Didactical and Semiotic Affordance of GeoGebra in a Productive Mathematical Discourse

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***Abstract*—**Using technology to expand the learning space is critical for a productive mathematical discourse. This is a case study of two teachers who developed and enacted GeoGebra-based mathematics lessons following their engagement in a two-year professional development. The didactical and semiotic affordance of GeoGebra in widening the learning space for a productive mathematical discourse was explored. The approach of thematic analysis was used for lesson artefact, lesson observation, and interview data. The results indicated that constructing tools in GeoGebra provided a didactical milieu where students used them to explore mathematical concepts with little or no support from their teacher. The prompt feedback from the GeoGebra motivated students to practice mathematical concepts repeatedly in which they privately rethink their solutions before comparing their answers with that of their colleagues. The constructing tools enhanced self-discovery, team spirit, and dialogue among students. With regards to the semiotic construct, the tools widened the physical and psychological atmosphere of the classroom by providing animations that served as virtual concrete to enhance the recording, manipulation, testing of a mathematical idea, construction, and interpretation of geometric objects. These findings advance the discussion of widening the classroom for a productive mathematical discourse within the context of the mathematics curriculum of Ghana and similar sub-Saharan African countries.

***Keywords*—**GeoGebra, theory of didactical situation, semiotic mediation, mathematics laboratory, mathematical discussion.

I. Introduction

THERE is a growing use of digital technologies such as handheld calculators, software applications and online resources in the mathematics classroom. The use of technology (e.g., GeoGebra) to mediate mathematical discussion is central in creating a setting where students will have the opportunity to manipulate a tool and use the feedback from the tool to generate argument leading to a meaningful construction of concepts [1], [2]. However, one often articulated issue is how technology such as GeoGebra can be used as an instructional tool to engage students in a shared mathematical discourse when constructing concepts [3], [4].

The mathematical discourse is explained as anything that goes on in the classroom that initiates students' negotiation and plenary discussion of mathematical concepts; students work individually and or in small groups with a tool and the teacher moves round to probe students thought process and respond to their questions [5].

From the theory of didactical situations and semiotic mediation, a tool is important to both physical (externally oriented) and psychological (internally oriented) atmosphere in the classroom where abstraction of mathematical concepts is promoted [6]. Physically, a productive mathematical discourse is enhanced by the availability of resources that can enable the learners to count objects, manipulate and test ideas, classify objects into subsets, record their observations, solve problems and puzzles through games, construct geometric patterns, and draw and interpret graphs. Thus, through concrete objects learners can practice and experiment mathematical concepts [7]. Psychologically, a productive mathematical discourse is enhanced in a setting where emphasis is on self-directed inquiry and exploration of mathematical concepts. These features of a tool are consistent with Vygotskian perspective of using artefacts as didactical and semiotic tool to mediate the construction of mathematics concepts in a socio-cultural setting [8].

Considering the growing of the digital technologies including handheld calculators, software applications, and online resources in mathematics education, it is critical to connect these new tools to classroom practices to promote not only the physical and psychological atmosphere, but also to enhance the cultural aspects of mathematics learning and dialogic construction of concepts [7]. These new tools provide new ways to create conditions of a mathematics laboratory inside an ''ordinary'' classroom [9]. They also have the potential for connecting the teacher and the students. For example, GeoGebra software has the features of algebraic, probability, statistics, and constructing tools; 2D and 3D graphics; CAS calculator; virtual whiteboard; and virtual keyboard which operate in a network. When these features are connected with a video projector, they provide a common workspace on the class screen for a whole class discussion [9]. Thus, from the theory of didactical situations and semiotic mediation, this study explores how GeoGebra expands the learning space for a productive mathematical discourse in the classroom. This exploration is guided by the research question: How do teachers use GeoGebra to provide didactic and semiotic affordance towards a productive mathematical discourse in the classroom following their engagement in a professional development programme?

