Determinate Fuzzy Set Ranking Analysis for Combat Aircraft Selection with Multiple Criteria Group Decision Making

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***Abstract*—**Using the aid of Hausdorff distance function and Minkowski distance function, this study proposes a novel method for selecting combat aircraft for Air Force. In order to do this, the proximity measure method was developed with determinate fuzzy degrees based on the relationship between attributes and combat aircraft alternatives. The combat aircraft selection attributes were identified as payloadability, maneuverability, speedability, stealthability, and survivability. Determinate fuzzy data from the combat aircraft attributes was then aggregated using the determinate fuzzy weighted arithmetic average operator. For the selection of combat aircraft, correlation analysis of the ranking order patterns of options was also examined. A numerical example from military aviation is used to demonstrate the applicability and effectiveness of the proposed method.

***Keywords*—**Combat aircraft selection, multiple criteria decision making, fuzzy sets, determinate fuzzy sets, intuitionistic fuzzy sets, proximity measure method, Hausdorff distance function, Minkowski distance function, PMM, MCDM.

I. Introduction

Fuzzy sets (FS) were introduced to address imprecision and uncertainty in real-life problems, and a lot of extensions of fuzzy set theory were developed [1-49].

Definition 1.[1] A fuzzy set A on a universe $X = \{x_1, x_2, ..., x_n\}$ is an object of the form:

$$A = \{< x_i, \mu_A(x_i) > \ | \forall x_i \in X\} \quad (1)$$

where $\mu_A(x_i) \in [0,1]$ is called the degree of membership of an element x_i to X, $\mu_{\bar{A}}(x_i) = 1 - \mu_A(x_i) \in [0,1]$ is called the degree of nonmembership an element x_i to X, and $\mu_A(x_i) \in [0,1]$, and $\mu_{\bar{A}}(x_i) = 1 - \mu_A(x_i) \in [0,1]$ satisfy the following condition:

$$A = \{< x_i, \mu_A(x_i), 1 - \mu_A(x_i) > \ | \forall x_i \in X\} \quad (2)$$

$$\mu_A(x_i) + 1 - \mu_A(x_i) = 1 \ | \forall x_i \in X \quad (3)$$

In a fuzzy set, the degree of indeterminacy or hesitation of element $x_i \in X$ to set A is $\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i) = 0$.

Definition 2. [1] Let $X = \{x_1, x_2, ..., x_n\}$ be a non-empty set in the unit interval [0,1] and fuzzy sets $A = \{< x_i, \mu_A(x_i) > \ | \forall x_i \in X\}$ and $B = \{< x_i, \mu_B(x_i) > \ | \forall x_i \in X\}$ are of the form. Then, fuzzy set aggregative operators are defined as follows:

Union: $A \vee B = \max\{\mu_A(x_i), \mu_B(x_i)\}$
Intersection: $A \wedge B = \min\{\mu_A(x_i), \mu_B(x_i)\}$
Complement: $\mu_{\bar{A}}(x_i) = 1 - \mu_A(x_i)$

Definition 2. Refined fuzzy set (RFS). A refined fuzzy set A on a universe $X = \{x_1, x_2, ..., x_n\}$ is an object of the form:

$$A = \{< x_i, \mu_A^1(x_i), \mu_A^2(x_i), ..., \mu_A^p(x_i) > | \ p \geq 2, \forall x_i \in X\} \quad (4)$$

where the membership degree $\mu_A(x_i)$ is refined /split into sub-membership degrees. $\mu_A^1(x_i)$ is a sub-membership degree of type 1 of the element x_i with respect to the set A, $\mu_A^2(x_i)$ is a sub-membership degree of type 2 of the element x_i with respect to the set A, $\mu_A^p(x_i)$ is a sub-membership degree of type p of the element x_i with respect to the set A, and $\mu_A^J(x_i) \subseteq [0,1]$ for $1 \leq j \leq p$, and $\sum_{j=1}^{p} \sup \mu_A^j(x_i) \leq 1$ for all $\forall x_i \in X$.

Definition 3. Determinate Fuzzy Set (DFS). A determinate fuzzy set A on a universe $X = \{x_1, x_2, ..., x_n\}$ is an object of the form:

$$A = \{< x_i, \mu_A(x_i), \nu_A(x_i) > \ | \forall x_i \in X\} \quad (5)$$

where $\mu_A(x_i) \in [0,1]$ is called the degree of membership of an element x_i in A, $\nu_A(x_i) = 1 - \mu_A(x_i)$ is called the degree

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of nonmembership an element x_i in *A*, $\mu_A(x_i)\in[0,1]$ and $\nu_A(x_i)\in[0,1]$ satisfy the following condition:

$$\mu_A(x_i)+\nu_A(x_i)=1 \ \mid \forall x_i \in X \tag{6}$$

where $\pi_A(x_i)=1-\mu_A(x_i)+\nu_A(x_i)=0$, $\mid \forall x_i \in X$ is called the degree of indeterminacy of x_i to *X*.

