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Aerial Firefighting Aircraft Selection with Standard Fuzzy Sets using Multiple Criteria Group Decision Making Analysis

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***Abstract*—**Aircraft selection decisions can be challenging due to their multidimensional and interdisciplinary nature. They involve multiple stakeholders with conflicting objectives and numerous alternative options with uncertain outcomes. This study focuses on the analysis of aerial firefighting aircraft that can be chosen for the Air Fire Service to extinguish forest fires. To make such a selection, the characteristics of the fire zones must be considered, and the capability to manage the logistics involved in such operations, as well as the purchase and maintenance of the aircraft, must be determined. The selection of firefighting aircraft is particularly complex because they have longer fleet lives and require more demanding operation and maintenance than scheduled passenger air service. This paper aims to use the fuzzy proximity measure method to select the most appropriate aerial firefighting aircraft based on decision criteria using multiple attribute decision making analysis. Following fuzzy decision analysis, the most suitable aerial firefighting aircraft is ranked and determined for the Air Fire Service.

***Keywords*—**Aerial firefighting aircraft selection, multiple criteria decision making, fuzzy sets, standard fuzzy sets, determinate fuzzy sets, indeterminate fuzzy sets, proximity measure method, Minkowski distance family function, Hausdorff distance function, MCDM, PMM, PMM-F.

I. Introduction

Wildfires are a common occurrence in Türkiye during the hot, dry months of May to October. Türkiye is one of the most vulnerable forested countries in Eurasia due to its natural forest cover. The forests are home to a diverse range of flora and fauna, including indigenous plant species. As many forested areas cannot be accessed by road, it is crucial to establish an aerial firefighting fleet to combat wildfires effectively.

Protecting forests and biodiversity is essential to maintaining a sustainable forest nature in Türkiye. Fleet planning for aerial firefighting is a challenging task that involves several variables such as aircraft economics, market analysis, performance, finance, and environmental factors. In literature reviews, multiple criteria analysis is frequently used to select an optimal aerial firefighting aircraft for fleet planning.

Choosing an aircraft involves complex decision-making as it has socio-political, environmental, economic, technical, and ethical implications. Multiple criteria decision making (MCDM) methods have gained interest in supporting decision-makers to evaluate decisions systematically, providing transparency, traceability, and reproducibility in the decision-making process for aircraft selection [1-54].

The aim of decision analysis is to provide a formal approach to solving decision problems that are too complex to be resolved informally. These problems can arise when there are multiple decision makers, conflicting goals, various possible courses of action, uncertainty about outcomes, unfamiliarity, and different subjective valuations of risks and consequences. Making decisions can also be complicated by overall goals, which often require comparing alternatives that are incommensurate and high dimensional. One-off decisions with high stakes in complex contexts can be particularly challenging since the objectives must first be identified jointly with the decision-makers. Aircraft selection decisions are often of this complex decision type.

To tackle this complexity, decision analysis methods combine problem structuring and multiple attribute decision making analysis models. When presented with a set of alternatives, these methods help structure the decision problem, systematize goals, and evaluate the measurable qualities of the alternatives. This allows for comparison between alternatives and supports decision makers to explore the decision space and model outcomes. As a result, decisions become more transparent, traceable, and repeatable.

MCDM analysis methods are used to determine how well an alternative satisfies a set of criteria or objectives. MCDM methods are typically classified based on how goal attainment is defined: overall value (score or rank), goal or aspiration level, and outranking. Summarizing value methods use a numerical score to indicate the preference of an alternative compared to others, such as Multiple Attribute Utility and Value (MAUT and MAVT), AHP, PARIS, TOPSIS, and VIKOR. Aspiration level methods assess solutions based on the level of attainment of a set of goals, such as goal programming. Outranking methods use pairwise comparison of alternatives to identify a ranking of preferences, such as ELECTRE, ORESTE and PROMETHEE methods [11-25].

MCDM models are used after the decision problem is structured, which requires clarity about who participates in the decision, which objectives and criteria are considered, and which alternatives are feasible. For a summarizing value approach, it is necessary to understand and quantify the impacts of the alternatives on the attributes. The impact of

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different alternatives on the attributes can be obtained from conceptual or mathematical models that make these assumed relationships explicit or from estimates obtained from data or expert knowledge. An assessment model is used to map the alternatives to the expected outcomes on the attributes.