II. Theoretical Underpinnings

A. Didactical Situations

A productive mathematics discussion is grounded in the theory of didactical situations and semiotic mediation. Both theories share a common root from the Vygotsky's perspective

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of socio-cultural learning where an artefact (a tool) can be used to open up a learning space for worthwhile mathematical tasks, cognitive processes, and discourses.

The Theory of Didactical Situations (TDS) [10] connotes the interaction between students, teacher and milieu when constructing mathematical knowledge [11]. The milieu describes the environment (e.g., a tool or artefact) and it must be accessible and relevant to the student. The interaction involves the dialogue between the student and the feedback from the environment. From the notion of TDS, the student exploits both the positive and negative feedback from the artefact towards achieving the instructional objective of the lesson without heavily relying on the teacher. Brousseau referred to this instruction of TDS as an *adidactical situation* or *adidactical milieu*, in which the student is expected to use the milieu to achieve a desirable learning goal, as if the teacher is absent [11], [12].

In a typical adidactical situation, the milieu provides new knowledge which is socially debated in the classroom for refinement [12]. Thus, a milieu needs to have the potential to widen the learning space such that a student can make epistemic strategies of using the feedback while using the tool to progressively determine the solution to a problem through dialogue [13]. For example, a tool can have game-like features for productive activities suitable for the development of both cognitive and social skills [14].

One of the key notions of TDS is the relationship between the students and the teacher, a didactical contrast [10]. The teacher has the responsibility to ensure that the subject matter the students construct through the artefact aligns with the expectation of the curriculum. This is called the *institutionalisation* of the required knowledge [11], [12]. For example, the teacher needs to pedagogically consider possible epistemic interactions using different types of tools and resources and reconfigure them to enhance a smooth transition of the pre-existing socio-cultural knowledge of the students to the new learning. Although the teacher's role is paramount to enhance social interaction in the classroom, the kernel of the TDS trusts on a constructivist epistemology where the interaction between the students and the milieu primes a productive mathematical discourse in the classroom.

B. Semiotic Mediation

A semiotic mediation framework is proposed by [15]. Learners solve practical tasks by articulating what they see and what they use their hands to manipulate. From the seminal work of [8], this unity of perception of a tool that mediates human activity and cognitive process elaborates the theoretical construct of semiotic mediation. Semiotic mediation describes the connection between a tool, a task, and mathematical knowledge in the context of the task [15], [16]. The teacher has the responsibility of not only designing the learning task (called mathematics knowledge) but also to introduce it to the students and guides them in an engaged mathematical discussion.

The principle of semiotic potential diverges from the radical constructivists' perspective, where the faciliatory role of the teacher in the classroom is shadowed and focus is placed only on the students' active role in constructing mathematical concepts with the tool [15]. In the semiotic instruction approach, the teacher uses the artefact to repurpose the mathematics curriculum to meet the context of the students' learning. The tenets of semiotic mediation reflect the notion of physical and psychological tools of mathematics laboratory. These tools initiate manipulation of objects and symbols, communication of both formal and informal mathematical concepts, and self-discovery of mathematics knowledge in the classroom [17].

III. The Potential of GeoGebra

From the discussion of the above two sections, a tool is critical in productive mathematical discourse because it affords enactive (manipulative based), iconic (image based), and symbolic (language based) representation of mathematical concepts [17]. In a technology learning environment, for example, dynamic geometry software (e.g., GeoGebra) has multi-representational features of enhancing manipulative, iconic and symbolic activities in the classroom [18]. GeoGebra is a free software. It contains geometry, algebra, spreadsheets, tables, graphing, statistics, and calculus in one easy-to-use package [19].

GeoGebra offers several teaching and learning opportunities in the classroom. It allows students to work independently or in small groups to explore mathematical concepts [20], create of visual and spatial element of solid objects [19], [21], to experience the interrelated connection between the manipulative, image, and language of a mathematical concept simultaneously [18].

