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Aircraft Supplier Selection Process with Fuzzy Proximity Measure Method using Multiple Criteria Group Decision Making Analysis

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***Abstract*—**Being effective in every organizational activity has become necessary due to the escalating level of competition in all areas of corporate life. In the context of supply chain management, aircraft supplier selection is currently one of the most crucial activities. It is possible to choose the best aircraft supplier and deliver efficiency in terms of cost, quality, delivery time, economic status, and institutionalization if a systematic supplier selection approach is used. In this study, an effective multiple criteria decision-making methodology, proximity measure method (PMM), is used within a fuzzy environment based on the vague structure of the real working environment. The best appropriate aircraft suppliers are identified and ranked after the proposed multiple criteria decision making technique is used in a real-life scenario.

***Keywords*—**Aircraft supplier selection, multiple criteria decision making, fuzzy sets, proximity measure method, Minkowski distance family function, Hausdorff distance function, PMM, MCDM.

I. Introduction

Aircraft supplier selection can be considered as one of the most crucial actions of purchasing management in a supply chain. Supply chain management is a process that involves planning, organizing, implementing, and controlling the activities of the supply chain network in the most effective manner. In other words, it covers all activities connected to the movement and transformation of goods and services from the place of origin to the site of use.

The aircraft supplier selection process is occasionally viewed as a highly complex phase because there are so many uncontrollable and unpredictable factors influencing the decisions. In contrast to conventional cost-based techniques, supplier selection is not carried out solely based on cost. In reality, it is considered a multiple criteria decision making (MCDM) challenge.

In the literature, two types of supplier selection issues are mentioned.In the first, one supplier can meet all requirements. In contrast, the supplier in the second scenario can partially meet the requirements. That is, no single supplier can meet all the needs. These two problem categories are also known as single sourcing and multiple sourcing [1].

There have been numerous studies employing various criteria and approaches, depending on its significance practical applications. A comprehensive literature review study was made, and net price, delivery, and quality were selection factors for ranking of alternatives [2].The research methodology was primarily divided into three categories: linear weighing techniques, mathematical programming models, and probabilistic and statistical methodologies. Further significant literature review investigations were completed in the supplier selection problems [3-5].

There have been previous studies that completed thorough literature reviews, In the literature, the approaches used to solve supplier selection issues have been broadly categorized, albeit slightly differently [6]. There are six classes according to the classification methodology.

These include statistical/probabilistic approaches, intelligent approaches (neural networks, case-based reasoning, expert systems), mathematical programming (LP, GP, MIP, DEA), hybrid approaches (AHP-LP, ANP-MIP, ANP-TOPSIS, Fuzzy-QFD, and others), multiple attribute decision making techniques (MADM) (AHP, ANP, MAUT, TOPSIS, VIKOR) and outranking methods (ELECTRE, PROMETHEE, ORESTE) [7-25].

The selection criteria and procedures employed were primarily utilized to categorize the research in the literature reviews [26-53]. Given the multiple criteria nature of the aircraft supplier selection problem and the ambiguity in the actual environment, fuzzy proximity measure method (PMM) is an effective method for choosing the best aircraft supplier. Therefore, the proposed MCDM process is used to select the best aircraft supplier for an airline company [26-27].

Multiple criteria decision-making methods have grown in popularity recently and are routinely used in a wide variety of real-life circumstances. It is becoming less practical for one decision maker to consider all of the pertinent components of the choice problems due to the increasing complexity of the decision problems. As a result, a group of decision-makers examine a variety of real-life problems.

The objective of this study is to propose a novel MCDM method for exploiting fuzzy data to rank alternatives for group decision-making using the PMM method.The proposed method ranks the alternatives and chooses the best one after considering all the decision makers' unique decision information.

For situations involving collective decision-making, an expanded PMM technique based on fuzzy numbers has been given in this study. Using arithmetic mean, geometric mean, or their variants, most studies in the literature combine the individual decision matrices produced by the decision makers

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into a collective decision matrix as the basis for ranking the alternatives or choosing the best one. The proposed methodology is demonstrated using a numerical example.

The numerical example has demonstrated that the suggested method can produce a comparable result, both in terms of rating the alternatives and choosing the best one.

The rest of the paper is organized as follows. In Section 2 basic definitions and notations of fuzzy numbers are introduced. The PMM method and its fuzzy extension PMM are presented. Section 3 presents fuzzy PMM application for aircraft supplier selection process. In Section 4, concluding remarks were presented with future research directions.

II. Methodology

In this section, although fuzzy PMM is used in this study, both PMM (proximity measure method) and Fuzzy PMM theoretical information is provided. In many real-life scenarios, multiple criteria decision making (MCDM) techniques are extensively used in complex decision-making environments.

The proximity measure method (PMM), developed by Ardil [26-27], is one of the most effective and commonly used MCDM methods. The fundamental concept of this method is quite straightforward. It makes use of the ideal solution as a benchmark as a point of comparison. The alternative that is ultimately selected is the one that deviates the least from the ideal solution. The best option is one that maximizes every benefit criterion and reduces every cost criterion.

The information provided by the decision maker (DM) or expert as precise numerical values forms the basis of the traditional PMM technique. However, in some real-life circumstances, the DM might not be able to articulate the value of the ratings of alternatives regarding criteria accurately or else he / she may utilize linguistic terms. In such situations, when evaluations are based on unquantifiable, incomplete, or unobtainable information, the DM may use other data formats, such as: interval numbers, fuzzy numbers, ordered fuzzy numbers, hesitant fuzzy sets, intuitionistic fuzzy sets and other.

On the other hand, it is becoming less practical to analyze all the pertinent parts of a decision problem by a single DM due to the decision problems' growing complexity. Hence, a group of DMs consider a variety of real-life problems. In these circumstances, the individual decisions made by each DM are frequently combined to create a collective decision (typically in the form of an individual decision matrix) (also in the form of a collective decision matrix). This collective choice serves as the foundation for rating the options and choosing the best one.

Arithmetic mean is one of the most frequently used ways of aggregation in MCDM methods like PMM. It is also common practice to combine several decisions in this way. A set of referees (called DMs) evaluate each alternative, and the average of their scores is used to determine each alternative's final score.

The purpose of this study is to introduce a novel way for alternative ranking with fuzzy data for group decision making utilizing the PMM method. The proposed method involves ranking the alternatives and choosing the optimal one based on each DM's individual decision information. The transformation of the choice matrices provided by the decision makers into aggregated matrices of alternatives is the crucial step in this methodology. The evaluations of each alternative regarding each criterion made by each decision maker are organized into a matrix that corresponds to each alternative.

The optimal decision matrix / ideal solution vector in this approach is a matrix made up of maximal assessments since all individual decision matrices are earlier normalized regarding the type of criterion. In contrast to the traditional PMM and the technique based on the accumulation of the individual decisions made by each DM, the distances of alternatives from the ideal solution are the distances between matrices. The best alternative is identified after a ranking of the alternatives is made using the proximity measure value of each to the ideal solution.

A. Fuzzy Sets

Fuzzy sets (FS) were introduced to address imprecision and uncertainty in real-life problems, and many extensions of fuzzy set theory were developed and used in decision making processes [54-103].