MCDM models require a preference model that takes into account the different perspectives of stakeholders, trade-offs among competing objectives, risk attitudes, and ambiguity attitudes of decision-makers. The purpose of an MCDM model in addressing a complex decision problem is to provide a focus for discussion, not to prescribe a solution. The model is useful for learning about trade-offs among alternatives and constructing decision-maker preferences. Decision-making involves judgment and valuation, so subjectivity cannot be avoided, and the responsibility for the decision and its consequences remain with the decision-maker(s).

MCDM methods are generally used to evaluate among different alternatives. However, in many cases, the actions to be taken go beyond a single alternative, and a set of potential alternatives must be identified. The number and combination of alternatives are subject to constraints, such as the available budget or other factors.

Multiple attribute decision analysis (MADA) models are designed to estimate the utility or attainment of an objective based on a set of hierarchically structured attributes or criteria. This requires the construction of a preference model by aggregating different attributes. MADA models are simple to conceptualize and suitable for including risky choices.

Preference models based on multiple attribute value theory consist of three elements: an objectives hierarchy, the assessment of marginal utilities or values, and trade-offs among different objectives. The objectives hierarchy involves breaking down the overall objective of the decision into intermediate objectives, which can be further disaggregated into lowest-level objectives for which measurable attributes are defined. Only fundamental objectives should be included in the hierarchy.

Once the objective hierarchy is defined, marginal valuation functions over the attributes and trade-offs among the attributes and objectives are elicited. The valuation functions may or may not include the risk preferences of decision-makers regarding the attributes and objectives. Trade-offs are elicited by understanding desirable trade-offs, often expressed as importance weights, among the attributes and how they should be aggregated - value or utility aggregation function. This elicitation process may yield uncertain parameters due to preference instability and limited interaction with decision-makers.

After assessing utilities for attributes and objectives, utilities corresponding to attribute levels are aggregated in the intermediate objectives, and later towards the overall objective. This requires defining relative importance weights of each attribute or objective and the aggregation function.

Therefore, the evaluation of portfolios requires a separate analysis, where the aggregation function and importance weights are defined to determine how to combine the individual utilities of the alternatives into a single score. This aggregation function can be either compensatory or non-compensatory, depending on the preferences of the decision-makers. The analysis of portfolios often involves the consideration of constraints, such as budget or resource limitations, which may affect the feasibility of certain portfolios. Additionally, sensitivity analysis can be conducted to evaluate the robustness of the results to changes in the preference model or the assessment model. Overall, the evaluation of portfolios requires a comprehensive and structured approach that considers the interactions and trade-offs among the individual alternatives and their corresponding actions, as well as the preferences and objectives of the decision-makers.

The paper is structured as follows. Section 2 introduces the basic definitions and notations of standard fuzzy numbers. The PMM method and its fuzzy extension, PMM-F, are presented. Section 3 describes the application of fuzzy PMM for the selection process of aerial firefighting aircraft. Finally, Section 4 presents concluding remarks along with future research directions.

II. Methodology

Although this study utilizes fuzzy PMM, it provides theoretical information for both PMM (proximity measure method) and fuzzy PMM approach. MCDM techniques are widely used in complex decision-making environments. PMM, which was developed by Ardil [26-28], is one of the most effective MCDM methods. The method uses the ideal solution as a benchmark for comparison, with the alternative that deviates the least from the ideal solution being selected. The best option is one that maximizes every benefit criterion and reduces every cost criterion.

The traditional PMM technique is based on precise numerical values provided by the decision maker or expert. However, in some situations, the decision maker may not be able to express the ratings of alternatives accurately or may use linguistic terms. In such cases, other data formats such as interval numbers, fuzzy numbers, ordered fuzzy numbers, hesitant fuzzy sets, and intuitionistic fuzzy sets may be used. As decision problems become more complex, it becomes impractical for a single decision maker to analyze all relevant aspects of the problem. Therefore, a group of decision makers are involved in making decisions for real-life problems. The individual decisions made by each decision maker are often combined to create a collective decision, usually in the form of an individual or collective decision matrix, which serves as the basis for rating options and selecting the best one.

The PMM method is effectively used in MCDM to aggregate evaluations from multiple decision makers. The arithmetic mean is often used to combine the individual scores given by each decision maker to determine the final score of each alternative. However, in situations where decision makers provide fuzzy or imprecise data, this method involves transforming the individual choice matrices provided by each decision maker into aggregated matrices of alternatives. These matrices organize the evaluations of each alternative based on each criterion, allowing for the selection of the optimal alternative.