IV. Research Design

In a case study research design, 11 in-service mathematics teachers from a senior high school in Ghana agreed to take part in a two-year professional development where they explore the pedagogical use of GeoGebra. As stated earlier, the overarching hypothesis of this research was that the didactical and semiotic affordance of GeoGebra would widen the features of learning space as a rich environment for a productive mathematical discourse. Following the engagement of the teachers in a professional development, this hypothesis was explored from the Vygotsky's socio-cultural perspective of a tool as a functional mediator in constructing mathematical concepts. Before describing the details of the professional development organized for the teachers, the framework adapted for this research is graphically presented and the constructs operationalized.

A. Conceptual Framework of the Didactical and Semiotic Affordance of GeoGebra

Vygotsky's socio-cultural perspective of a didactical and semiotic use of a tool [8] frames the discussion of widening the learning space to enable students in an engaged mathematical discourse (see Fig. 1).

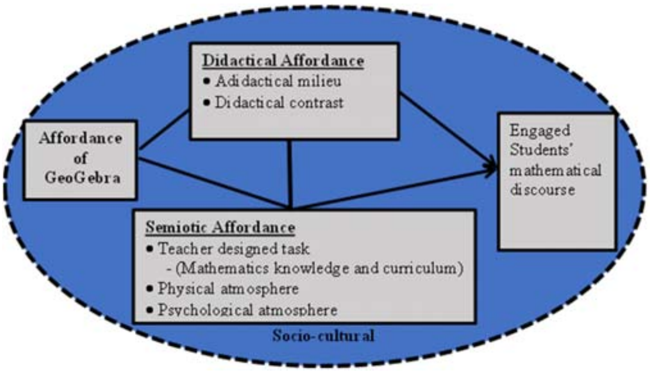

Fig. 1 Didactical and semiotic affordance of GeoGebra

Mathematical discourse is operationalized as a setting where students have the opportunity to manipulate GeoGebra and use the feedback from it to generate arguments leading to a meaningful construction of concepts. The adidactical affordance is the student's ability to use GeoGebra to explore mathematical concepts with little or no support from the teacher. This includes using the GeoGebra to conjuncture and generalize mathematical concepts. It also involves using the GeoGebra to either correct or confirm their solution to the tasks presented to them. The didactical contrast is operationalized as how the teacher uses the GeoGebra to scaffold students' mathematics learning. It includes using the GeoGebra to facilitate independent or pair work.

The semiotic affordance is explained to encompass how the teacher uses the GeoGebra to repurpose the learning goals in the mathematics curriculum to meet the needs of the individual learner. It also involves how the learners draw upon their prior knowledge with the GeoGebra. It also involves how teachers use GeoGebra to expand the physical and psychological atmosphere of the classroom. Physical atmosphere involves, the availability of resources that enable learners to count objects, manipulate and test ideas, classify objects into subsets, record their observations, solve problems and puzzles through games, construct geometric patterns, and draw and interpret graphs [6]. The psychological atmosphere involves a setting where the emphasis is on self-directed inquiry and exploration of mathematical concepts.

B. The Professional Development Intervention

A sequence of activities was designed and implemented in the professional development mediated with GeoGebra. The professional development followed three phases. The first phase involved the preliminary exploration from the teachers' perspective to understand how technology could be adopted in their school. The first phase lasted for two months. Based on the initial understanding, the second phase started, and it lasted for six months. A concrete example of an actual teaching experiment involving the principles of the TDS and semiotic mediation was modelled and presented to the participating teachers for practice and replication. The teachers worked in groups (2 or 3 members) to design GeoGebra-based mathematics lessons. The activities included in phase two were more cyclic and iterative. Each activity was used to inform the subsequent one. In situations where teachers had difficulties, remedial workshops were offered to address the preceding activity, thus making the activities of the professional development cyclic. The teachers used the support offered by the researchers who doubled as a facilitator to improve the quality of the lessons they developed (iterative). After repeated refinement, the final lesson artefacts were used in the classroom by the teachers (phase three). The researcher observed and recorded key moments of the lesson in the field notebook.