Definition 1.[54] A fuzzy set A on a universe $X=\{x_1,x_2,...,x_n\}$ is an object of the form:

$$A=\{<x_i,\mu_A(x_i)> \mid \forall x_i \in X\} \tag{1}$$

where $\mu_A(x_i)\in[0,1]$ is called the degree of membership of an element x_i to X, $\mu_{\bar{A}}(x_i)=1-\mu_A(x_i)\in[0,1]$ is called the degree of nonmembership of an element x_i to X, and $\mu_A(x_i)\in[0,1]$, and $\mu_{\bar{A}}(x_i)=1-\mu_A(x_i)\in[0,1]$ satisfy the following condition:

$$A=\{<x_i,\mu_A(x_i),1-\mu_A(x_i)> \mid \forall x_i \in X\} \tag{2}$$

$$\mu_A(x_i)+1-\mu_A(x_i)=1 \mid \forall x_i \in X \tag{3}$$

In a fuzzy set, the degree of indeterminacy or hesitation of element $x_i \in X$ to set A is $\pi_A(x_i)=1-\mu_A(x_i)-\nu_A(x_i)=0$.

Definition 2. The support of a fuzzy set A is the ordinary subset of X supp $A=\{\mu_A(x_i)>0 \mid x\in X\}$.

Definition 3. A fuzzy set A is normalized iff $\exists \in X \mid \mu_A(x_i)=1$.

Definition 4. A fuzzy set A is convex

$$iff\ \forall x_1,x_2\in\mathbb{R},\lambda\in[0,1]\,\mu_A(\lambda x_1)+(1-\lambda)x_2)\geq\min(\mu_A(x_1),\mu_A(x_2)).$$

Definition 5. A fuzzy number A is a convex, normalized fuzzy subset of the real line $\mathbb{R}$ such that:

a) there exists exactly one $x_0 \in \mathbb{R}, \mu_A(x_0) = 1$ (x_0 is called the mean value of A,

b) $\mu_A(x_i)$ piecewise continuous.

If fuzzy subset A of the real line $\mathbb{R}$ is convex and normalized, its membership function is piecewise continuous, and there exists more than one element $x_0 \in \mathbb{R}, \mu_A(x_0) = 1$ then A is called a flat fuzzy number.

In many practical applications of fuzzy numbers, positive triangular fuzzy numbers are used. Fig. 1 shows the characteristic points of such numbers, which describe them uniquely. A triangular fuzzy number is represented as a triplet A positive triangular fuzzy number A is represented as a triplet $A = (a_A, b_A, c_A)$, and $0 \le a_A \le b_A \le c_A$, and its membership function is of the form.

$$\mu_A(x) = \begin{cases} 0 & for \quad x \le a_A \\ \dfrac{x - a_A}{b_A - a_A} & for \quad a_A \le x \le b_A \\ \dfrac{c_A - x}{c_A - b_A} & for \quad b_A \le x \le c_A \\ 0 & for \quad x \ge c_A \end{cases} \tag{4}$$

$$f(x, a_A, b_A, c_A) = \max\left(\min\left(\frac{x - a_A}{b_A - a_A}, \frac{c_A - x}{c_A - b_A}, 0 \right) \right)$$

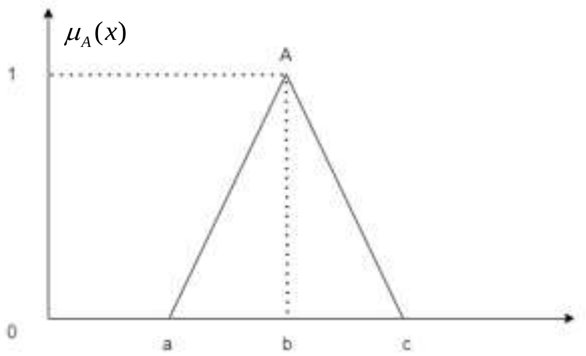

Fig. 1 A triangular positive fuzzy number A

Definition 6. [59] Let $a_i = (a_1, a_2, a_3)$, and $b_i = (b_1, b_2, b_3)$ be the two triangular fuzzy numbers. The main arithmetic operations between two triangular fuzzy number are described as

Addition

$$a + b = (a_1 + b_1, a_2 + b_2, a_3 + b_3), a_1 \ge 0, b_1 \ge 0$$

Multiplication

$$a\,x\,b = (a_1xb_1, a_2xb_2, a_3xb_3), a_1 \ge 0, b_1 \ge 0$$

Subtraction

$$a - b = (a_1 - b_1, a_2 - b_2, a_3 - b_3), a_1 \ge 0, b_1 \ge 0$$

Division

$$a / b = (a_1 / b_1, a_2 / b_2, a_3 / b_3), a_1 \ge 0, b_1 \ge 0$$

Inverse of a triangular fuzzy number

$$a^{-1} = (1/a_1, 1/a_2, 1/a_3), a_1 \ge 0$$

Scaler Multiplication

$$\lambda\,x\,a = (\lambda x a_1, \lambda x a_2, \lambda x a_3), a \ge 0, \lambda \ge 0$$

Symmetric image

$$a = (-a_1, -a_2, -a_3), a_1 \ge 0$$

Definition 7. Let $X = \{x_1, x_2, ..., x_n\}$ be a non-empty set in the unit interval [0,1] and fuzzy sets $A = \{< x_i, \mu_A(x_i) > \mid \forall x_i \in X\}$ and $B = \{< x_i, \mu_B(x_i) > \mid \forall x_i \in X\}$ are of the form. Then, fuzzy set aggregative operators are defined as

Union: $A \vee B = \max\{\mu_A(x_i), \mu_B(x_i)\}$
Intersection: $A \wedge B = \min\{\mu_A(x_i), \mu_B(x_i)\}$
Complement: $\mu_{\bar{A}}(x_i) = 1 - \mu_A(x_i)$

Definition 8. Given any real number k and a triangular fuzzy number a, the operations of the two numbers are given as

(1) Multiplication of a triangular fuzzy number by a constant

$$k * a = (ka_1, ka_2, ka_3), a_1 \ge 0, k \ge 0$$

(2) Division of a triangular fuzzy number by a constant

$$a / k = (a_1 / k, a_2 / k, a_3 / k), a_1 \ge 0, k \ge 0$$

(3) Division of a constant by a triangular fuzzy number

$$k / a = (k / a_1, k / a_2, k / a_3), a_1 \ge 0, k \ge 0$$

Definition 9. Given two triangular fuzzy numbers (a, b) and any real number k, the commutative operations of these two numbers are expressed as

$$a + b = b + a, a \ge 0, b \ge 0, k \ge 0$$

$$a\,x\,b = b\,x\,a, a \ge 0, b \ge 0, k \ge 0$$

$$a - b = b - a, a \ge 0, b \ge 0, k \ge 0$$

$$k * a = a * k, a \ge 0, k \ge 0$$

Definition 10. Let $a_i = (a_1, a_2, a_3)$, and $b_i = (b_1, b_2, b_3)$ be the two triangular fuzzy numbers (Fig.2). The distance between them using the vertex method is given as

$$d(a,b) = \left(\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - c_3)^2] \right)^{1/2} \quad (5)$$

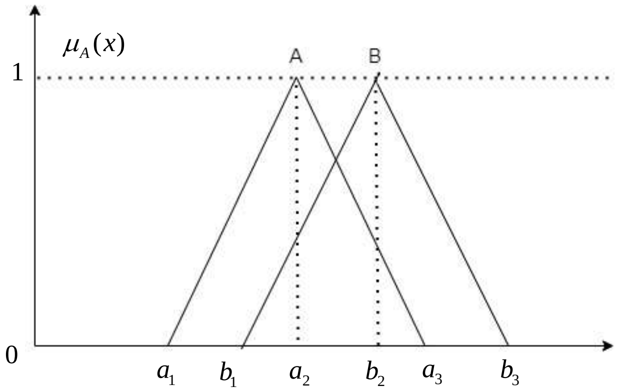

Fig. 2 Two triangular fuzzy numbers

Definition 11. The defuzzification of a triangular fuzzy number $a_i = (a_1, a_2, a_3)$ is given as

$$J(a_i) = \frac{[(a_3 - a_1) + (a_2 - a_1)]}{3} + a_1 \quad (6)$$

B. Linguistic variables and fuzzy set theory

In fuzzy set theory, conversion scales are applied to transform the linguistic variables into fuzzy numbers. The conversion scales are used to rate the criteria and the alternatives. Table 1 presents the linguistic variables and fuzzy ratings used for the criteria and Table 2 presents the linguistic variables and fuzzy ratings used for the alternatives.