In this approach, the optimal decision matrix or ideal solution vector is a matrix composed of maximal assessments, as all individual decision matrices are normalized based on the criterion type. Unlike traditional PMM and techniques that rely on the accumulation of individual decisions, the distances between matrices represent the distances of alternatives from the ideal solution. The best alternative is identified by ranking the alternatives using the proximity measure value of each alternative to the ideal solution.

The paper briefly reviews and classifies standard fuzzy sets [55-104] as determinate fuzzy sets and indeterminate fuzzy sets and provides numerical examples of multiple criteria decision making for an aerial firefighting aircraft selection problem.

Definition 1.[55] Fuzzy set. For any universal set X , fuzzy set (FS) is of the form

$$F=\left\{< x_i, p_F(x_i) > \ | \forall x_i \in X\right\} \tag{1}$$

where $p_F(x_i)\in[0,1]$ is called the degree of membership of an element x_i to X, $q_F(x_i)=1-p_F(x_i)\in[0,1]$ denotes the degree of nonmembership of an element x_i to X, and $p_F(x_i)\in[0,1]$, and $q_F(x_i)=1-p_F(x_i)\in[0,1]$ satisfy the following condition:

$$F=\left\{< x_i, p_F(x_i), 1-p_F(x_i) > \ | \forall x_i \in X\right\} \tag{2}$$

$$p_F(x_i)+1-p_F(x_i)=1 \ | \forall x_i \in X \tag{3}$$

In a fuzzy set, the degree of indeterminacy or hesitation of element $x_i \in X$ to set F is $i_F(x_i)=1-p_F(x_i)-q_F(x_i)=0$.

Definition 2. Determinate fuzzy set. For any universal set X , determinate fuzzy set (DFS) is of the form

$$D=\left\{< x_i, p_D(x_i), q_D(x_i) > \ | \forall x_i \in X\right\} \tag{4}$$

where $p_D(x_i)\in[0,1]$ denotes the degree of membership of an element x_i to D, $q_D(x_i)=1-p_D(x_i)$ denotes the degree of nonmembership an element x_i to D, $p_D(x_i)\in[0,1]$ and $q_D(x_i)\in[0,1]$ satisfy the following condition:

$$p_D(x_i)+q_D(x_i)=1 \ | \forall x_i \in X \tag{5}$$

where $i_D(x_i)=1-p_D(x_i)+q_D(x_i)=0, \ | \forall x_i \in X$ denotes the degree of indeterminacy of x_i to X.

For the given element x_i, $< x_i, p_D(x_i), q_D(x_i) >$ denotes determinate fuzzy number (DFN), and for convenience, $a=(p_\alpha, q_a)$ denotes a DFN, which meets the conditions $p_D(x_i), q_D(x_i)\in[0,1]$ and $p_D(x_i)+q_D(x_i)=1$.

Definition 3. Indeterminate fuzzy set. For any universal set X, indeterminate fuzzy set (IFS) is of the form

$$I=\left\{< x_i, p_I(x_i), i_I(x_i), q_I(x_i) > \ | \forall x_i \in X\right\} \tag{6}$$

a) where the triplet components , $p,i,q \to [0,1]$, represent the degree of membership, the degree of indeterminacy, and the degree of nonmembership, respectively, provided that $p_I(x_i)$, $i_I(x_i)$ and $q_I(x_i)$ satisfy the following conditions:

1) when all three components are independent;

$$0\le p_I(x_i)+i_I(x_i)+q_I(x_i)\le 3 \ | \forall x_i \in X \tag{7}$$

2) when two components are dependent, while the third one is independent from them;

$$0\le p_I(x_i)+i_I(x_i)+q_I(x_i)\le 2 \ | \forall x_i \in X \tag{8}$$

3) when all three components are dependent;

$$0\le p_I(x_i)+i_I(x_i)+q_I(x_i)\le 1 \ | \forall x_i \in X \tag{9}$$

where $r_I(x_i)=1-p_I(x_i)+i_I(x_i)+q_I(x_i)=0$, $\forall x_i \in X$, denotes the degree of refusal, and also, $r_I(x_i): X \to [0,1]$, $\forall x_i \in X$.

When three or two of the components (p,i,q) are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1)[104].