For the given element x_i, $<x_i,\mu_A(x_i),\nu_A(x_i)>$ is called determinate fuzzy number (DFN), and for convenience, one can utilize $a=(\mu_\alpha,\nu_a)$ to denote a DFN, which meets the conditions $\mu_A(x_i),\nu_A(x_i)\in[0,1]$ and $\mu_A(x_i)+\nu_A(x_i)=1$.

Definition 4.[5] Intuitionistic Fuzzy Set (IFS). Let $X=\{x_1,x_2,...,x_n\}$ be a nonempty set, an IFS *A* in *X* is given by

$$A=\{<x_i,\mu_A(x_i),\nu_A(x_i)>\mid \forall x_i\in X\} \tag{7}$$

where $\mu_A(x_i)\in[0,1]$ and $\nu_A(x_i)\in[0,1]$ with the condition

$$0\le \mu_A(x_i)+\nu_A(x_i)\le 1 \ \mid \forall x_i\in X \tag{8}$$

The numbers $\mu_A(x_i)$ and $\nu_A(x_i)$ denote the membership degree and nonmembership degree of the element x_i to *X*.

In addition, $\pi_A(x_i)=1-\mu_A(x_i)+\nu_A(x_i) \ \mid \forall x_i\in X$ denotes indeterminacy degree of the element x_i to *X*. It is evident that $0\le \pi_A(x_i)\le 1 \ \mid \forall x_i\in X$.

For the given element x_i, $<x_i,\mu_A(x_i),\nu_A(x_i)>$ is called intuitionistic fuzzy number (IFN), and for convenience, one can utilize $a=(\mu_\alpha,\nu_a)$ to denote a IFN, which meets the conditions $\mu_A(x_i),\nu_A(x_i)\in[0,1]$ and $0\le\mu_A(x_i)+\nu_A(x_i)\le 1$.

Definition 5. [24],[28] Let $a_i=(\mu_{\alpha_i},\nu_{a_i})$, $a_1=(\mu_{\alpha_1},\nu_{a_1})$ and $a_2=(\mu_{\alpha_2},\nu_{a_2})$ be three IFN/DFNs, $\delta\ge 0$, then the operations of IFN/DFNs are defined as follows:

$$\alpha_1\subset\alpha_2 \ \ iff\, \mu_{\alpha_1}\le\mu_{\alpha_2} \,\&\, \nu_{\alpha_1}\ge\nu_{\alpha_2}$$

$$\alpha_1\cup\alpha_2=(\max\{\mu_{\alpha_1},\mu_{\alpha_2}\},\min\{\nu_{\alpha_1},\nu_{\alpha_2}\})$$

$$\alpha_1\cap\alpha_2=(\min\{\mu_{\alpha_1},\mu_{\alpha_2}\},\max\{\nu_{\alpha_1},\nu_{\alpha_2}\})$$

$$a_1\oplus a_2=(\mu_{\alpha_1}+\mu_{a_2}-\mu_{\alpha_1}.\nu_{a_2},\mu_{\alpha_1}.\nu_{a_2})$$

$$a_1\otimes a_2=(\mu_{\alpha_1}.\mu_{a_2},\mu_{\alpha_1}+\nu_{a_2},\mu_{\alpha_1}.\nu_{a_2})$$

$$\delta a_i=(1-(1-\mu_{a_i})^\delta,(\nu_{a_i})^\delta)$$

$$a_i^\delta=((\mu_{a_i})^\delta,1-(1-\nu_{a_i})^\delta)$$

$$a_i^C=(\nu_{a_i},\mu_{a_i})$$

Definition 6. For IFN/DFVs, the score function $s(\alpha_i)$ is defined as the difference of membership and nonmembership function, as follows:

$$s(\alpha_i)=(\mu(\alpha_i)-\nu(\alpha_i)) \tag{9}$$

where $s(\alpha_i)\in[-1,1]$. The larger the score $s(\alpha_i)$, the greater the IFVDFV α_i.

Note that the score function alone cannot differentiate many IFV/DFVs even though they are obviously different. To make the comparison method more discriminatory, an accuracy function $h(\alpha_i)$, which is defined as the sum of the membership and nonmembership function, was introduced as follows:

$$h(\alpha_i)=(\mu(\alpha_i)+\nu(\alpha_i)) \tag{10}$$

where $h(\alpha_i)\in[0,1]$. When the scores are the same, the larger the accuracy $h(\alpha_i)$, the greater the IFV/DFV α_i.