Table 1. Linguistic variables for criteria ratings

Linguistic variable	Fuzzy number
Very Low (VL)	(0.0, 0.0, 0.1)
Low (L)	(0.0, 0.1, 0.3)
Medium Low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium High (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1)
Very High (VH)	(0.9, 1, 1)

Table 2. Linguistic variables for alternative ratings

Linguistic variable	Fuzzy number
Very Poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium Poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium Good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very Good (VG)	(9, 10, 10)

C. PMM (Proximity Measure Method)

PMM (proximity measure method) is an effective MCDM method for multiple criteria decision making. The technique determines the optimum alternative based on proximity measure value to the ideal solution. The following are the steps of the PMM methodology [26-27]:

Step 1. Decision matrix is established.

The decision matrix $X = [x_{ij}]_{mxn}$ for the alternatives (a_i), the decision criteria (c_j), and the criteria weights (ω_j) is constructed as

MCDM Model	ω_1	ω_2	$\cdots$	ω_j	$\cdots$	ω_n
	c_1	c_2	$\cdots$	c_j	$\cdots$	c_n
a_1	x_{11}	x_{12}	$\cdots$	x_{1j}	$\cdots$	x_{1n}
a_2	x_{21}	x_{22}	$\cdots$	x_{2j}	$\cdots$	x_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_i	x_{i1}	x_{i2}	$\cdots$	x_{ij}	$\cdots$	x_{in}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_m	x_{m1}	x_{m2}	$\cdots$	x_{mj}	$\cdots$	x_{mn}

Step 2. Decision matrix is normalized.

The raw data are normalized using vector scale transformation to bring the various criteria scales into a comparable scale. The normalized decision matrix $R = [r_{ij}]_{mxn}$ is given as

$$r_{ij} = \frac{x_{ij}}{\left(\sum_{i=1}^{m} x_{ij}^2 \right)^{1/2}} \quad (7)$$

where r_{ij} represents the normalized criteria rating.

Step 3. Weighted normalized decision matrix is determined.

The weighted normalized matrix $V = [v_{ij}]_{mxn}$ for criteria is computed by multiplying the weights (ω_j) of evaluation criteria with the normalized decision matrix r_{ij}

$$v = r_{ij}\omega_j, i = 1, ..., m; j = 1, ..., n \quad (8)$$

Step 4. Ideal solution is obtained.

The ideal solution of the alternatives is computed as follows:

$$v^* = \left\{ v_1^*, ..., v_n^* \right\} = \left\{ \max_i v_{ij} \mid \Omega_b, \min_i v_{ij} \mid \Omega_c \right\} \quad (9)$$

where $i = 1,...,m; j = 1,...,n$, Ω_b denotes the benefit type criterion, whereas Ω_c denotes the cost type criterion.

Step 5. The distance of each alternative from ideal solution is determined. The distance $d_i(v_{ij}, v_j^*)$ of each weighted alternative $i = 1,2,..,m$ from the ideal solution is computed as

$$d_i(v_{ij}, v_j^*) = \sum_{j=1}^{n} d_v(v_{ij}, v_{ij}^*), \ i = 1,2,..,m \tag{10}$$

where $d_v(v_{ij}, v_{ij}^*)$ is the distance measure between two fuzzy numbers v_{ij} and v_{ij}^*. The Minkowski distance family is given as

$$d_i = \left(\frac{1}{n} \sum_{j=1}^{n} | v_{ij} - v_j^* |^{\delta} \right)^{1/\delta}, i = 1,...,m; j = 1,...,n \tag{11}$$

where $\delta = \{1,2,3,\infty\}$, $\delta = 1$ denotes Manhattan distance, $\delta = 2$ denotes Euclidean distance, $\delta = 3$ denotes Minkowski distance. Hausdorff distance is the maximum distance of a set to the nearest point in the other set.Hausdorff distance from set *A* to set *B* is a maximin function, defined as where *a* and *b* are points of sets *A* and *B* respectively, and $d(a,b)$ is any metric between these points. The Hausdorff distance is given as

$$d_i(A,B) = \max_{a \in A} \left\{ \min_{b \in B} \left\{ d(a,b) \right\} \right\} \tag{12}$$

Step 5. Compute the proximity measure value d_i of each alternative. The alternative having the least proximity measure value of d_i is identified as the best alternative d_i^*.

$$d_i^* = \min d_i = d(\arg \min_{i \in I} d_i) \tag{13}$$

Step 6. Correlation analysis of the ranking orders of alternatives is carried out using the Spearman correlation coefficient that is defined as the Pearson correlation coefficient between the rank variables. The *n* raw scores { x_i , y_i } are converted to ranks { $R(x_i)$, $R(y_i)$ }, and r_s is computed as

$$r_s = \rho_{R(x_i)}, \rho_{R(y_i)} = \frac{\operatorname{cov}(R(x_i), R(y_i))}{\rho_{R(x_i)}, \rho_{R(y_i)}} \tag{14}$$

where ρ denotes the Pearson correlation coefficient applied to the rank variables; $\operatorname{cov}(R(x_i), R(y_i))$ is the covariance of the rank variables, $\rho_{R(x_i)}, \rho_{R(y_i)}$ are the standard deviations of the rank variables.

D.Fuzzy PMM (Proximity Measure Method)

The steps of fuzzy PMM can be expressed as follows:

Step 1. The importance of criteria and the ratings of alternatives with respect to the criteria are evaluated. The decision makers $D_k = \{D_1, D_2, D_3\}$ evaluate each criterion $C_j = \{C_1, C_2, .., C_j\}$ by using linguistic variables as shown in Table 1 and rate the alternatives according to Table 2.

Step 2. Linguistic terms are transformed into triangular fuzzy numbers by benefiting from the Table 1. Rating of alternatives (x_{ij}) and the importance of the criteria (ω_j) are obtained as

$$x_{ij} = \frac{1}{K}\left(x_{ij}^1 + x_{ij}^2 + ... + x_{ij}^K \right), x_{ij} = \left(a_{ij}, b_{ij}, c_{ij} \right) \tag{15}$$

$$\omega_j = \frac{1}{K}\left(\omega_j^1 + \omega_j^2 + ... + \omega_j^K \right), \omega_j = \left(\omega_{j1}, \omega_{j2}, \omega_{j3} \right) \tag{16}$$

Step 3. Normalization of fuzzy decision mattix is performed by using the linear scale transformation $R = [r_{ij}]_{mxn}$. The related linear data transformations are shown as

If criterion is benefit Ω_b, then $c^* = \max_i c_{ij}$ is used for data transformation.

$$r_s = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*} \right) \tag{17}$$

if criterion is cost Ω_c, and $a_j^- = \max_i a_{ij}$ is used for data transformation.

$$r_s = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}} \right) \tag{18}$$

Step 4. Firstly, aggregated weight matrix is obtained then the weighted normalized fuzzy decision matrix is found as

$$V = [v_{ij}]_{mxn}, i = 1,2,...,m; j = 1,...,n$$

$$v_{ij} = r_{ij}\omega_j \tag{19}$$

Step 4. Fuzzy ideal solution is obtained.