C. Sources of Data and Participants

Interviews, lesson artefacts, and lesson observation were collected. At each phase of the professional development programme, the participants were interviewed (lasting 20-30 minutes) to gain an understanding of how they used the GeoGebra to promote productive mathematical discourse in the classroom. The teachers developed lesson artefacts which contain objects, graphs, and or animations in the GeoGebra window to facilitate concept formation. The lesson artefacts also outline the sequence of instructional activities to engage the students in both minds- and hands-on practice of the concepts.

For this article, data from Teacher 1 and Teacher 2 are reported. The names are pseudonyms. They were the teachers committed to using their artefacts in the first and second years upon multiple iterations and refinement. Table I illustrates the background of Teacher 1, and Teacher 2. The study followed the protocols of the Human Ethics Committee, University of Otago (Number 17/014).

TABLE I
BACKGROUND INFORMATION ABOUT THE TEACHERS

Background information	Teacher 1	Teacher 2
Class taught	SHS 2 (students aged 16 years)	SHS 3 (students aged 17 years)
Topic taught	Circle theorem	Enlargement
Age	40-45 years	25-30 years
Qualification	Bachelor of education in mathematics	BSc Maths with a diploma in the teaching profession Master's degree (In progress)
Teaching experience	14 years	6 years
Prior experience with technology	MS Word, PowerPoint, and Corel Draw	MS Word, Excel, SPSS and MATLAB

D. Data Analysis

The approach of thematic analysis involving iterative, data-driven inductive, and deductive coding techniques were used [22]. Deductively, an open-ended coding technique was used to label the significant features of the data that reflect the affordance of GeoGebra towards a productive mathematical discourse. Iteratively, the text was read repeatedly to check for possible multiple meanings of codes and recognition of patterns and themes in relation to the research question. The initial inductive codes such as dialogue, group work, self-discovery, feedback, hands-on practice, manipulation, drill and practice, verification of solution, and access were then grouped and compared with the deductive codes of the didactical and semiotic framework [10], [16].

V. Results

The findings are reported under the broad headings of didactical and semiotic affordance of GeoGebra. Three sources of data: interviews (I), lesson artefact (LA), and lesson observation (LO) from each participant were integrated into reporting the findings. An annotation is provided for the excerpts used in this report to indicate the source of the data. For example, Teacher 2_ I means the data relate to an interview from Teacher 2. Similarly, Teacher 2_LA means the data relate to a lesson artefact from Teacher 2, and Teacher 2_LO means the data are a lesson observation from Teacher 2.

A. Didactical Affordance of GeoGebra

Evidence in the data showed that the teachers were able to use GeoGebra to provide adidactical and didactical mediation for students towards the construction of enlargement and circle theorem concepts. For example, in Teacher 2's lesson on enlargement with a scale factor, k (Fig. 2), he provided the students with a pre-designed artefact in which the slider feature in the GeoGebra was used to determine the size and position of an object with respect to the center of enlargement (COE) when it is enlarged by a scale factor (e.g., k = 0.3, 1.5, 1, etc.). The slider allowed students to practice repeatedly the conception of the orientation of an object when it is enlarged with scale factor, k.

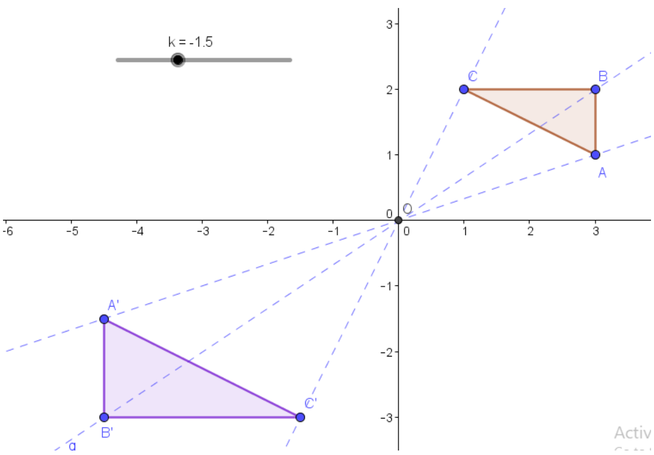

Fig. 2 Adidactical milieu affordance of GeoGebra [Teacher 2_LA]