Definition 7. Let $\alpha_1=(\mu_{\alpha_1},\nu_{\alpha_1})$ and $\alpha_2=(\mu_{\alpha_2},\nu_{\alpha_2})$ be two DFVs, $s(\alpha_1)=(\mu(\alpha_1)-\nu(\alpha_2))$ and $s(\alpha_2)=(\mu(\alpha_3)-\nu(\alpha_2))$ be the scores of α_1 and α_2, respectively, and $h(\alpha_1)=(\mu(\alpha_1)+\nu(\alpha_1))$, and $h(\alpha_2)=(\mu(\alpha_2)+\nu(\alpha_2))$ be the accuracy degrees of α_1 and α_2, respectively; then,

1) If $s(\alpha_i)<s(\alpha_2)$, then α_1 is smaller than α_2, i.e., $\alpha_1<\alpha_2$.
2) If $s(\alpha_i)=s(\alpha_2)$, then
   a) if $h(\alpha_1)<h(\alpha_2)$, then α_1 is smaller than α_2, i.e., $\alpha_1<\alpha_2$;
   b) if $h(\alpha_1)=h(\alpha_2)$, then α_1 and α_2 represent the same information, i.e., $\mu_{\alpha_1}=\mu_{\alpha_2}$ and $\nu_{\alpha_1}=\nu_{\alpha_2}$, denoted by $\alpha_1=\alpha_2$.

Definition 8. Let $\alpha_1=<\mu_1,\nu_1>$ and $\alpha_2=<\mu_2,\nu_2>$ be two IFSs/DFSs. Then, the generalized Minkowski distance between them is defined as follows:

$$d_\gamma(\alpha_1,\alpha_2)=\left(\frac{1}{n}\sum_{j=i}^{n}\left(\mid\mu_{a_1}-\mu_{a_2}\mid^\gamma+\mid\nu_{a_1}-\nu_{a_2}\mid^\gamma\right)\right)^{1/\gamma} \tag{11}$$

when $\gamma=1$, the $d_1(\alpha_1,\alpha_2)$ is called the Manhattan distance between α_1 and α_2; when $\gamma=2$, the $d_2(\alpha_1,\alpha_2)$ is called the Euclidean distance between α_1 and α_2; and when $\gamma=\infty$, the $d_\infty(\alpha_1,\alpha_2)$ is called the Chebyshev distance between α_1 and α_2.

Definition 9. Let $\alpha_1 =< \mu_1, \nu_1 >$ and $\alpha_2 =< \mu_2, \nu_2 >$, $\alpha_1 \neq \alpha_2$ be two IFSs/DFSs. The similarity measure between two IFSs/DFSs α_1 and α_2 is a fuzzy relation that expresses the degree to which α_1 and α_2 are equal. Then, the similarity between them is defined as follows:

$$S(\alpha_1, \alpha_2) = \frac{1}{n} \sum_{j=i}^{n} \frac{\alpha_1 \cap \alpha_2}{\alpha_1 \cup \alpha_2} \tag{12}$$

Definition 10. Determinate fuzzy entropy is defined as follows. $A \in IFS(X) / DFS(X)$ represents the number of elements in *X*.

$$E(A) = \frac{1}{n} \sum_{i=1}^{n} \frac{\min(\mu(\alpha_i), \nu(\alpha_i))}{\max(\mu(\alpha_i), \nu(\alpha_i))} \tag{13}$$

Definition 11. Let $\alpha_1 =< \mu_1, \nu_1 >$ and $\alpha_2 =< \mu_2, \nu_2 >$ be two IFS/DFS. Then, the weighted distance between them is defined as follows:

$$d(\alpha_1, \alpha_2) = \frac{1}{n} \sum_{j=i}^{n} \frac{|\mu_1 - \mu_2| + |\nu_1 - \nu_2|}{(\mu_1 + \nu_1) + (\mu_2 + \nu_2)} \tag{14}$$

Definition 12. Let $\alpha_1 =< \mu_1, \nu_1 >$ and $\alpha_2 =< \mu_2, \nu_2 >$ be two IFS/DFS. Hausdorff distance function is defined as follows:

$$d_H(\alpha_1, \alpha_2) = \left(\frac{1}{n} \sum_{j=i}^{n} \max\left(|\mu_{a_1} - \mu_{a_2}|, |\nu_{a_1} - \nu_{a_2}| \right) \right) \tag{15}$$

II. Methodology

A. Determinate Fuzzy Proximity Measure Method (PMM)

Proximity measure method (PMM) evaluates alternatives based on a proximity measure value that represents the deviation from the best alternative. The proximity measure value used to rank the options represents the minimum deviation from the best option. The ranking of the alternatives commences from the alternative with the lowest proximity measure value and decreases as the proximity measure value increases. The procedural steps of proposed multiple criteria group decision making method are given as follows:

Step 1. Decision matrix is structured.

MCDM Model	ω_1	ω_2	$\cdots$	ω_j	$\cdots$	ω_n
	c_1	c_2	$\cdots$	c_j	$\cdots$	c_n
a_1	x_{11}	x_{12}	$\cdots$	x_{1j}	$\cdots$	x_{1n}
a_2	x_{21}	x_{22}	$\cdots$	x_{2j}	$\cdots$	x_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_i	x_{i1}	x_{i2}	$\cdots$	x_{ij}	$\cdots$	x_{in}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_m	x_{m1}	x_{m2}	$\cdots$	x_{mj}	$\cdots$	x_{mn}

Step 2. Criteria weights are determined.