4) when two components are considered: the degree of membership, and the degree of nonmembership.

$$0\le p_I(x_i)+q_I(x_i)\le 1 \ | \forall x_i \in X \tag{10}$$

where $i_I(x_i)=1-p_I(x_i)+q_I(x_i)=0$, $\forall x_i \in X$, denotes the degree of indeterminacy, and also, $i_I(x_i): X \to [0,1]$, $\forall x_i \in X$ [59].

In the literature reviews, there are various types of sets that have been derived from the standard fuzz sets (SFSs), including but not limited to intuitionistic fuzzy sets, vague sets, fermatean fuzzy sets, fuzzy soft sets, picture fuzzy sets, cubic fuzzy sets, circular fuzzy sets, Pythagorean fuzzy sets, q-rung orthopair fuzzy sets, spherical fuzzy sets, t-spherical fuzzy sets, and neutrosophic sets [55-104].

Definition 4. Given three IFNs, $a=< p,q >$, $a_1=< p_1,q_1 >$, and $a_2=< p_2,q_2 >$, then the basic arithmetic operations can be defined as

(1) $\bar{a}=< q,p >$

(2) $a_1 \vee a_2=\langle \max\{p_1,p_2\}, \min\{q_1,q_2\}\rangle$

(3) $a_1 \wedge a_2=\langle \min\{p_1,p_2\}, \max\{q_1,q_2\}\rangle$

(4) $a_1 \oplus a_2 = \langle (p_1 + p_2 - p_1 p_2)\ , q_1 q_2 \rangle$

(5) $a_1 \otimes a_2 = \langle (p_1 p_2, (p_1 + p_2 - p_1 p_2) \rangle$

(6) $\lambda a_1 = \langle 1-(1-p_1)^{\lambda}, q_1{}^{\lambda} \rangle$

(7) $a_1^{\lambda} = \langle p_1{}^{\lambda}, 1-(1-q_1)^{\lambda} \rangle$

Definition 5. Given three IFNs, $a =< p, q >$, $a_1 =< p_1, q_1 >$, and $a_2 =< p_2, q_2 >$, and $n, n_1, n_2 > 0$ then

(1) $a_1 \oplus a_2 = a_2 + a_1$

(2) $a_1 \otimes a_2 = a_2 \otimes a_1$

(3) $n(a_1 \oplus a_2) = na_1 \oplus na_2$

(4) $n_1 a \otimes n_2 a = (n_1 \oplus n_2) a$

(5) $a^{n_1} \otimes a^{n_2} = a^{n_1 + n_2}$

(6) $a_1{}^n \otimes a_2{}^n = (a_1 \otimes a_2)^n$

Definition 6. Let $I =< p_I, q_I >$ is a IFN, then a score function S of I is defined as

$$S(I) = p_I - q_I \tag{11}$$

where $S(I) \in [-1,1]$. The larger the score $S(I)$ is, the greater the IFN I is. However, it should be noted that the scoring function cannot differentiate many IFNs in some cases. To solve this problem, the accuracy function is introduced.

Definition 7. Let $I =< p_I, q_I >$ is a IFN, then a score function H of C is defined as

$$H(I) = p_I + q_I \tag{12}$$

where $H(I) \in [0,1]$. The larger the accuracy degree $H(I)$ is, the greater the IFN I is. Based on the above score function and the accuracy function, a comparison method of IFNs is proposed, which is shown as follows.

Definition 8. Let $a_1 =< p_1, q_1 >$, and $a_2 =< p_2, q_2 >$ be any two IFNs, and $S(a_1)$, $S(a_2)$ are the score functions of a_1 and a_2, and $H(a_1)$, $H(a_2)$ are the accuracy functions of a_1 and a_2, respectively, then

(1) *If* $S(a_1) > S(a_2)$, then $a_1 > a_2$

(2) *If* $S(a_1) = S(a_2)$, then

If $H(a_1) > H(a_2)$, then $a_1 > a_2$

If $H(a_1) = H(a_2)$, then $a_1 = a_2$

Definition 9. Let $a_k = \langle p_k, q_k \rangle$ $(k = 1, 2, ..., n)$ is a collection of IFNs, and $M^n \rightarrow M$, then the aggregation result is still a IFN as

$$IFWA(a_1, a_2, ..., a_n) = \omega_1 a_1 \oplus \omega_2 a_2 \oplus ... \oplus \omega_n a_n$$

$$IFWA(a_1, a_2, ..., a_n) = \left\langle 1 - \prod_{k=1}^{n} (1-p_k)^{\omega_k}, \prod_{k=1}^{n} q_k^{\omega_k} \right\rangle \tag{13}$$

where M is the set of all IFNs, and $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is a weight vector of $(a_1, a_2, ..., a_n)$, such that $0 \le \omega_k \le 1$ and $\sum_{k=1}^{n} \omega_k = 1$. Then, the IFWA is called the indeterminate fuzzy weighted averaging operator.

