Specifically, analyze how introducing or removing an alternative from the bounds affects the results using an augmented or reduced decision matrix, and assess how changes in preference ordering, such as adjusting the relative importance of criteria, affect the outcome of the method. This process enables the evaluation of the robustness and reliability of the RLC method and the identification of any weaknesses that require addressing to improve its performance.

III. Application

A. Determining the Aircraft Selection Criteria

The process of fleet planning involves acquiring the appropriate model of aircraft that aligns with an airline's strategic, tactical, and operational requirements, ensuring the right number of aircraft are stationed at the correct location(s), and then trading the aircraft at the most advantageous time.

In this scenario, the evaluation criteria were determined after conducting a literature review and consulting professional opinions. Typically, airline businesses make decisions that either directly or indirectly minimize unit costs and maximize unit benefits.

The aircraft fleet planning team considered the usual criteria for analyzing long-term investments. A group of three experts were consulted to complete the initial decision matrix, resulting in the identification of six decision-making criteria. The following are the criteria and their definitions:

Flight range (km) is, C_1 criterion to be maximized, the distance that is flown by an aircraft without refueling. The range must be as high as possible to increase the number of locations that can be served by an aircraft.

Number of seats is, C_2 criterion to be maximized, the maximum number of seats that can sit on an aircraft, both in terms of the physical space available and the limitations set by law. Aircraft with the largest number of seats are preferred among aircraft with similar specifications.

Maximum takeoff weight (MTOW) (kg) is, C_3 criterion to be maximized, the maximum weight at which the aircraft is certified for takeoff due to structural or other limits.

Luggage volume (m^3) is, C_4 criterion to be maximized, the maximum space available for keeping luggage in an aircraft. The aircraft with the largest luggage volume is preferred, among the aircrafts with the similar technical characteristics.

Fuel consumption (kg/km), C_5 criterion to be minimized, measures the amount of fuel an aircraft consumes to fly a specific distance. Fuel consumption is a cost-effective measure for reducing CO_2 emissions for environmental sustainability.

Purchase cost (\$ x 10^6) is, C_6 criterion to be minimized, the amount of a customer is willing to pay for purchasing an aircraft. The cost incurred when purchasing the aircraft from the manufacturer or supplier. The average market price was used the for assessment of the aircraft.

The management of airline fleet planning recommends the acquisition of narrow-body commercial passenger aircraft as a means of fulfilling the latest strategic, tactical, and operational requirements, while simultaneously improving capabilities and capacities with minimum maintenance and operating expenses.

The MCDM problem is assessed by a panel of three experts, who identify the decision criteria and narrow down the choices to six aircraft. Additionally, two test aircraft alternatives with the lowest and highest target values are included in the set of alternatives to evaluate the rank reversal performance of the proposed RLC approach.

B. Determining the Aircraft Alternatives

Narrow-body commercial passenger aircraft, which are widely used in the aviation market, have been chosen as alternatives: Airbus aircraft {A19N (A_1), A20N (A_2), A21N (A_3)}, Boeing aircraft {B37M (A_4), B38M (A_5), B39M (A_6)}. Benefit criteria are flight range (C_1), number of seats (C_2), maximum takeoff weight (C_3), luggage volume (C_4), cost criteria are fuel consumption (C_5) and purchase cost (C_6). To decide which of the alternatives is best, it is desired to assess them all using multiple criteria analysis.

C. Application of Aircraft Selection Problem

Table 1 shows the initial decision making matrix including six criteria and six aircraft alternatives.

Table 1. Initial decision matrix

Options (A_i)	Decision criteria (C_j)					
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	6850	160	75500	27,70	2,82	101,5
A_2	6300	194	79000	37,40	2,79	110,6
A_3	7400	244	97000	51,70	3,30	129,5
A_4	7130	172	80286	32,45	2,85	99,7
A_5	6570	210	82191	43,69	3,04	121,6
A_6	6570	220	88314	51,37	3,30	128,9
S_j^{max}	8000	300	100000	60	2	90
S_j^{min}	6000	130	70000	20	3,5	130

Using Eq. (3), Table 2 shows the normalized form of the initial decision-making matrix for the evaluation of alternatives.