The fuzzy entropy measure is used to determine the expert's criteria weight vector. The calculation formula for determining the criteria weights by using information entropy is as follows:

$$\omega_k = \frac{1 - H_k}{\sum_{k=1}^{k} H_k} \tag{16}$$

$$H_k = \sum_{j=1}^{n} \omega_j \left(\frac{1}{m} \sum_{i=1}^{m} E(a_{ij}^k \right)$$

$$E(a_{ij}^k) = \frac{\mu(a_{ij}^k) \wedge \nu(a_{ij}^k)}{\mu(a_{ij}^k) \vee \nu(a_{ij}^k)}$$

Step 2. Decision matrix is normalized.

$$r_{ij}(x) =< \mu_{ij}, \nu_{ij} >= \begin{cases} < \mu_{ij}, \nu_{ij} >, \text{ for } c_j \in \Omega_b \\ < \nu_{ij}, \mu_{ij} >, \text{ for } c_j \in \Omega_c \end{cases} \tag{17}$$

where Ω_b and Ω_c are the sets of benefit criteria and cost criteria, respectively.

Step 3. The normalized values r_{ij} are multiplied by the weights of criteria ω_k.

$$\eta_{ij} = \omega_k r_{ij} = (1 - (1 - \mu_{r_{ij}})^{\omega_k}, \nu_{r_{ij}}^{\omega_k}), \omega_k > 0 \tag{18}$$

Step 4. The weighted proximity measure ε_{ij} and the ideal proximity measure vector are obtained.

a. $$\varepsilon_\delta(\eta_j^*, \eta_{ij}) = \left(\frac{1}{n} \sum_{j=i}^{n} \left(|\mu_{\eta_j^*} - \mu_{\eta_{ij}}|^\delta + |\nu_{\eta_j^*} - \nu_{\eta_{ij}}|^\delta \right) \right)^{1/\delta} \tag{19}$$

where $\eta_j^* = (\mu_j^*, \nu_j^*)$, $j = 1, ..., n$

$$\mu_j^* = \{\max_i \mu_{ij} \mid j \in \Omega_b, \min_i \nu_{ij} \mid j \in \Omega_b\}$$

$$\nu_j^* = \{\min_i \nu_{ij} \mid j \in \Omega_b, \max_i \nu_{ij} \mid j \in \Omega_b\}$$

$\eta_j^* = (\mu_j^*, \nu_j^*)$ is the ideal proximity measure vector, Ω_b and Ω_c are the sets of benefit criteria and cost criteria, respectively. The Manhattan distance, Euclidean distance, and Chebyshev distance are the reduced forms of the Minkowski distance function with $\delta \in \{1, 2, \infty\}$ parameters.

b. Manhattan distance $\delta = 1$.

$$\varepsilon_1(\eta_j^*, \eta_{ij}) = \left(\frac{1}{n} \sum_{j=i}^{n} \left(|\mu_{\eta_j^*} - \mu_{\eta_{ij}}| + |\nu_{\eta_j^*} - \nu_{\eta_{ij}}| \right) \right) \tag{20}$$

c. Euclidean distance $\delta = 2$.

$$\varepsilon_2\left(\eta_j^*,\eta_{ij}\right) = \left(\frac{1}{n}\sum\nolimits_{j=i}^{n}\left(|\,\mu_{\eta_j^*} - \mu_{\eta_{ij}}\,|^2 + |\,\nu_{\eta_j^*} - \nu_{\eta_{ij}}\,|^2\right)\right)^{1/2} \quad (21)$$

d. Chebyshev distance $\delta = \infty$.

$$\varepsilon_\infty\left(\eta_j^*,\eta_{ij}\right) = \max_i\left(|\,\mu_{\eta_j^*} - \mu_{\eta_{ij}}\,| + |\,\nu_{\eta_j^*} - \nu_{\eta_{ij}}\,|\right) \quad (22)$$

e. Hausdorff distance ε_H.

$$\varepsilon_H\left(\eta_j^*,\eta_{ij}\right) = \left(\frac{1}{n}\sum\nolimits_{j=i}^{n}\max\left(|\,\mu_{\eta_j^*} - \mu_{\eta_{ij}}\,|, |\,\nu_{\eta_j^*} - \nu_{\eta_{ij}}\,|\right)\right) \quad (23)$$

Step 5. The alternative having the least proximity measure value of ε_i is identified as the best alternative ε_i^*.

$$\varepsilon_i^* = \min \varepsilon_i = \varepsilon(\arg\min_{i\in I} \varepsilon_i) \quad (24)$$

Step 6. Correlation analysis of the ranking orders of alternatives is carried out using the Spearman correlation coefficient that is defined as the Pearson correlation coefficient between the rank variables. The n raw scores $\{x_i, y_i\}$ are converted to ranks $\{R(x_i), R(y_i)\}$, and r_s is computed as

$$r_s = \rho_{R(x_i)}, \rho_{R(y_i)} = \frac{\operatorname{cov}(R(x_i), R(y_i))}{\rho_{R(x_i)}, \rho_{R(y_i)}} \quad (25)$$

where ρ denotes the Pearson correlation coefficient applied to the rank variables; $\operatorname{cov}(R(x_i), R(y_i))$ is the covariance of the rank variables, $\rho_{R(x_i)}, \rho_{R(y_i)}$ are the standard deviations of the rank variables.