The fuzzy ideal solution of the alternatives is computed as

$$v^* = \left\{ v_1^*, ..., v_n^* \right\} = \left\{ \max_i v_{ij} \mid i = 1,...,m; j = 1,...,n \right\} \tag{20}$$

where $v_j^* = \max_i v_{ij} = (1,1,1)$.

Step 5. The distance of each alternative from fuzzy ideal solution is determined. The distance $d_i(v_{ij}, v_j^*)$ of each weighted alternative $i = 1,2,..,m$ from the fuzzy ideal solution is computed as

$$d_i(\nu_{ij}, \nu_j^*) = \sum_{j=1}^{n} d_\nu(\nu_{ij}, \nu_{ij}^*), \ i = 1,2,..,m \tag{21}$$

where $d_\nu(\nu_{ij}, \nu_{ij}^*)$ is the distance measure between two fuzzy numbers ν_{ij} and v_{ij}^*. The Minkowski distance family is given as

$$d_i = \left(\frac{1}{n} \sum_{j=1}^{n} | \nu_{ij} - \nu_j^* |^\delta \right)^{1/\delta}, i = 1,...,m; j = 1,...,n \tag{22}$$

where $\delta = \{1,2,3,\infty\}$, $\delta = 1$ denotes Manhattan distance, $\delta = 2$ denotes Euclidean distance, $\delta = 3$ denotes Minkowski distance. Hausdorff distance is the maximum distance of a set to the nearest point in the other set.Hausdorff distance from set *A* to set *B* is a maximin function, defined as where *a* and *b* are points of sets *A* and *B* respectively, and $d(a,b)$ is any metric between these points. The Hausdorff distance is given as

$$d_i(A,B) = \max_{a \in A} \left\{ \min_{b \in B} \left\{ d(a,b) \right\} \right\} \tag{23}$$

$$d_i = \left(\frac{1}{n} \sum_{j=1}^{n} \max \left\{ | a_{ij} - a_j^* |, | b_{ij} - b_j^* |, | c_{ij} - c_j^* | \right\} \right) \tag{24}$$

Step 5. Compute the proximity measure value d_i of each alternative.The alternative having the least proximity measure value of d_i is identified as the best alternative d_i^*.

$$d_i^* = \min d_i = d(\arg \min_{i \in I} d_i) \tag{25}$$

Step 6. Correlation analysis of the ranking orders of alternatives is carried out using the Spearman correlation coefficient that is defined as the Pearson correlation coefficient between the rank variables. The *n* raw scores { x_i , y_i } are converted to ranks { $R(x_i)$, $R(y_i)$ }, and r_s is computed as

$$r_s = \rho_{R(x_i)}, \rho_{R(y_i)} = \frac{\operatorname{cov}(R(x_i), R(y_i))}{\rho_{R(x_i)}, \rho_{R(y_i)}} \tag{26}$$

where ρ denotes the Pearson correlation coefficient applied to the rank variables; $\operatorname{cov}(R(x_i), R(y_i))$ is the covariance of the rank variables, $\rho_{R(x_i)}, \rho_{R(y_i)}$ are the standard deviations of the rank variables.

III. Application

In this section, the fuzzy PMM approach is applied to the MCDM problem. The proposed framework for evaluating aircraft supplier selection under uncertainty consists of three steps.

1. Selection of evaluation criteria.
2. Evaluation and selection of best alternative using selected criteria.
3. Conduct sensitivity analysis to determine the influence of criteria weights on decision making.

These steps are presented in detail as follows.

a) Criteria selection

The first step involves selection of criteria for evaluating aircraft supplier selection problem. The criteria were identified from literature review [27], discussion with transportation experts and practical experience with supply chain management. The final list contains five criteria Cost (C1), Quality (C2), Delivery Time (C3), Economic Status (C4), and Institutionalization (C5). The criterion C1 is the cost (Ω_c) category criteria that is, the lower the value, the more optimal the alternative (or aircraft supplier). The remaining criteria are benefit (Ω_b) type criteria, that is, the higher the value, the more sustainable the alternative (or aircraft supplier).

b) Alternatives evaluation and selection using fuzzy PMM

The second step involves allocation of linguistic ratings to the five criteria and the potential alternatives for each of the criteria by the decision makers or experts. The criteria ratings are provided from Table 2 and the alternative ratings for each of the criteria from Table 1. The linguistics terms are then transformed to fuzzy triangular numbers. Then, fuzzy PMM is applied to aggregate the criteria and the alternative ratings to generate an overall score for assessing the performance of the alternatives (or aircraft suppliers). The alternative with the lowest score is selected as the best alternative for the fleet planning and recommended for airline acquisition.

c) Validity analysis

To determine the value of different distance measures in choosing the best option among the various options, the validity analysis uses Hausdorff and Minskowski distance family measures (or aircraft suppliers). The validity analysis examines the overall decision to several distance metrics in the individual alternative comparison process to determine its validity. Changing the distance measures and analyzing how they impact the decision will reveal the solution to this MCDM problem. This is helpful when there are uncertainties surrounding the relative importance of various distance measures. A validity analysis is carried out in this instance to determine the significance of distance measures in choosing the best alternative from the given alternatives (or aircraft suppliers).

d) Numerical illustration

Let us assume that an airline fleet planning group is interested in implementing an enlargement of the fleet capacity. The aircraft supplier alternatives provide short, medium, and long-haul aircraft types in the civil aviation sector.

Let us assume that an expert committee of three decision makers D1, D2 and D3 is formed to select the best aircraft supplier alternative. The five criteria used for evaluation are already identified. The committee used linguistic assessments (Tables 1 and 2) to rate the five criteria and the three aircraft supplier alternatives (A_i): (A1), (A2), (A3). The assessment results are shown in Tables 3 and 4 respectively.

The steps of the fuzzy PMM are performed as follows:

Step 1. Evaluation of the criteria and alternative ratings with respect to the criteria assessed by the decision makers are shown in the Tables 3 and 4.

Table 3. The evaluation of decision makers for the importance weight of the criteria

Criteria	D1	D2	D3
C1(Cost)	H	VH	H
C2(Quality)	VH	H	VH
C3 (Delivery Time)	M	M	VH
C4(Economic Status)	MH	VH	MH
C5(Institutionalization	H	M	H

Table 4.The evaluation of decision makers for alternative ratings

		C1	C2	C3	C4	C5
D1	A1	MG	VG	VG	MG	MP
	A2	VG	MP	G	G	MG
	A3	MP	MG	MP	VG	G
D2	A1	VG	MG	VG	G	VG
	A2	MG	VG	G	F	MP
	A3	G	MP	MP	MP	F
D3	A1	VG	F	MG	G	MP
	A2	MG	MG	G	MP	G
	A3	MP	G	MG	G	MG

Step 2. Linguistic terms are transformed into triangular fuzzy numbers, and fuzzy decision matrix and fuzzy weights of criteria are, respectively, shown as in Table 5 and Table 6.