It was observed during the lesson enactment that as Teacher 2 dragged the slider (see Fig. 2) to adjust the values of k, the students realized without heavily relying on the teacher's verbal explanation (adidactical milieu) that when $k < -1$, the size of the image (triangle A1B1C1) is enlarged, but it is at the opposite side of the object (triangle ABC) with respect to the COE. On the other hand, when $k = -1$, the size of the image is the same as the object, but it is on the opposite side of the object with respect to the COE. When $-1 < k < 0$, the size of the image is reduced but it is at the opposite side of the object with respect to COE; when $k = 0$, no image is formed; $k = 1$, the image size is the same as the object and at the same side of the object with respect to COE; and when $0 < k < 1$, the size of the image is reduced, it is at the same side of the object with respect to COE (see Fig. 2).

Again, it was observed that the prompt feedback from the GeoGebra as the slider was dragged motivated the students to do repeated practice on their own. The repeated practice made them privately rethink about their solutions before comparing their answers with that of their friends. This observation was consistent with what Teacher 2 recounted during the interview after his lesson:

> Students seemed happy and enthused playing with the tools in the GeoGebra. They talk about their answers with their friends. The GeoGebra helps them to confirm their solutions. They are happy when they get their answers correct. [Teacher 2_I]

It was also apparent in the lesson plan of Teacher 2 and Teacher 1 how they use the animations in GeoGebra to build students' trust in self-discovery, team spirit and dialogue. These trusts are characterized as didactical contrast. For example, Teacher 2 used the GeoGebra to scaffold students' mathematics learning by using tools to facilitate independent or pair work. In Fig. 3 for example, Teacher 2 offered the opportunity for students to work in small groups (2 or 3 students) by observing the animations of solid figures. Teacher 2 activated the 2D, 3D, and algebra views simultaneously which facilitated the students to write the formula connecting the scale factor (k) of an enlargement, volume of the image of the object (enlarged figure) and volume of the object (original figure).

It was observed during the lesson enactment that, not only did the pre-designed artefact in Fig. 3 provided a more readily usable and accurate object for the students to manipulate, but it also made the students appreciate both the algebraic and geometric connection between the areas and volumes of similar plane figures and their corresponding images. From the comment of Teacher 2, when objects are hurriedly drawn on the chalkboard during the class session, it sometimes distorts the basic principles behind the concept.

> Using the GeoGebra, the students can make a proper connection between the concept to the animation. In the normal session [using chalkboard], the diagrams we draw sometimes do not bring the full meaning of the concepts we want to put across because we rush in drawing them on the board. [Teacher 2_I]

Commenting on the didactical affordance of GeoGebra in teaching enlargement, Teacher 2 drew our attention to the fact that the students need to have technical skills in ICT to enhance the smooth navigation of the tools in the GeoGebra window.

> In investigating areas and volumes of similar figures, students were supposed to do more of the activities by themselves and that requires a lot of skills and technicalities in the use of ICT. Some of the students struggled to switch between the algebra, 2D and 3D views. [Teacher 2_I]

B. Semiotic Affordance of GeoGebra

As earlier explained, the teacher design task involves content-specific instructional technology where the teacher has the responsibility of linking technology with specific content in the mathematics curriculum for instructional purposes.

Following the activities of the professional development, the teachers demonstrated varied use of GeoGebra to enact the content of the mathematics curriculum. They were able to use the GeoGebra to facilitate the transition of the prior knowledge of their students to the new mathematical concepts. In the following excerpt from the lesson plan, Teacher 2 was able to repurpose the mathematics curriculum by providing a sequence of learning activities that led the students to explore the properties of enlargement with a scale factor (*teacher design task*).