III. Application

In this section, combat aircraft selection problem is considered using the determinate fuzzy sets and the proximity measure method. The fighter aircraft selection process is considered as a multiple criteria group decision analysis problem from the literature review [50-90].

For the group decision problem, combat aircraft candidates are evaluated by three experts $E_k = \{E_1, E_2, E_3\}$ and the weight of each expert (λ_k) is set equal to ($\lambda_k = 1/K$) and the best aircraft is selected according to the proposed proximity measure method (PMM) under fuzzy environment.

In the group decision-making process, the aircraft alternatives are evaluated according to five benefit type of criteria: payloadability (C1), maneuverability (C2), speedability (C3), stealthability (C4), and survivability (C5).

The experts evaluate the ten combat aircraft candidates (A_i) using the determinate fuzzy numbers (DFN) $< x_i, \mu_A(x_i), \nu_A(x_i) >$, where $\mu_A(x_i), \nu_A(x_i) \in [0,1]$, $\nu_A(x_i) = 1 - \mu_A(x_i)$, and $\mu_A(x_i) + \nu_A(x_i) = 1$. The evaluation ratings of the experts are reflected in the initial determinate fuzzy decision matrix as shown in Table 1. The objective criteria weight vector was calculated according to formula (16) as ω_k =(0.207, 0.203, 0.189, 0.209, 0.192), and the calculated entropic criteria weights are given in Table 2 and Fig.1.

Table 1. Determinate fuzzy decision matrix

E_i	A_i	C1		C2		C3		C4		C5	
E1	A1	0,7	0,3	0,3	0,7	0,4	0,6	0,7	0,3	0,6	0,4
	A2	0,6	0,4	0,8	0,2	0,4	0,6	0,2	0,8	0,5	0,5
	A3	0,4	0,6	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
	A4	0,3	0,7	0,3	0,7	0,7	0,3	0,7	0,3	0,3	0,7
	A5	0,6	0,4	0,5	0,5	0,4	0,6	0,5	0,5	0,9	0,1
	A6	0,2	0,8	0,7	0,3	0,9	0,1	0,3	0,7	0,4	0,6
	A7	0,5	0,5	0,2	0,8	0,6	0,4	0,8	0,2	0,8	0,2
	A8	0,4	0,6	0,8	0,2	0,6	0,4	0,2	0,8	0,7	0,3
	A9	0,4	0,6	0,1	0,9	0,7	0,3	0,9	0,1	0,4	0,6
	A10	0,8	0,2	0,8	0,2	0,3	0,7	0,2	0,8	0,2	0,8
E2	A1	0,3	0,7	0,6	0,4	0,7	0,3	0,7	0,3	0,9	0,1
	A2	0,2	0,8	0,5	0,5	0,6	0,4	0,8	0,2	0,4	0,6
	A3	0,1	0,9	0,5	0,5	0,9	0,1	0,9	0,1	0,4	0,6
	A4	0,6	0,4	0,3	0,7	0,4	0,6	0,4	0,6	0,3	0,7
	A5	0,5	0,5	0,9	0,1	0,5	0,5	0,5	0,5	0,7	0,3
	A6	0,6	0,4	0,4	0,6	0,6	0,4	0,4	0,6	0,3	0,7
	A7	0,7	0,3	0,8	0,2	0,8	0,2	0,3	0,7	0,4	0,6
	A8	0,4	0,6	0,4	0,6	0,6	0,4	0,6	0,4	0,1	0,9
	A9	0,7	0,3	0,4	0,6	0,7	0,3	0,3	0,7	0,6	0,4
	A10	0,3	0,7	0,2	0,8	0,7	0,3	0,7	0,3	0,5	0,5
E3	A1	0,8	0,2	0,6	0,4	0,2	0,8	0,4	0,6	0,4	0,6
	A2	0,9	0,1	0,5	0,5	0,2	0,8	0,4	0,6	0,4	0,6
	A3	0,2	0,8	0,7	0,3	0,8	0,2	0,5	0,5	0,5	0,5
	A4	0,3	0,7	0,2	0,8	0,6	0,4	0,7	0,3	0,7	0,3
	A5	0,4	0,6	0,6	0,4	0,6	0,4	0,4	0,6	0,4	0,6
	A6	0,6	0,4	0,7	0,3	0,4	0,6	0,9	0,1	0,9	0,1
	A7	0,5	0,5	0,5	0,5	0,6	0,4	0,6	0,4	0,6	0,4
	A8	0,6	0,4	0,4	0,6	0,4	0,6	0,6	0,4	0,6	0,4
	A9	0,7	0,3	0,9	0,1	0,5	0,5	0,7	0,3	0,7	0,3
	A10	0,9	0,1	0,8	0,2	0,3	0,7	0,3	0,7	0,3	0,7

Table 2. Objective criteria weight vector

	C1	C2	C3	C4	C5
H_k	0,512	0,519	0,552	0,507	0,546
$1-H_k$	0,488	0,481	0,448	0,493	0,454
ω_k	0,207	0,203	0,189	0,209	0,192

The combat aircraft evaluation criteria are considered as benefit type of attributes. Therefore, the normalization procedure was performed according to the formula (17) and the normalized determinate fuzzy matrix is presented in Table 3.