Table 5. Fuzzy weights of criteria

Criteria	D1	D2	D3	ω_J
C1	0.7, 0.9, 1	0.9, 1, 1	0.7, 0.9, 1	0.77,.93,1.00
C2	0.9, 1, 1	0.7, 0.9, 1	0.9, 1, 1	0.83,0.97,1.00
C3	0.3, 0.5, 0.7	0.3, 0.5, 0.7	0.9, 1, 1	0.50,0.67,0.80
C4	0.5, 0.7, 0.9	0.9, 1, 1	0.5, 0.7, 0.9	0.63,0.80,0.93
C5	0.7, 0.9, 1	0.3, 0.5, 0.7	0.7, 0.9, 1	0.57,0.77,0.90

Table 6. Fuzzy decision matrix

		C1	C2	C3	C4	C5
D1	A1	5, 7, 9	9, 10, 10	9, 10, 10	5, 7, 9	1, 3, 5
	A2	9, 10, 10	1, 3, 5	7, 9, 10	7, 9, 10	5, 7, 9
	A3	1, 3, 5	5, 7, 9	1, 3, 5	9, 10, 10	7, 9, 10
D2	A1	9, 10, 10	5, 7, 9	9, 10, 10	7, 9, 10	9, 10, 10
	A2	5, 7, 9	9, 10, 10	7, 9, 10	3, 5, 7	1, 3, 5
	A3	7, 9, 10	1, 3, 5	1, 3, 5	1, 3, 5	3, 5, 7
D3	A1	9, 10, 10	3, 5, 7	5, 7, 9	7, 9, 10	1, 3, 5
	A2	5, 7, 9	5, 7, 9	7, 9, 10	1, 3, 5	7, 9, 10
	A3	1, 3, 5	7, 9, 10	5, 7, 9	7, 9, 10	5, 7, 9

Step 3. Normalization of fuzzy decision matrix is performed as shown in Table 7.

Table 7. Normalized fuzzy decision matrix

		C1	C2	C3	C4	C5
D1	A1	0.2, 0.14, 0.11	0.9, 1, 1	0.9, 1, 1	0.5, 0.7, 0.9	0.1, 0.3, 0.5
	A2	0.11,0.10,0.10	0.1, 0.3, 0.5	0.7, 0.9, 1	0.7, 0.9, 1	0.5, 0.7, 0.9
	A3	1, 0.33, 0.20	0.5, 0.7, 0.9	0.1, 0.3, 0.5	0.9, 1, 1	0.7, 0.9, 1
D2	A1	0.56, 0.5, 0.5	0.5, 0.7, 0.9	0.9, 1, 1	0.7, 0.9, 1	0.9, 1, 1
	A2	1, 0.71, 0.56	0.9, 1, 1	0.7, 0,9, 1	0.3, 0.5, 0.7	0.1, 0.3, 0.5
	A3	0.71,0.56,0.5	0.1, 0.3, 0.5	0.1, 0.3, 0.5	0.1, 0.3, 0.5	0.3, 0.5, 0.7
D3	A1	0.11, 0.1, 0.1	0.3, 0.5, 0.7	0.5, 0.7, 0.9	0.7, 0.9, 1	0.1, 0.3, 0.5
	A2	0.20,0.14,0.11	0.5, 0.7, 0.9	0.7, 0.9, 1	0.1, 0.3, 0.5	0.7, 0.9, 1
	A3	1, 0.33, 0.2	0.7, 0.9, 1	0.5, 0.7, 0.9	0.7, 0.9, 1	0.5, 0.7, 0.9

Step 4. The weighted normalized fuzzy decision matrix is calculated as shown in Table 8.

Table 8. Weighted normalized fuzzy decision matrix

		C1	C2	C3	C4	C5
D1	A1	0.15,0.13,0.11	0.75,0.97,1.00	0.45,0.67,0.80	0.32,0.56,0.84	0.06,0.23,0.45
	A2	0.09,0.09,0.10	0.08,0.29,0.50	0.35,0.60,0.80	0.44,0.72,0.93	0.28,0.54,0.81
	A3	0.77,0.31,0.20	0.42,0.68,0.90	0.05,0.20,0.40	0.57,0.80,0.93	0.40,0.69,0.90
D2	A1	0.43,0.47,0.50	0.42,0.68,0.90	0.45,0.67,0.80	0.44,0.72,0.93	0.51,0.77,0.90
	A2	0.77,0.67,0.56	0.75,0.97,1.00	0.35,0.60,0.80	0.19,0.40,0.65	0.06,0.23,0.45
	A3	0.55,0.52,0.50	0.08,0.29,0.50	0.05,0.20,0.40	0.06,0.24,0.47	0.17,0.38,0.63
D3	A1	0.09,0.09,0.10	0.25,0.48,0.70	0.25,0.47,0.72	0.44,0.72,0.93	0.06,0.23,0.45
	A2	0.15,0.13,0.11	0.42,0.68,0.90	0.35,0.60,0.80	0.06,0.24,0.47	0.40,0.69,0.90
	A3	0.77,0.31,0.20	0.58,0.87,1.00	0.25,0.47,0.72	0.44,0.72,0.93	0.28,0.54,0.81

Step 5. Aggregated weighted normalized decision matrix for the alternatives is shown in Table 9.

Table 9. Aggregated weighted normalized decision matrix for the alternatives

	C1	C2	C3	C4	C5
A1	0.22,0,23,0.24	0.47,0.71,0.87	0.38,0,60,0.77	0.40,0.67,0.90	0.21,0.41,0.60
A2	0.20,0.22,0.23	0.25,0.48,0.70	0.35,0.58,0.77	0.44,0.72,0.93	0.28,0.51,0.72
A3	0.31,0.28,0.25	0.36,0.58,0.73	0.32,0.56,0.77	0.36,0.61,0.84	0.13,0.33,0.57

Step 6. The fuzzy ideal solution vector v^* is determined as shown in Table 10.

Table 10. Fuzzy ideal solution vector

	C1	C2	C3	C4	C5
v^*	0.31,0.28,0.25	0.47,0.71,0.87	0.38,0.60,0.77	0.44,0.72,0.93	0.28,0.51,0.72

Step 7. The fuzzy proximity measure d_i for all the alternatives is calculated and shown in Table 11. The distances from fuzzy ideal solutions are calculated.

Proximity measure values d_i and ranking order patterns R_i of aircraft supplier alternatives are shown in Table 11. Regarding the proximity measure values d_i, the best aircraft supplier d_i^* is A1.

Table 11. The distances from fuzzy ideal solutions v^*. Proximity measure values d_i and ranking order patterns R_i of aircraft supplier alternatives

	d_1	R_i	d_2	R_i	d_3	R_i	d_∞	R_i	d_H	R_i
A1	0.194	1	0.206	1	0.214	1	0,033	1	0,088	1
A2	0.290	2	0.307	2	0.317	2	0,068	3	0,124	2
A3	0.416	3	0.427	3	0.432	3	0,053	2	0,162	3

Step 8. The correlation analysis of the ranking order patterns of alternatives is given as shown in Table 12 and the graphical representation of the ranking order patterns of aircraft supplier alternatives is given as shown in Fig. 3.