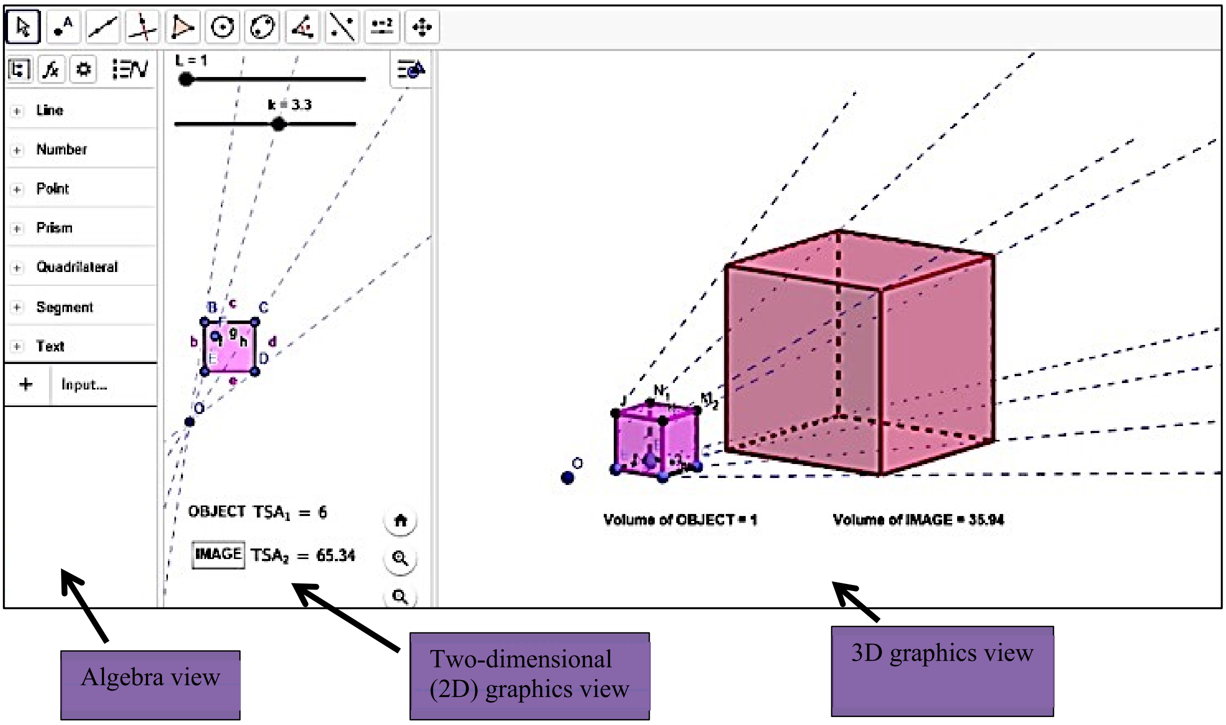

Fig. 3 Didactical contrast affordance of GeoGebra in enlargement lesson [Teacher 2_LA]

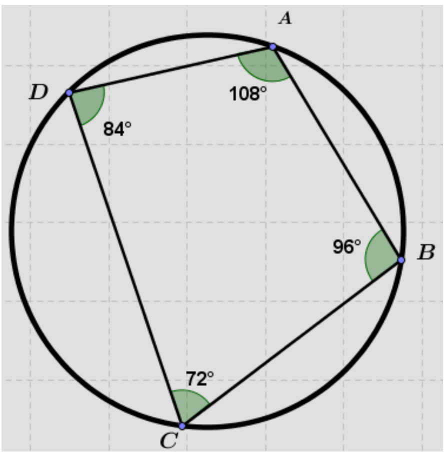

Fig. 4 Semiotic affordance of GeoGebra in circle theorem lesson [Teacher 1_LA]

> The teacher drew various objects in the GeoGebra window for exploring the properties of enlargement with a scale factor (k). For each given scale factor (k), guide students to observe the image cube on the GeoGebra 3D graphics view and record its total surface area and volume in the table provided in the worksheet.