Table 3. Normalized determinate fuzzy decision matrix

E_i	A_i	C1		C2		C3		C4		C5	
E1	A1	0,7	0,3	0,3	0,7	0,4	0,6	0,7	0,3	0,6	0,4
	A2	0,6	0,4	0,8	0,2	0,4	0,6	0,2	0,8	0,5	0,5
	A3	0,4	0,6	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
	A4	0,3	0,7	0,3	0,7	0,7	0,3	0,7	0,3	0,3	0,7
	A5	0,6	0,4	0,5	0,5	0,4	0,6	0,5	0,5	0,9	0,1
	A6	0,2	0,8	0,7	0,3	0,9	0,1	0,3	0,7	0,4	0,6
	A7	0,5	0,5	0,2	0,8	0,6	0,4	0,8	0,2	0,8	0,2
	A8	0,4	0,6	0,8	0,2	0,6	0,4	0,2	0,8	0,7	0,3
	A9	0,4	0,6	0,1	0,9	0,7	0,3	0,9	0,1	0,4	0,6
	A10	0,8	0,2	0,8	0,2	0,3	0,7	0,2	0,8	0,2	0,8
E2	A1	0,3	0,7	0,6	0,4	0,7	0,3	0,7	0,3	0,9	0,1
	A2	0,2	0,8	0,5	0,5	0,6	0,4	0,8	0,2	0,4	0,6
	A3	0,1	0,9	0,5	0,5	0,9	0,1	0,9	0,1	0,4	0,6
	A4	0,6	0,4	0,3	0,7	0,4	0,6	0,4	0,6	0,3	0,7
	A5	0,5	0,5	0,9	0,1	0,5	0,5	0,5	0,5	0,7	0,3
	A6	0,6	0,4	0,4	0,6	0,6	0,4	0,4	0,6	0,3	0,7
	A7	0,7	0,3	0,8	0,2	0,8	0,2	0,3	0,7	0,4	0,6
	A8	0,4	0,6	0,4	0,6	0,6	0,4	0,6	0,4	0,1	0,9
	A9	0,7	0,3	0,4	0,6	0,7	0,3	0,3	0,7	0,6	0,4
	A10	0,3	0,7	0,2	0,8	0,7	0,3	0,7	0,3	0,5	0,5
E3	A1	0,8	0,2	0,6	0,4	0,2	0,8	0,4	0,6	0,4	0,6
	A2	0,9	0,1	0,5	0,5	0,2	0,8	0,4	0,6	0,4	0,6
	A3	0,2	0,8	0,7	0,3	0,8	0,2	0,5	0,5	0,5	0,5
	A4	0,3	0,7	0,2	0,8	0,6	0,4	0,7	0,3	0,7	0,3
	A5	0,4	0,6	0,6	0,4	0,6	0,4	0,4	0,6	0,4	0,6
	A6	0,6	0,4	0,7	0,3	0,4	0,6	0,9	0,1	0,9	0,1
	A7	0,5	0,5	0,5	0,5	0,6	0,4	0,6	0,4	0,6	0,4
	A8	0,6	0,4	0,4	0,6	0,4	0,6	0,6	0,4	0,6	0,4
	A9	0,7	0,3	0,9	0,1	0,5	0,5	0,7	0,3	0,7	0,3
	A10	0,9	0,1	0,8	0,2	0,3	0,7	0,3	0,7	0,3	0,7

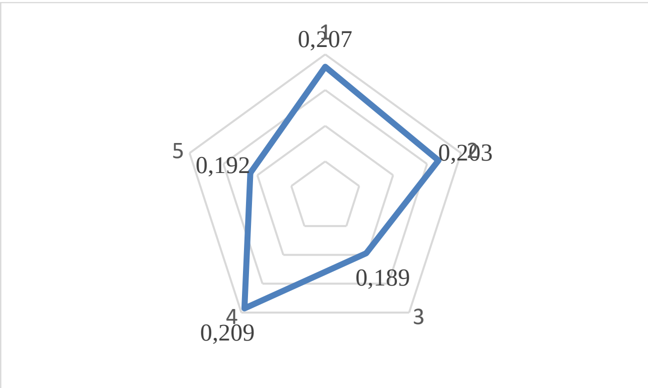

Fig.1 Distribution of calculated entropic criteria weights

The weighed normalized determinate fuzzy decision matrix is calculated using the formula (18) and the weighted normalized determinate fuzzy matrix is presented in Table 4.