Table 12. Correlation analysis of the ranking order patterns of alternatives

	d_1	d_2	d_3	d_∞	d_H
d_1	1				
d_2	1	1			
d_3	1	1	1		
d_∞	0,50	0,50	0,50	1	
d_H	1	1	1	0,50	1

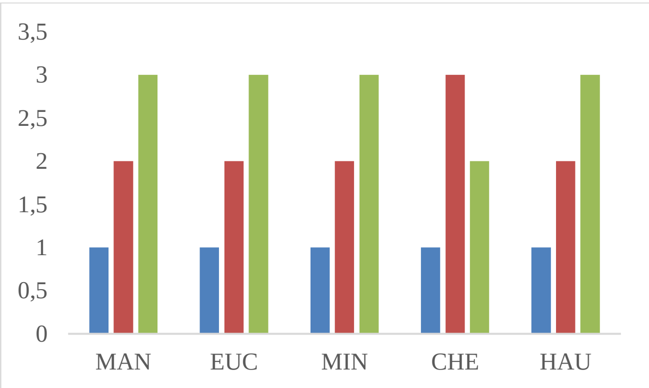

Fig. 3 Ranking order patterns of aircraft supplier alternatives

Evaluating the fuzzy PMM - MCDM ranking analysis of aircraft supplier alternatives, it is seen that the best aircraft supplier is alternative A1, which is ranked as first by five distance functions. The aircraft supplier A2 is selected as the second alternative, while aircraft supplier A3 is selected as the worst alternative. Finally, the effectiveness and viability of the proposed method was validated in fuzzy ranking analysis.

IV. Conclusion

The supplier selection process is one of the most crucial supply chain management processes. Using a sound supplier selection approach is essential for creating a sustainable system. The proposed fuzzy MCDM technique considers several variables, as opposed to traditional methods that simply consider cost, such as cost, quality, delivery time, **e**conomic status**,** and institutionalization.

Based on the triangular fuzzy numbers for linguistic variables, a Minkowski distance family function and Hausdorff distance function on fuzzy set have been constructed in this study. Numerical example is used to demonstrate the distinctiveness and benefit of the proposed distance metrics in the challenge of choosing an aircraft supplier. Comparing ranking order patterns of the alternatives is made easier by the provided distance measures.

References

[1] Ghodsypour, S. H., O'Brien C. (1998). A decision support system for supplier selection using an integrated analytic hierarchy process and linearprogramming.*International Journal of Production Economics,* 5 6-57, 199-212.

[2] Weber, C. A., Current, J. R., , Benton, W. C. (1991). Vender selection criteria and methods. *European Journal of Operational Research*, 50, 2-18.

[3] Degraeve, Z., Labro, E., Roodhooft, F. (2000). An evaluation of supplier selection methods from a total cost of ownership perspective. *European Journal of Operational Research*, 125(1), 34-59.

[4] De Boer, L., Labro, E., Morlacchi, P. (2001). A review of methods supporting supplier selection *European Journal of Purchasing & Supply Management*, 7, 75-89.

[5] Ho, W., Xu, X. D., Prasanta K. (2010). Multi-criteria decision making approaches for supplier evaluation and selection: A literature review. *European Journal of Operational Research,* 202, 16-24.

[6] Sanayei, A., Mousavi, S. F., Yazdankhah, A. (2010). Group decision making process for supplier selection with VIKOR under fuzzy environment. *Expert Systems with Applications*, 37, 24-30.

[7] Chen,C. T., Lin,C. T., Huang, S. F.(2006). A fuzzy approach for supplier evaluation and selection in supply chain management, *International Journal of Production Economics*, vol. 102(2), 289–301.

[8] Min, H. (1994). International supplier selection: a multi-attribute utility approach, International Journal of Physical Distribution and Logistics Management, vol. 24(5), 24–33.

[9] Boran, FE., Genç, S., Kurt, M., Akay, D., (2009). A Multi-Criteria Intuitionistic Fuzzy Group Decision Making For Supplier Selection With TOPSIS Method", *Expert Systems with Applications*, 36(8), pp.11363-11368, 2009.

[10] Izadikhah, M. (2012). Group Decision Making Process for Supplier Selection with TOPSIS Method under Interval-Valued Intuitionistic Fuzzy Numbers, *Advances in Fuzzy Systems*, vol. 2012, Article ID 407942.

[11] Saaty, T. L. (1990). How to make a decision: The Analytic Hierarchy Process. *European Journal of Operational Research*, 48(1), 9-26.

[12] Saaty, T. L. (2008). Decision making with the analytic hierarchy process. *International Journal of Services Sciences*, 1(1), 83-98.

[13] Buckley,J.J. (1985). Fuzzy hierarchical analysis, *Fuzzy Sets and Systems*, 17, 233–247.

[14] Dyer, J.S. (2016). Multiattribute Utility Theory (MAUT). In: Greco, S., Ehrgott, M., Figueira, J. (eds) Multiple Criteria Decision Analysis. *International Series in Operations Research & Management Science*, vol 233. Springer, New York, NY. https://doi.org/10.1007/978-1-4939-3094-4_8.

[15] Hwang, C.L.; Yoon, K. (1981). Multiple Attribute Decision Making: Methods and Applications. New York: Springer-Verlag.

[16] Chu, T.C. (2002. Facility location selection using fuzzy TOPSIS under group decisions, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, Vol. 10 No. 6, pp. 687-701.

[17] Opricovic, S. (2007). A fuzzy compromise solution for multicriteria problems. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 15(3), 363–380.

[18] Opricovic, S., Tzeng, G.-H. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, 156(2), 445–455.

[19] Roy, B. (1991). The outranking approach and the foundation of ELECTRE methods. *Theory and Decision*, 31(1), 49–73.

[20] Fei, L., Xia, J., Feng, Y., Liu, L. (2019) An ELECTRE-Based Multiple Criteria Decision Making Method for Supplier Selection Using Dempster-Shafer Theory. *IEEE Access*, 7, 84701-84716.

[21] Brans JP., Mareschal B. (2005). Promethee Methods. In: Multiple Criteria Decision Analysis: State of the Art Surveys. *International Series in Operations Research & Management Science*, vol 78, pp 163-