Table 2. Normalized decision matrix for the evaluation of alternatives

Options (A_i)	Decision criteria (C_j)					
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0,575	0,824	0,817	0,808	0,547	0,288
A_2	0,850	0,624	0,700	0,565	0,527	0,515
A_3	0,300	0,329	0,100	0,208	0,867	0,988
A_4	0,435	0,753	0,657	0,689	0,567	0,243
A_5	0,715	0,529	0,594	0,408	0,693	0,790
A_6	0,715	0,471	0,390	0,216	0,867	0,973

In MCDM analysis, the order of preference for alternatives in the Reference Linear Combination (RLC) method can be determined using two approaches: unweighted and weighted normalized RLC.

The unweighted approach assigns equal importance $\omega_j = 1/J$ to all criteria, while the weighted approach incorporates objective or subjective weights for each criterion. The weighted approach was based on the criteria weight vector given by:

$$\omega_j = \{0.2, 0.18, 0.14, 0.16, 0.17, 0.15\}$$

The resulting preference order obtained from either approach reflects the relative performance of alternatives based on their distance from the reference solution point across all criteria considered as shown in Table 3.

Table 3. Unweighted and weighted normalized RLC based preference order of alternatives

	Unweighted Normalized RLC		Weighted normalized RLC	
Options (A_i)	$d = d(A_i, s^*)$	Rank R_i	$d = d(A_i, s^*)$	Rank R_i
A_1	0,643	6	0,643	6
A_2	0,630	5	0,637	5
A_3	0,465	1	0,462	1
A_4	0,557	2	0,557	2
A_5	0,622	4	0,623	4
A_6	0,605	3	0,610	3

Sorting the distances vector $d = d(A_i, s^*)$ in ascending order one gets

$$d(A_3, s^*) \le d(A_4, s^*) \le d(A_6, s^*) \le d(A_5, s^*) \le d(A_2, s^*) \le d(A_1, s^*)$$

which means that A_3 is the closest alternative to the reference solution point. The final preference order result of RLC method for this aircraft selection case is therefore:

$$A_3 \ge A_4 \ge A_6 \ge A_5 \ge A_2 \ge A_1 .$$

Suppose now that one takes out one alternative, say A_5, of the MCDM problem for this aircraft selection case. Then, one must now consider the following modified (reduced) score matrix in Table 4.

Table 4. Reduced decision matrix

Options (A_i)	Decision criteria (C_j)					
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	6850	160	75500	27,70	2,82	101,5
A_2	6300	194	79000	37,40	2,79	110,6
A_3	7400	244	97000	51,70	3,30	129,5
A_4	7130	172	80286	32,45	2,85	99,7
A_6	6570	220	88314	51,37	3,30	128,9
S_j^{max}	8000	300	100000	60	2	90
S_j^{min}	6000	130	70000	20	3,5	130

Applying RLC steps 3 and 4 one gets the same normalized distances, unweighted and weighted average distances for the reduced MCDM problem in Tables 5 - 6.

Table 5. Normalized decision matrix for the evaluation of reduced MCDM problem

Options (A_i)	Decision criteria (C_j)					
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0,575	0,824	0,817	0,808	0,547	0,288
A_2	0,850	0,624	0,700	0,565	0,527	0,515
A_3	0,300	0,329	0,100	0,208	0,867	0,988
A_4	0,435	0,753	0,657	0,689	0,567	0,243
A_6	0,715	0,471	0,390	0,216	0,867	0,973

Table 6. Unweighted and weighted normalized RLC based preference order of alternatives from the reduced decision matrix

	Unweighted Normalized RLC		Weighted normalized RLC	
Options (A_i)	$d = d(A_i, s^*)$	Rank R_i	$d = d(A_i, s^*)$	Rank R_i
A_1	0,643	6	0,643	6
A_2	0,630	5	0,637	5
A_3	0,465	1	0,462	1
A_4	0,557	2	0,557	2
A_6	0,605	3	0,610	3

Sorting the distances vector $d = d(A_i, s^*)$ in ascending order one gets

$$d(A_3, s^*) \le d(A_4, s^*) \le d(A_6, s^*) \le d(A_2, s^*) \le d(A_1, s^*)$$

and one deduces the final preference order of reduced MCDMA problem

$$A_3 \ge A_4 \ge A_6 \ge A_2 \ge A_1 .$$

which is naturally consistent with the previous result, i.e. there is no rank reversal.