Table 4. Weighted normalized determinate fuzzy matrix

E_i	A_i	C1		C2		C3		C4		C5	
E1	A1	0,220	0,783	0,070	0,930	0,092	0,908	0,222	0,778	0,161	0,839
	A2	0,172	0,830	0,279	0,721	0,092	0,908	0,045	0,955	0,125	0,875
	A3	0,100	0,901	0,131	0,869	0,123	0,877	0,135	0,865	0,125	0,875
	A4	0,071	0,930	0,070	0,930	0,204	0,796	0,222	0,778	0,066	0,934
	A5	0,172	0,830	0,131	0,869	0,092	0,908	0,135	0,865	0,357	0,643
	A6	0,045	0,956	0,217	0,783	0,354	0,646	0,072	0,928	0,093	0,907
	A7	0,133	0,869	0,044	0,956	0,159	0,841	0,285	0,715	0,266	0,734
	A8	0,100	0,901	0,279	0,721	0,159	0,841	0,045	0,955	0,206	0,794
	A9	0,100	0,901	0,021	0,979	0,204	0,796	0,381	0,619	0,093	0,907
	A10	0,283	0,721	0,279	0,721	0,065	0,935	0,045	0,955	0,042	0,958
E2	A1	0,071	0,930	0,170	0,830	0,204	0,796	0,222	0,778	0,357	0,643
	A2	0,045	0,956	0,131	0,869	0,159	0,841	0,285	0,715	0,093	0,907
	A3	0,022	0,979	0,131	0,869	0,354	0,646	0,381	0,619	0,093	0,907
	A4	0,172	0,830	0,070	0,930	0,092	0,908	0,101	0,899	0,066	0,934
	A5	0,133	0,869	0,374	0,626	0,123	0,877	0,135	0,865	0,206	0,794
	A6	0,172	0,830	0,099	0,901	0,159	0,841	0,101	0,899	0,066	0,934
	A7	0,220	0,783	0,279	0,721	0,263	0,737	0,072	0,928	0,093	0,907
	A8	0,100	0,901	0,099	0,901	0,159	0,841	0,174	0,826	0,020	0,980
	A9	0,220	0,783	0,099	0,901	0,204	0,796	0,072	0,928	0,161	0,839
	A10	0,071	0,930	0,044	0,956	0,204	0,796	0,222	0,778	0,125	0,875
E3	A1	0,283	0,721	0,170	0,830	0,041	0,959	0,101	0,899	0,093	0,907
	A2	0,378	0,626	0,131	0,869	0,041	0,959	0,101	0,899	0,093	0,907
	A3	0,045	0,956	0,217	0,783	0,263	0,737	0,135	0,865	0,125	0,875
	A4	0,071	0,930	0,044	0,956	0,159	0,841	0,222	0,778	0,206	0,794
	A5	0,100	0,901	0,170	0,830	0,159	0,841	0,101	0,899	0,093	0,907
	A6	0,172	0,830	0,217	0,783	0,092	0,908	0,381	0,619	0,357	0,643
	A7	0,133	0,869	0,131	0,869	0,159	0,841	0,174	0,826	0,161	0,839
	A8	0,172	0,830	0,099	0,901	0,092	0,908	0,174	0,826	0,161	0,839
	A9	0,220	0,783	0,374	0,626	0,123	0,877	0,222	0,778	0,206	0,794
	A10	0,378	0,626	0,279	0,721	0,065	0,935	0,072	0,928	0,066	0,934

The aggregated weighted normalized determinate fuzzy decision matrix is given in Table 5.

Table 5. Aggregated weighted normalized determinate fuzzy decision matrix

A_i	C1		C2		C3		C4		C5	
A1	0,174	0,807	0,079	0,862	0,044	0,885	0,110	0,816	0,083	0,788
A2	0,197	0,792	0,143	0,816	0,045	0,901	0,046	0,850	0,073	0,896
A3	0,049	0,945	0,117	0,839	0,123	0,748	0,085	0,774	0,084	0,886
A4	0,046	0,895	0,038	0,938	0,124	0,847	0,152	0,816	0,093	0,884
A5	0,092	0,866	0,095	0,767	0,084	0,875	0,079	0,876	0,163	0,773
A6	0,072	0,870	0,148	0,820	0,161	0,790	0,164	0,802	0,162	0,816
A7	0,087	0,839	0,056	0,843	0,104	0,805	0,160	0,818	0,148	0,823
A8	0,092	0,877	0,133	0,837	0,085	0,862	0,072	0,867	0,127	0,867
A9	0,105	0,820	0,146	0,820	0,110	0,822	0,215	0,764	0,100	0,845
A10	0,233	0,749	0,195	0,792	0,042	0,886	0,038	0,883	0,035	0,922

The ideal proximity measure vector is computed using the formula (19) as shown in Table 6.