186. Springer, New York, NY. https://doi.org/10.1007/0-387-23081-5_5.
[22] Brans, J., Ph. Vincke. (1985). A Preference Ranking Organisation Method: (The PROMETHEE Method for Multiple Criteria Decision-Making). *Management Science*, 31(6), 647-656.
[23] Brans, J.P., Macharis, C., Kunsch, P.L., Chevalier, A., Schwaninger, M., (1998). Combining multicriteria decision aid and system dynamics for the control of socio-economic processes. An iterative real-time procedure. *European Journal of Operational Research* 109, 428-441.
[24] Brans, J.P., Vincke, Ph., Mareschal, B., (1986). How to select and how to rank projects: the PROMETHEE method. *European Journal of Operational Research*, 24, 228-238.
[25] Taherdoost, H., Madanchian, M. (2023). Multi-Criteria Decision Making (MCDM) Methods and Concepts. *Encyclopedia*, 3(1), 77–87.
[26] Ardil, C. (2023). Aircraft Supplier Selection using Multiple Criteria Group Decision Making Process with Proximity Measure Method for Determinate Fuzzy Set Ranking Analysis. *International Journal of Industrial and Systems Engineering*, 17(3), 127 - 135.
[27] Ardil, C. (2023). Determinate Fuzzy Set Ranking Analysis for Combat Aircraft Selection with Multiple Criteria Group Decision Making. *International Journal of Computer and Information Engineering*, 17(3), 272 - 279.
[28] Ardil, C. (2019). Fighter Aircraft Selection Using Technique for Order Preference by Similarity to Ideal Solution with Multiple Criteria Decision Making Analysis. *International Journal of Transport and Vehicle Engineering*, 13(10), 649 - 657.
[29] Ardil, C. (2019). Aircraft Selection Using Multiple Criteria Decision Making Analysis Method with Different Data Normalization Techniques. *International Journal of Industrial and Systems Engineering*, 13(12), 744 - 756.
[30] Ardil, C. (2019). Military Fighter Aircraft Selection Using Multiplicative Multiple Criteria Decision Making Analysis Method. *International Journal of Mathematical and Computational Sciences*, 13(9), 184 - 193.
[31] Ardil, C. (2020). A Comparative Analysis of Multiple Criteria Decision Making Analysis Methods for Strategic, Tactical, and Operational Decisions in Military Fighter Aircraft Selection. *International Journal of Aerospace and Mechanical Engineering*, 14(7), 275 - 288.
[32] Ardil, C. (2020). Aircraft Selection Process Using Preference Analysis for Reference Ideal Solution (PARIS). *International Journal of Aerospace and Mechanical Engineering*, 14(3), 80 - 93.
[33] Ardil, C. (2020). Regional Aircraft Selection Using Preference Analysis for Reference Ideal Solution (PARIS). *International Journal of Transport and Vehicle Engineering*, 14(9), 378 - 388.
[34] Ardil, C. (2020). Trainer Aircraft Selection Using Preference Analysis for Reference Ideal Solution (PARIS). *International Journal of Aerospace and Mechanical Engineering*, 14(5), 195 - 209.
[35] Ardil, C. (2021). Advanced Jet Trainer and Light Attack Aircraft Selection Using Composite Programming in Multiple Criteria Decision Making Analysis Method. *International Journal of Aerospace and Mechanical Engineering*, 15(12), 486 - 491.
[36] Ardil, C. (2021). Airline Quality Rating Using PARIS and TOPSIS in Multiple Criteria Decision Making Analysis. *International Journal of Industrial and Systems Engineering*, 15(12), 516 - 523.
[37] Ardil, C. (2021). Comparison of Composite Programming and Compromise Programming for Aircraft Selection Problem Using Multiple Criteria Decision Making Analysis Method. *International Journal of Aerospace and Mechanical Engineering*, 15(11), 479 - 485.
[38] Ardil, C. (2021). Fighter Aircraft Evaluation and Selection Process Based on Triangular Fuzzy Numbers in Multiple Criteria Decision Making Analysis Using the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). *International Journal of Computer and Systems Engineering*, 15(12), 402 - 408.
[39] Ardil, C. (2021). Military Combat Aircraft Selection Using Trapezoidal Fuzzy Numbers with the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). *International Journal of Computer and Information Engineering*, 15(12), 630 - 635.
[40] Ardil, C. (2021). Freighter Aircraft Selection Using Entropic Programming for Multiple Criteria Decision Making Analysis. *International Journal of Mathematical and Computational Sciences*, 15(12), 125 - 132.
[41] Ardil, C. (2021). Neutrosophic Multiple Criteria Decision Making Analysis Method for Selecting Stealth Fighter Aircraft. *International Journal of Aerospace and Mechanical Engineering*, 15(10), 459 - 463.
[42] Ardil, C. (2022). Aircraft Selection Problem Using Decision Uncertainty Distance in Fuzzy Multiple Criteria Decision Making Analysis. *International Journal of Mechanical and Industrial Engineering*, 16(3), 62 - 69.
[43] Ardil, C. (2022). Aircraft Selection Using Preference Optimization Programming (POP).*International Journal of Aerospace and Mechanical Engineering*, 16(11), 292 - 297.
[44] Ardil, C. (2022). Fighter Aircraft Selection Using Fuzzy Preference Optimization Programming (POP). *International Journal of Aerospace and Mechanical Engineering*, 16(10), 279 - 290.
[45] Ardil, C. (2022). Fighter Aircraft Selection Using Neutrosophic Multiple Criteria Decision Making Analysis. *International Journal of Computer and Systems Engineering*, 16(1), 5 - 9.
[46] Ardil, C. (2022). Military Attack Helicopter Selection Using Distance Function Measures in Multiple Criteria Decision Making Analysis. *International Journal of Aerospace and Mechanical Engineering*, 16(2), 20 - 27.
[47] Ardil, C. (2022). Multiple Criteria Decision Making for Turkish Air Force Stealth Fighter Aircraft Selection. *International Journal of Aerospace and Mechanical Engineering*, 16(12), 369 - 374.
[48] Ardil, C. (2022). Vague Multiple Criteria Decision Making Analysis Method for Fighter Aircraft Selection. *International Journal of Aerospace and Mechanical Engineering*, 16(5),133-142.
[49] Ardil, C. (2022).Fuzzy Uncertainty Theory for Stealth Fighter Aircraft Selection in Entropic Fuzzy TOPSIS Decision Analysis Process. *International Journal of Aerospace and Mechanical Engineering*, 16(4), 93 - 102.
[50] Ardil, C. (2023). Fuzzy Multiple Criteria Decision Making for Unmanned Combat Aircraft Selection Using Proximity Measure Method. *International Journal of Computer and Information Engineering*, 17(3), 193 - 200.
[51] Ardil, C. (2023). Unmanned Combat Aircraft Selection using Fuzzy Proximity Measure Method in Multiple Criteria Group Decision Making. *International Journal of Computer and Systems Engineering*, 17(3), 238 - 245.
[52] Ardil, C. (2023). Using the PARIS Method for Multiple Criteria Decision Making in Unmanned Combat Aircraft Evaluation and Selection. *International Journal of Aerospace and Mechanical Engineering*, 17(3), 93 - 103.
[53] Ardil, C. , Pashaev, A. , Sadiqov, R. , Abdullayev, P. (2019). Multiple Criteria Decision Making Analysis for Selecting and Evaluating Fighter Aircraft. *International Journal of Transport and Vehicle Engineering*, 13(11), 683 - 694.
[54] Zadeh, L. A. (1965). Fuzzy sets. *Inf. Control*. 8(3), 338–353.
[55] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning. *Inf. Sci.* 8(3), 199–249.
[56] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-II. *Inf. Sci.* 8(4), 301–357.
[57] Zadeh, L. A. (1975).The concept of a linguistic variable and its application to approximate reasoning-III. *Inf. Sci.* 9(1), 43–80.
[58] Atanassov, K. (1986).Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* 20(1), 87–96.
[59] Awasthi, A., Chauhan, S.S., Omrani, H. (2011). Application of fuzzy TOPSIS in evaluating sustainable transportation systems. *Expert Syst. Appl*., 38, 12270-12280.
[60] Ecer, F., Pamucar, D. (2021).MARCOS technique under intuitionistic fuzzy environment for determining the COVID-19 pandemic performance of insurance companies in terms of healthcare services. *Appl. Sof Comput.* 104, 107199.
[61] Verma, R. (2021). On intuitionistic fuzzy order-alpha divergence and entropy measures with MABAC method for multiple attribute group decision-making. *J. Intell. Fuzzy. Syst. Appl. Eng. Technol.* 40(1), 1191–1217.
[62] Ilbahar, E., Kahraman, C., Cebi, S. (2022). Risk assessment of renewable energy investments: A modifed failure mode and efect analysis based on prospect theory and intuitionistic fuzzy AHP. *Energy* 239, 121907.
[63] Verma, R. , Merig, J. M. (2020). A new decision making method using interval-valued intuitionistic fuzzy cosine similarity measure based on the weighted reduced intuitionistic fuzzy sets. *Informatica* 31(2), 399–433.
[64] Wang, Z., Xiao, F. , Ding, (2022).W. Interval-valued intuitionistic fuzzy Jenson–Shannon divergence and its application in multi-attribute decision making. *Appl. Intell.* 1–17.
[65] Verma, R. , Merigó, J. M. (2021). On Sharma-Mittal's entropy under intuitionistic fuzzy environment. *Cybern. Syst.* 52(6), 498–521.
[66] Zhao, M., Wei, G. , Wei, C. (2021). Extended CPT-TODIM method for interval-valued intuitionistic fuzzy MAGDM and its application to

urban ecological risk assessment. *J. Intell. Fuzzy Syst.* 40(3), 4091–4106.