Definition 10. Let $a_k = \langle p_k, p_k \rangle$ $(k = 1, 2, ..., n)$ is a collection of IFNs, and $M^n \rightarrow M$ then the aggregation result is still a IFN as

$$IFWG(a_1, a_2, ..., a_n) = a_1{}^{\omega_1} \oplus a_2{}^{\omega_2} \oplus ... \oplus a_n{}^{\omega_n}$$

$$IFWG(a_1, a_2, ..., a_n) = \left\langle \prod_{k=1}^{n} p_k^{\omega_k}, 1 - \prod_{k=1}^{n} (1-q_k)^{\omega_k} \right\rangle \tag{14}$$

where M is the set of all IFNs, and $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is a weight vector of $(a_1, a_2, ..., a_n)$, such that $0 \le \omega_k \le 1$ and $\sum_{k=1}^{n} \omega_k = 1$. Then, the IFWG is called the indeterminate fuzzy weighted geometric operator.

Definition 11. Let $a_1 =< p_1, i_{1,}, q_1 >$, and $a_2 =< p_2, i_1, q_2 >$ be any two IFNs, then the Minkowski distance family function is defined as

$$d_{\delta}(a_1, a_2) = \begin{pmatrix} \frac{1}{n} \sum_{j=1}^{n} | p_1(x) - p_2(x) |^{\delta} + \\ | i_1(x) - i_2(x) |^{\delta} + | q_1(x) - q_2(x) |^{\delta} \end{pmatrix}^{1/\delta} \tag{15}$$

where $\delta = \{1, 2, 3, \infty\}$ values are often used to define Minkowski distance family function {Manhattan ($\delta = 1$), Euclidean ($\delta = 2$), Minkowski ($\delta = 3$), Chebyshev ($\delta = \infty$).

Hausdorff distance is the maximum distance of a set to the nearest point in the other set.Hausdorff distance from set A to set B is a maximin function, defined as where a and b are points of sets A and B respectively, and $d(a,b)$ is any metric between these points. The Hausdorff distance is given as

$$d_H(a_1,a_2)=\begin{pmatrix}\frac{1}{n}\sum_{j=1}^{n}\max(|p_1(x)-p_2(x)|,\\ |i_1(x)-i_2(x)|,|q_1(x)-q_2(x)|)\end{pmatrix} \quad (16)$$

III. Application

In this section, the aerial firefighting aircraft selection problem for multiple criteria decision-making analysis employs two types of standard fuzzy sets: determinate fuzzy sets (DFSs) and indeterminate fuzzy sets (IFSs).

To manage the multiple criteria decision-making process, fuzzy decision analysis is utilized, with the development of two methods of multiple criteria decision making (MCDM) based on either the IFWA operator or the IFWG operator.

In the MCDM problem, there are m alternatives $X_i=\{x_1,x_2,...,x_m\}$, which are evaluated with respect to n attributes $A_j=\{a_1,a_2,...,a_n\}$, the weight vector of the attributes is $\omega_j=\{\omega_1,\omega_2,...,\omega_n\}$ satisfying $\omega_j\ge 0$, $(j=1,2,...,n)$, $\sum_{j=1}^{n}\omega_j=1$.

Also, $R=[r_{ij}]_{mxn}$ is the decision matrix, where $r_{ij}=(p_{ij},q_{ij})$ is the evaluation value of alternative X_i for attribute A_j, which is expressed by IFN, such that $p_{ij}\in[0,1]$, $q_{ij}\in[0,1]$, $p_{ij}+q_{ij}\le 1$. Then, the alternatives can be ranked.

The decision steps based on the IFWA operator or IFWG operator are shown as follows:

Step 1. Normalize the decision matrix

To mitigate the impact of the distinct attribute types, which are categorized as either benefit type or cost type, the cost type attributes are transformed into benefit type attributes.

$$r_{ij}=(p_{ij},q_{ij})=\begin{cases}(p_{ij},q_{ij})\ for\ \Omega_b\\(q_{ij},p_{ij})\ for\ \Omega_c\end{cases}$$

where Ω_b denotes benefit attribute A_j and Ω_c denotes cost attribute A_j.