Also, there was evidence across the three data that the teachers were able to use GeoGebra to widen the physical atmosphere of the classroom. The excerpt below is an illustrative example of how the teachers used the GeoGebra to provide animations which served as virtual concrete to enhance recording, manipulation and testing of mathematical ideas, construction, and interpretation of geometric objects.

> Students, in their small groups, observe the animations in the GeoGebra window. Add the interior opposite angles (< ABC and < ADC; < BAD and < BCD) as any of the points is dragged (see Fig. 3). Write down your observation for a discussion. [Teacher 1_LA]

When asked about the affordance of GeoGebra in exploring mathematics concepts, Teacher 2 and Teacher 1 were unanimous in the view that it is a kind of software that promotes a self-directed and inquiry approach to learning. For example, Teacher 1's comment indicates that GeoGebra does not only reduce the abstract nature of some of the concepts in mathematics, but it also brings a kind of playing atmosphere in the classroom where students freely interact with the tool which leads to the formation of mathematical concepts (*psychological atmosphere*). It was observed that Teacher 1 was able to reconfigure the existing everyday objects within the students' socio-cultural environment to build their knowledge of the circle theorem.

> Using the artefact [GeoGebra] I noticed that it really helped the students to grasp the concept because it brings out the whole concept to the students. Whilst the children

> are playing with it or working with it, it isn't something that is more abstract, it brings the whole concept to them. [Teacher 1_I]

Similarly, it can be inferred from the comment of Teacher 2 that GeoGebra can be used not only for the purpose of reference and verification of solutions, but also as a source to get the correct answers and deduction of mathematical formulae.

> I wanted the learners to find out the formulae for the scale factor. For example, finding the scale factor for the solid figure, the total surface area of the image and that of the object. I adjusted the sliders several times. The learners then recorded the results. They compared with their colleagues and realised how the formulae came about. That's how I used the GeoGebra. [Teacher 2_1]

From Teacher 2's comment above, adjusting the slider several times provided multiple embodiments of the concept of scale factor within a short time. The three-dimensional (3D) feature in the GeoGebra enabled different orientations of the solid (cube) to be explored in relation to the scale factor given. It is impracticable to achieve multiple embodiments of a concept in a normal classroom where objects have to be drawn on a static two-dimensional chalkboard.

VI. Discussion

This study explored the didactical and semiotic affordance of GeoGebra in promoting a productive mathematical discourse. Overall, the results indicated that following the engagement of the teachers in a two-year professional development involving designing and enacting technology-based mathematics lessons, they were able to use the construction tools in the GeoGebra to provide adidactical, didactical, and semiotic mediation for students to construct the concepts of enlargement and circle theorems.

As earlier explained the adidactical milieu is the student's ability to use a tool to explore mathematical concepts with little or no support from the teacher. Consistent with the notion of the adidactical milieu, the results of the study indicated that the slider feature in the GeoGebra offered opportunities for students to practice repeatedly the concepts of enlargement and circle theorems without heavily relying on the teacher's verbal explanation. This finding is consistent with that of Uwurukundo et al. [20] who reported that GeoGebra enhanced students to work independently in exploring the concept of linear functions with minimum assistance from their teacher.

With regard to the affordance of didactical contrast, the results of the study showed that GeoGebra provided an environment where self-discovery, team spirit, and dialogue were promoted. Teachers were able to use the GeoGebra to scaffold students' mathematics thinking where they observed animations of geometric and algebraic reasons of solid figures and drew conclusions thereafter. This study also reported that when objects are hurriedly drawn on the chalkboard during the class session, it sometimes distorts the basic principles behind the concept of the subject matter. This implies that as teachers pre-designed the lesson artefact using GeoGebra, it offers a more readily usable and accurate object for the students to manipulate with it. The literature reviewed for this study did not report this finding.