Similarly, suppose one introduces a new alternative A_7 compatible with min and max bounds of criteria in the MCDM problem so that the modified (augmented) MCDM problem is characterized by the following (augmented) score matrix in Table 7 as follows

Table 7. Augmented decision matrix

Options (A_i)	Decision criteria (C_j)					
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	6850	160	75500	27,70	2,82	101,5
A_2	6300	194	79000	37,40	2,79	110,6
A_3	7400	244	97000	51,70	3,30	129,5
A_4	7130	172	80286	32,45	2,85	99,7
A_5	6570	210	82191	43,69	3,04	121,6
A_6	6570	220	88314	51,37	3,30	128,9
A_7	7000	170	80000	35	2,5	120
S_j^{max}	8000	300	100000	60	2	90
S_j^{min}	6000	130	70000	20	3,5	130

From RLC steps 3 and 4 one now gets normalized distances, unweighted and weighted average distances for the augmented MCDM problem in Tables 8 - 9.

Table 8. Normalized decision matrix for the evaluation of augmented MCDM problem

Options (A_i)	Decision criteria (C_j)					
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0,575	0,824	0,817	0,808	0,547	0,288
A_2	0,850	0,624	0,700	0,565	0,527	0,515
A_3	0,300	0,329	0,100	0,208	0,867	0,988
A_4	0,435	0,753	0,657	0,689	0,567	0,243
A_5	0,715	0,529	0,594	0,408	0,693	0,790
A_6	0,715	0,471	0,390	0,216	0,867	0,973
A_7	0,500	0,765	0,667	0,625	0,333	0,750

Table 9. Unweighted and weighted normalized RLC based preference order of alternatives from the reduced decision matrix

	Unweighted Normalized RLC		Weighted normalized RLC	
Options (A_i)	$d = d(A_i, s^*)$	Rank R_i	$d = d(A_i, s^*)$	Rank R_i
A_1	0,643	7	0,643	7
A_2	0,630	6	0,637	6
A_3	0,465	1	0,462	1
A_4	0,557	2	0,557	2
A_5	0,622	5	0,623	5
A_6	0,605	3	0,610	4
A_7	0,607	4	0,600	3

Unweighted case: Sorting the distances vector $d = d(A_i, s^*)$ in ascending order one gets

$$d(A_3, s^*) \le d(A_4, s^*) \le d(A_6, s^*)$$
$$d(A_7, s^*) \le d(A_5, s^*) \le d(A_2, s^*) \le d(A_1, s^*)$$

and one deduces the final preference order of augmented MCDMA problem

$$A_3 \ge A_4 \ge A_6 \ge A_7 \ge A_5 \ge A_2 \ge A_1 .$$

which is also naturally consistent with the previous results of preference orderings, i.e. there is no rank reversal.

Weighted case: Sorting the distances vector $d = d(A_i, s^*)$ in ascending order one gets

$$d(A_3, s^*) \le d(A_4, s^*) \le d(A_7, s^*)$$
$$d(A_6, s^*) \le d(A_5, s^*) \le d(A_2, s^*) \le d(A_1, s^*)$$

and one deduces the final preference order of augmented MCDMA problem

$$A_3 \ge A_4 \ge A_7 \ge A_6 \ge A_5 \ge A_2 \ge A_1 .$$

which is also naturally consistent with the previous results of preference orderings, i.e. there is no rank reversal.

The RLC method is simple to apply and can be used with any reference solution point chosen within the bounds of the criteria. The effectiveness of the proposed method was demonstrated on how it works using a numerical aircraft selection problem. Using the RLC multiple criteria analysis approach, the study analyzed the dependable outcomes without any instances of rank reversal by examining both unweighted and weighted decision matrices, as well as reduced and augmented decision matrices.

IV. Conclusion

In this paper, the Reference Linear Combination (RLC) method, a new stable preference ordering method was proposed for solving multiple criteria decision-making (MCDM) problems. The method is based on the Reference Linear Combination (RLC) approach, which ensures that the method is free of rank reversal by introducing minimum and maximum bounds for each criterion in the well-formulated MCDM problem.

One of the advantages of the RLC method is its simplicity and flexibility. It can be applied to any reference solution point within the bounds of the criteria, making it easy to use in practice. The effectiveness of the method was demonstrated on a numerical aircraft selection problem, comparing the results obtained using unweighted and weighted decision matrices, as well as the reduced and augmented decision matrices.

Overall, the RLC method is a promising approach for solving MCDM problems. Its simplicity and ability to handle different types of decision matrices make it a useful tool for decision-making in a wide range of fields. Future research could explore the method's performance on larger and more complex problems to fully evaluate its potential in MCDM applications.

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