Step 2. Aggregate all attribute value $r_{ij}(j=1,2,...,n)$ to the comprehensive value z_i by IFWA operator.

$$z_i=IFWA(r_{i1,},r_{i2},...,r_{in})$$

or by IFWG operator

$$z_i=IFWG(r_{i1,},r_{i2},...,r_{in})$$

Step 3.Rank z_i $(i=1,2,...,m)$ based on the score function $S(z_i)$ and accuracy function $H(z_i)$.

Step 4. Rank all the alternatives. The bigger the IFN z_i is, the better the alternative X_i is.

A. Numerical Illustration

B. Application of Determinate Fuzzy Sets (DFSs)

To demonstrate the practical application of the proposed method in this paper, an example of multiple criteria decision making (MCDM) is presented. In the given scenario, the Air Fire Service is considering purchasing an aerial firefighting aircraft to combat agriculture and forest fires, with three potential aerial firefighting aircraft alternatives: $X_i=\{X_1,X_2,X_3\}$.

A committee of three experts $E_i=\{E_1,E_2,E_3\}$ was tasked with identifying the decision attributes, resulting in the selection of five attributes for evaluating the alternatives:

A1: Price, whether new or used, considering that some of the aircraft versions may no longer be in production but are still available on the market.

A2: Operating expenses per hour, encompassing fuel costs, maintenance, and flight crew labor.

A3: Water volume, which is affected by the distance between the fire and the water intake site and has a significant impact on the amount of water that can be used per hour.

A4: Performance of the aircraft, including takeoff run, climb rate, engine power, and airspeed.

A5: Aircraft survivability, referring to its capacity to withstand and/or avoid a hostile environment.

The attribute weight vector was determined by the committee as follows:

$$\omega_j=(0.15,\ 0.18,\ 0.20,\ 0.25,\ 0.22)^T$$

The attribute values of each alternative are evaluated using determinate fuzzy numbers (DFNs), assuming that the decision matrix is presented in Table 1.

Table 1. The decision matrix

	A1	A2	A3	A4	A5
X1	(0.4, 0.6)	(0.8, 0.2)	(0.7, 0.3)	(0.8, 0.2)	(0.5, 0.5)
X2	(0.6, 0.4)	(0.7, 0.3)	(0.6, 0.4)	(0.7, 0.3)	(0.6, 0.6)
X3	(0.5, 0.5)	(0.6, 0.4)	(0.9, 0.1)	(0.6, 0.4)	(0.8, 0.2)

(1) The decision-making steps based on IFWA operator

Step 1. Normalize the decision matrix.

As the attributes A1 and A2 are of cost type, they are converted into benefit type attributes, after which the normalized decision matrix is obtained as presented in Table 2.

Table 2. The normalized decision matrix

	A1	A2	A3	A4	A5
X1	(0.6, 0.4)	(0.2, 0.8)	(0.7, 0.3)	(0.8, 0.2)	(0.5, 0.5)
X2	(0.4, 0.6)	(0.3, 0.7)	(0.6, 0.4)	(0.7, 0.3)	(0.6, 0.4)
X3	(0.5, 0.5)	(0.4, 0.6)	(0.1, 0.9)	(0.6, 0.4)	(0.2, 0.8)

Step 2. Aggregate all attribute values r_{ij} to the comprehensive value z_i by IFWA operator and the score function ranking R_i of alternatives are shown in Table 3.

Table 3. The ranking order of the alternatives based on the score function

	$\sum_{j=1}^{n} p_I$	$\sum_{j=1}^{n} q_I$	z_i	R_i
X1	0,706	0,294	0,411	1
X2	0,562	0,438	0,125	2
X3	0,551	0,449	0,101	3

The ranking order of the alternatives based on the score function is as follows:

$$z_3 \prec z_2 \prec z_1$$

According to the MCDM analysis ranking, the best alternative for an aerial firefighting aircraft was selected as A1.

(2) The decision-making steps based on IFWG operator

Step 1. Normalize the decision matrix.

As the attributes A1 and A2 are categorized as cost type, they are converted into benefit type attributes, after which the normalized decision matrix is obtained (as shown in Table 4).