[67] Liu, P., Pan, Q., Xu, H. (2021). Multi-attributive border approximation area comparison (MABAC) method based on normal q-rung orthopair fuzzy environment. J. Intell. Fuzzy Syst. Appl. *Eng. Technol.* 5, 40.

[68] Atanassov, K. , Gargov, G. (1989).Interval-valued intuitionistic fuzzy sets. *Fuzzy Syst.* 31(3), 343–349.

[69] Hajiagha, S. H. R., Mahdiraji, H. A., Hashemi, S. S. , Zavadskas, E. K. (2015).Evolving a linear programming technique for MAGDM problems with interval valued intuitionistic fuzzy information. *Expert Syst. Appl.* 42(23), 9318–9325.

[70] You, P., Liu, X. H. , Sun, J. B. (2021).A multi-attribute group decision making method considering both the correlation coefficient and hesitancy degrees under interval-valued intuitionistic fuzzy environment. *Inf. Sci.* 104, 107187.

[71] Ye, F. (2010).An extended TOPSIS method with interval-valued intuitionistic fuzzy numbers for virtual enterprise partner selection. *Expert Syst. Appl.* 37(10), 7050–7055.

[72] Chen, X., Suo, C. F. , Li, Y. G. (2021). Distance measures on intuitionistic hesitant fuzzy set and its application in decision-making. *Comput. Appl. Math.* 40(3), 63–84.

[73] Hou, X. Q. et al. (2016).Group decision-making of air combat training accuracy assessment based on interval-valued intuitionist fuzzy set. *Syst. Eng. Electron.* 38(12), 2785–2789.

[74] Liu, Y., Jiang, W. (2020).A new distance measure of interval-valued intuitionistic fuzzy sets and its application in decision making. *Sof. Comput*. 24(9), 6987–7003.

[75] Garg, H., Kumar, K. (2020).A novel exponential distance and its based TOPSIS method for interval-valued intuitionistic fuzzy sets using connection number of SPA theory. *Artif. Intell. Rev* 53(1), 595–624.

[76] Zhang, Z. M. , Chen, S. M. (2021).Optimization-based group decision making using interval-valued intuitionistic fuzzy preference relations. *Inf. Sci.* 561, 352–370.

[77] Atanassov, K. (1994).Operator over interval-valued intuitionistic fuzzy sets. *Fuzzy Syst.* 64(2), 159–174.

[78] Xu, Z. S. ,Yager, R. R. (2006).Some geometric aggregation operators based on intuitionistic fuzzy sets. *Int. J. Gen. Syst.* 35(4), 417–433.

[79] Xu, Z. S. , Chen, J. (2007).An approach to group decision making based on interval–valued intuitionistic judgment matrices. *Syst. Eng. Theory Pract.* 27(4), 126–133.

[80] Kong, D. P. et al. (2019).A decision variable-based combinatorial optimization approach for interval-valued intuitionistic fuzzy MAGDM. *Inf. Sci.* 484(5), 197–218.

[81] Yao, R. P. (2019).An Approach to variable weight group decision making based on the improved score function of interval-valued intuitionistic sets. *Stat. Decis.* 35(11), 36–38.

[82] Xu, Z. S. (2007).Method for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. *Control Decis.* 22(2), 215–219.

[83] Da, Q. , Liu, X. W. (1999).Interval number linear programming and its satisfactory solution. *Syst. Eng. Teory Pract.* 19(4), 3–7.

[84] Liu, H. C., Chen, X. Q., Duan, C. Y. , Wang, Y. M. (2019). Failure Mode and Effect Analysis Using Multi Criteria Decision Making Methods; A Systematic Literature Review. *Computers and Industrial Engineering*, 135, 881-897.

[85] Chen, M., Tzeng, G. (2004). Combining grey relation and TOPSIS concepts for selecting an expatriate host country. *Math. Comput. Model*., 40, 1473-1490.

[86] Gupta, R., Kumar, S. (2022). Intuitionistic fuzzy scale-invariant entropy with correlation coefficients-based VIKOR approach for multi-criteria decision-making. *Granular Computing*, 7, 77-93.

[87] Tuğrul, F. (2022). An Approach Utilizing The Intuitionistic Fuzzy TOPSIS Method to Unmanned Air Vehicle Selection.*Ikonion Journal of Mathematics* 4(2) 32-41.

[88] Altuntas,G.,Yildirim, B.F. (2022).Logistics specialist selection with intuitionistic fuzzy TOPSIS method, *International Journal of Logistics Systems and Management*, vol. 42(1), 1-34.

[89] Yao, R., Guo, H. (2022). A multiattribute group decision-making method based on a new aggregation operator and the means and variances of interval-valued intuitionistic fuzzy values. *Sci Rep* 12, 22525.

[90] Wang, Y., Lei, Y.J. (2007). A Technique for Constructing intuitionistic Fuzzy Entropy. *J. Control Decis*. 12, 1390–1394.

[91] Fu, S., Xiao, Yz., Zhou, Hj. (2022). Interval-valued intuitionistic fuzzy multi-attribute group decision-making method considering risk preference of decision-makers and its application. Sci Rep 12, 11597.

[92] Liu, P., Gao, H. (2018), An overview of intuitionistic linguistic fuzzy information aggregations and applications. Marine Economics and Management, Vol. 1 No. 1,55-78.

[93] Yager, RR. (2013. Pythagorean fuzzy subsets. *Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)* 57–61 6.

[94] Yager, R. R. (2013). Pythagorean membership grades in multi-criteria decision making. *IEEE Trans Fuzzy Syst* 22(4):958–965.

[95] Yager, R. R. (2017). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 25(5), 1222– 1230.

[96] Tian, X., Niu, M., Zhang, W., Li, L., Herrera-Viedma, E. (2021). A novel TODIM based on prospect theory to select green supplier with q-rung orthopair fuzzy set. *Technological and Economic Development of Economy*, 27(2), 284-310.

[97] Cuong, B. C., Kreinovich, V. (2013). Picture Fuzzy Sets - a new concept for computational intelligence problems. Departmental Technical Reports (CS). 809. *In Proceedings of the Third World Congress on Information and Communication Technologies WICT'2013*, Hanoi, Vietnam, December 15-18, 2013, pp. 1-6.

[98] Cuong, B. C. (2014). Picture Fuzzy Sets. *Journal of Computer Science and Cybernetics*, V.30, N.4 (2014), 409–420.

[99] Gündogdu, FK, Kahraman, C. (2019). Spherical fuzzy sets and spherical fuzzy TOPSIS method. *J Intell Fuzzy Syst* 36(1):337–352.

[100] Mahmood, T.; Ullah, K.; Khan, Q.; Jan, N. An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Comput Appl*. 2018, 1–13.

[101] Ullah, K., Mahmood, T., Jan, N. (2018). Similarity Measures for T-Spherical Fuzzy Sets with Applications in Pattern Recognition. *Symmetry*, 10(6), 193.

[102] Smarandache, F. (2003). A unifying field in logics neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability. (3rd ed.). Xiquan, Phoenix: American Research Press.

[103] Smarandache, F. (2003).Neutrosophic Logic - Generalization of the Intuitionistic Fuzzy Logic. https://arxiv.org/abs/math/0303009