As a semiotic tool, the results of the study indicated that GeoGebra offered teachers opportunities to repurpose the objectives of the mathematics curriculum to meet the needs of their learners (teacher design task) and expanded the physical and psychological atmosphere of the classroom. Following the activities of the professional development, the teachers demonstrated varied use of GeoGebra to enact the content of the mathematics curriculum. For example, they were able to provide a sequence of learning activities that facilitate the transition of the prior knowledge of students to the new mathematical concepts. As mentioned in the literature review, in the notion of semiotic mediation, an artefact may be used by both students and teachers to accomplish mathematical tasks [13].

Another important finding was that as students adjusted the slider several times, it provided multiple embodiments of the concept of scale factor within a short time (physical atmosphere) which enhanced self-directed and inquiry approach to learning (psychological atmosphere). A prior study [18] indicated that GeoGebra has a feature which allows the user to insert real-world pictures that embody mathematical concepts such as parabola, circle, parallel lines, slope, and Pythagoras theorem [18]. This, according to the authors, enables students to see and manipulate the grid view in the GeoGebra as an extension of a concrete geoboard which allows them to draw, sketch, measure, and test mathematical theories and model geometric patterns. Again, as students drag the slider, it dynamically provides the quintessential visual and spatial element of solid objects [18]. Thus, the dynamic spatial ability of the GeoGebra creates a pedagogical space of experiencing the interrelated connection between the manipulative, image, and language of a mathematical concept simultaneously. GeoGebra offered prompt feedback which motivated students to repeatedly practice concepts on their own where they privately rethink about their solutions before comparing their answers to their friend's. These opportunities are difficult to achieve through the use of a static two-dimensional chalkboard.

While it was interesting to show in this study that GeoGebra afforded both didactical and semiotic affordance in engaging students in a deep discourse of learning the concepts of enlargement and circle theorems, caution is needed in adopting this technology in the classroom. The results of the study showed that the students need to have basic technical skills in ICT to enhance smooth navigation of the tools in the GeoGebra window. This result reflects that of [20] which also found that there are some possible pitfalls the users of GeoGebra need to be aware of: focus on the visual levels to the detriment of mathematical proofs of some concepts, some valuable time required to learn the basics of the software, focus on technology and not the mathematical concept, focus on the answer obscures the details and structure of the concept being explored, lack of basic procedural skills to operate the software, and it requires a number of design decisions to come out with a concrete artefact.

VII. Conclusions and Implications

Teachers were able to use the construction tools in the GeoGebra to provide adidactical milieu, didactical contrast, and semiotic mediation for students to construct concepts of enlargement and circle theorems. Teachers achieved this pedagogical success following their engagement in a two-year professional development involving designing and enacting technology-based mathematics lessons. The findings reported in this study have two key pedagogical implications. First, using technology to engage students in an interactive setting is critical to many mathematics educators. Therefore, the didactical and semiotic affordance of GeoGebra articulated in this article is not to persuade the readers to accept it as a new substitute for teachers' explanation of concepts in the classroom, but to have them understand certain features of GeoGebra that can be employed to (i) enhance self-discovery, team spirit, and dialogue; (ii) provide multiple embodiments of the mathematical concepts, (iii) provide dynamic visual and spatial element of solid objects; and (iv) provide animations towards geometric and algebraic reasons. Second, professional development is important towards enhancing teachers' competence in implementing curriculum innovation. This study provides an example of technology-oriented professional development within the context of Ghanaian mathematics curriculum. The pedagogy of didactical and semiotic mediation rooted in the socio-cultural learning theory was modelled and presented to the participating teachers for practice and replication. The results reported in this study indicated that this approach of professional development is potential in enhancing teachers' competence in using technology to orchestrate a productive mathematical discourse.

VIII. Limitation and Future Research

The study reported a caution toward effective use of GeoGebra in mathematical discussion. Students need to have basic technical skills in ICT to enhance smooth navigation of the tools in the GeoGebra window. This implies that not only teachers need orientation prior to the use of GeoGebra, but students too. To advance the discussion of effective use of technology in teaching and learning, an understanding of students' readiness in terms of basic knowledge and skills in computer window through research is critical.

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