Table 4. The normalized decision matrix

	A1	A2	A3	A4	A5
X1	(0.2, 0.4)	(0.2, 0.5)	(0.7, 0.1)	(0.3, 0.3)	(0.5, 0.4)
X2	(0.3, 0.6)	(0.2, 0.7)	(0.6, 0.1)	(0.4, 0.4)	(0.4, 0.3)
X3	(0.3, 0.5)	(0.2, 0.7)	(0.6, 0.2)	(0.3, 0.4)	(0.4, 0.5)

Step 2. Aggregate all attribute values r_{ij} to the comprehensive value z_i by IFWG operator and the score function ranking R_i of alternatives are shown in Table 5.

Table 5. The ranking order of the alternatives based on the score function

	$\sum_{j=1}^{n} p_I$	$\sum_{j=1}^{n} q_I$	z_i	R_i
X1	0,524	0,476	0,048	1
X2	0,518	0,482	0,036	2
X3	0,298	0,702	-0,404	3

The ranking order of the alternatives based on the score function is as follows:

$$z_3 \prec z_2 \prec z_1$$

According to the MCDM analysis ranking, the best alternative for an aerial firefighting aircraft was selected as A1.

In the aerial firefighting aircraft selection problem, the proximity measure method (PMM) was utilized [26-28], with the ranking orders of the alternatives determined using various Minkowski distance family functions, including the Manhattan distance (z_1), Euclidean distance (z_2), Minkowski distance (z_3), Chebyshev distance (z_∞), and Hausdorff distance (z_H) functions, as shown in Table 6.

Table 6. The ranking orders of the alternatives based on the Minkowski distance family functions and the Hausdorff distance function

	z_1	R_i	z_2	R_i	z_3	R_i	z_∞	R_i	z_H	R_i
X1	0,062	2	0,133	2	0,076	2	0,025	2	0,238	2
X2	0,060	1	0,113	1	0,056	1	0,016	1	0,211	1
X3	0,126	3	0,246	3	0,192	3	0,056	3	0,278	3

The ranking order of the alternatives based on the proximity measure method (PMM) is presented as follows:

$$z_3 \prec z_1 \prec z_2$$

According to the MCDM analysis ranking, the best alternative for an aerial firefighting aircraft was selected as A2.

C. Application of Indeterminate Fuzzy Sets (DFSs)

To demonstrate the practical application of the proposed method in this paper, an example of multiple criteria decision making (MCDM) is presented. The Air Fire Service is considering purchasing an aerial firefighting aircraft to combat agriculture and forest fires, with three potential aerial firefighting aircraft alternatives: $X_i = \{X_1, X_2, X_3\}$.

A committee of three experts $E_i = \{E_1, E_2, E_3\}$ was tasked with identifying the decision attributes, resulting in the selection of five attributes for evaluating the alternatives:

A1: Price, whether new or used, considering that some of the aircraft versions may no longer be in production but are still available on the market.

A2: Operating expenses per hour, encompassing fuel costs, maintenance, and flight crew labor.

A3: Water volume, which is affected by the distance between the fire and the water intake site and has a significant impact on the amount of water that can be used per hour.

A4: Performance of the aircraft, including takeoff run, climb rate, engine power, and airspeed.

A5: Aircraft survivability, referring to its capacity to withstand and/or avoid a hostile environment.

The attribute weight vector was determined by the committee as follows:

$$\omega_j = (0.15,\ 0.18,\ 0.20,\ 0.25,\ 0.22)^T$$

The attribute values of each alternative are evaluated using indeterminate fuzzy numbers (IFNs), assuming that the decision matrix is presented in Table 7.

Table 7. The decision matrix

	A1	A2	A3	A4	A5
X1	(0.4, 0.2)	(0.5, 0.2)	(0.7, 0.1)	(0.3, 0.3)	(0.5, 0.4)
X2	(0.6, 0.3)	(0.7, 0.2)	(0.6, 0.1)	(0.4, 0.4)	(0.4, 0.3)
X3	(0.5, 0.3)	(0.7, 0.2)	(0.6, 0.2)	(0.3, 0.4)	(0.4, 0.5)

(1) The decision-making steps based on IFWA operator

Step 1. Normalize the decision matrix.

As the attributes A1 and A2 are of cost type, they are converted into benefit type attributes, after which the normalized decision matrix is obtained as presented in Table 8.