Table 6. The ideal proximity measure vector

	C1		C2		C3		C4		C5	
η_j^*	0,233	0,749	0,195	0,767	0,161	0,748	0,215	0,764	0,163	0,773

Using formulas (19)-(23) Hausdorff distance function and Minkowski distance function based proximity measure values were computed and the ranking (R_i) order patterns of alternatives were determined using the formula (24) as shown in Table 7. The visualization of ranking order patterns of combat aircraft alternatives is shown in Fig. 2.

Table 7. Proximity measure values and ranking order patterns of combat aircraft alternatives

A_i	ε_1	(R_i)	ε_2	(R_i)	ε_3	(R_i)	ε_∞	(R_i)	ε_H	(R_i)
A1	0,166	5	0,128	4	0,206	4	0,254	3	0,099	5
A2	0,183	8	0,144	6	0,238	7	0,270	6	0,108	7
A3	0,180	7	0,155	9	0,265	9	0,380	10	0,111	8
A4	0,219	10	0,170	10	0,279	10	0,333	9	0,126	10
A5	0,162	4	0,139	5	0,228	5	0,258	4	0,101	6
A6	0,111	1	0,103	2	0,187	2	0,282	7	0,070	2
A7	0,148	3	0,118	3	0,198	3	0,236	2	0,089	3
A8	0,193	9	0,145	7	0,231	6	0,270	5	0,112	9
A9	0,112	2	0,094	1	0,157	1	0,199	1	0,065	1
A10	0,171	6	0,153	8	0,257	8	0,296	8	0,098	4

Using the formula (25), the correlation analysis of the ranking order patterns of alternatives was conducted and the correlation coefficients are shown in Table 8.

Table 8. Correlation analysis of the ranking order patterns of alternatives

	ε_1	ε_2	ε_3	ε_∞	ε_H
ε_1	1				
ε_2	0,88	1			
ε_3	0,87	0,99	1		
ε_∞	0,54	0,79	0,81	1	
ε_H	0,93	0,85	0,83	0,56	1

In MCDM analysis, as indicated in Table 8, the correlation analysis reveals that the Minkowski distance function and Euclidean distance function have a higher correlation coefficient (0.99) than the other correlation coefficients. The correlation coefficient between Manhattan distance function and Hausdorff distance function is 0,93. The correlation coefficient between Manhattan distance function and Euclidean distance function is 0,88. The correlation coefficient between Euclidean distance function and Hausdorff distance function is 0,85. The lowest correlation coefficient 0,54 is found between Manhattan distance function and Chebyshev distance function.

Finally, from the ranking analysis, when the ranking patterns of combat aircraft alternatives are examined, it is seen that the best alternative is alternative A9. The Manhattan distance function lists alternative A9 as second, while alternative A6 as first. Whereas the Chebyshev distance ranks the alternative A6 as seventh, other distance functions list alternative A6 as second.

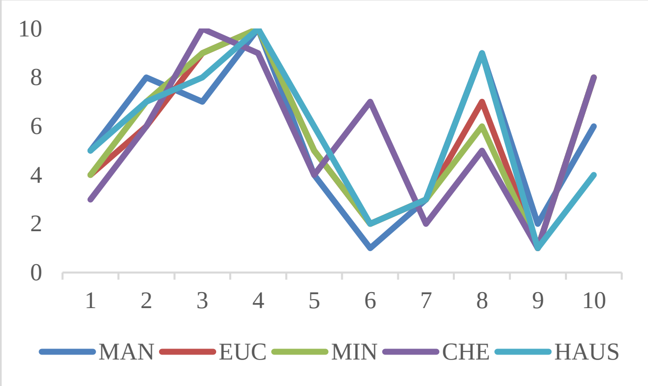

Fig. 2 Ranking order patterns of combat aircraft alternatives

The Manhattan distance function, Euclidean distance function, Minkowski distance function, and Hausdorff distance function rank the alternative A7 in the third place, while the Chebyshev distance function ranks the alternative A7 in the second place.

IV. Conclusion

In this paper, determinate fuzzy set theory has been applied to select combat aircraft as a new approach on decision support practice in military aviation.

For selection of combat aircraft, the ratings of aircraft alternatives were performed using determinate fuzzy set degrees based on the relation among decision attributes and combat aircraft alternatives.

Second, determinate fuzzy set operations were utilized to aggregate fuzzy information from the aircraft attributes. Last, Minkowski distance function and Hausdorff distance function were proposed to rank the combat aircraft alternatives. The result of the example indicates that it is possible to rank combat aircraft using proximity measure method in multiple criteria group decision making analysis.

This paper presents a novel multiple criteria group decision making technique for combat aircraft selection process. Another novelty of the paper is proposing determinate fuzzy sets to evaluate the combat aircraft alternatives using Minkowski distance function and Hausdorff distance function.

The ranking order patterns indicate that the alternative A9 was selected as the best combat aircraft for Air Force. The proposed method can be extended to other multiple criteria decision making techniques to address the complex decision-making challenges in science and technology.

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