Table 8. The normalized decision matrix

	A1	A2	A3	A4	A5
X1	(0.2, 0.4)	(0.2, 0.5)	(0.7, 0.1)	(0.3, 0.3)	(0.5, 0.4)
X2	(0.3, 0.6)	(0.2, 0.7)	(0.6, 0.1)	(0.4, 0.4)	(0.4, 0.3)
X3	(0.3, 0.5)	(0.2, 0.7)	(0.6, 0.2)	(0.3, 0.4)	(0.4, 0.5)

Step 2. Aggregate all attribute values r_{ij} to the comprehensive value z_i by IFWA operator and the score function ranking R_i of alternatives are shown in Table 9.

Table 9. The ranking order of the alternatives based on the score function

	$\sum_{j=1}^{n} p_I$	$\sum_{j=1}^{n} q_I$	z_i	R_i
X1	0,427	0,294	0,133	1
X2	0,404	0,334	0,069	2
X3	0,380	0,418	-0,038	3

The ranking order of the alternatives based on the score function is as follows:

$$z_3 \prec z_2 \prec z_1$$

According to the MCDM analysis ranking, the best alternative for an aerial firefighting aircraft was selected as A1.

(2) The decision-making steps based on IFWG operator

Step 1. Normalize the decision matrix.

As the attributes A1 and A2 are of cost type, they are converted into benefit type attributes, after which the normalized decision matrix is obtained as presented in Table 10.

Table 10. The normalized decision matrix

	A1	A2	A3	A4	A5
X1	(0.2, 0.4)	(0.2, 0.5)	(0.7, 0.1)	(0.3, 0.3)	(0.5, 0.4)
X2	(0.3, 0.6)	(0.2, 0.7)	(0.6, 0.1)	(0.4, 0.4)	(0.4, 0.3)
X3	(0.3, 0.5)	(0.2, 0.7)	(0.6, 0.2)	(0.3, 0.4)	(0.4, 0.5)

Step 2. Aggregate all attribute values r_{ij} to the comprehensive value z_i by IFWG operator and the score function ranking R_i of alternatives are shown in Table 11.

Table 11. The ranking order of the alternatives based on the score function

	$\sum_{j=1}^{n} p_I$	$\sum_{j=1}^{n} q_I$	z_i	R_i
X1	0,348	0,346	0,002	1
X2	0,367	0,441	-0,074	2
X3	0,341	0,476	-0,134	3

The ranking order of the alternatives based on the score function is as follows:

$$z_3 \prec z_2 \prec z_1$$

According to the MCDM analysis ranking, the best alternative for an aerial firefighting aircraft was selected as A1.

In the aerial firefighting aircraft selection problem, the proximity measure method (PMM) was utilized [26-28], with the ranking orders of the alternatives determined using various Minkowski distance family functions, including the Manhattan distance (z_1), Euclidean distance (z_2), Minkowski distance (z_3), Chebyshev distance (z_∞), and Hausdorff distance (z_H) functions, as shown in Table 12.

Table 12. The ranking orders of the alternatives based on the Minkowski distance family functions and the Hausdorff distance function

	z_1	R_i	z_2	R_i	z_3	R_i	z_∞	R_i	z_H	R_i
X1	0,012	1	0,035	1	0,011	1	0,005	1	0,111	1
X2	0,027	2	0,063	2	0,026	2	0,007	2	0,163	2
X3	0,041	3	0,076	3	0,030	3	0,013	3	0,167	3

The ranking order of the alternatives based on the proximity measure method (PMM) is as follows:

$$z_3 \prec z_2 \prec z_1$$

According to the MCDM analysis ranking, the best alternative for an aerial firefighting aircraft was selected as A1.

IV. Conclusion

This study aims to evaluate aerial firefighting aircraft that could be selected for the aerial firefighting fleet to extinguish wildfires in Türkiye. When analyzing aerial firefighting aircraft, it is essential to consider the characteristics of fire zones across the country, as well as the capacity for purchasing, maintaining, and managing logistics associated with such aerial firefighting squadron operations.

Selecting aerial firefighting aircraft is challenging because they have a longer fleet life and require more operation and maintenance than aircraft used for regular commercial passenger flights.

This study proposes the use of standard fuzzy sets {determinate fuzzy sets, indeterminate fuzzy sets}, and the proximity measure approach to select the best aerial firefighting aircraft based on five identified conflicting attributes. A numerical example is used to demonstrate the uniqueness and effectiveness of the proposed methodologies in the challenge of choosing an aerial firefighting aircraft.

Standard fuzzy sets can be utilized in addressing various complex decision-making problems and can be combined with other MCDM methods to effectively manage uncertainty and ambiguity in a fuzzy environment.

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