

Modeling and Stability Analysis of Viral Propagation in Wireless Mesh Networking

Haowei Chen, Kaiqi Xiong

Abstract—We have developed a better model for understanding the dynamics of malware spread in WMNs in this paper. The suggested model provides an insight into how viral propagation with energy exhaustion and various dispersed node densities might function. Based on a theoretical examination of the suggested model, we conclude that the threshold parameter could be used to identify the dynamics of viral spread globally. When the threshold is less than 1, the virus may be contained, but if it is greater than 1, a pandemic may result. Lastly, we discuss the various viral propagation strategies in relation to the distributed node densities and communication radii in WMNs. The aforementioned numerical simulation findings could serve as a guarantee of the theoretical analyses' correctness.

Keywords—Bluetooth Security, Malware Propagation, Wireless Mesh Networks, Stability Analysis.

I. INTRODUCTION

WIRELESS mesh networks (WMNs) claim to provide improved wireless network connectivity and a greater coverage area. They have some interesting features [11]- [16]. However, a growing number of malicious programs have emerged in wireless networks due to the vulnerabilities of wireless networks, such as the openness of media, terminal mobility, dynamic changes of network topology, cooperative algorithm, lack of centralized monitoring and administrative points, and lack of centralized monitoring and administrative points, posing a serious threat to wireless network security [1]- [5].

Conventional wired networks have been hit by numerous harmful virus outbreaks, as the Melissa or Codered [6] worms. Networks with wireless interfaces on laptops have quickly increased in size and complexity as a result of developments in advanced wireless technology. Authors' attention-grabbing wireless ad hoc networks construct a topology of nodes that can travel without any pre-established structures. It was possible to observe an analysis and simulation of malware propagation across an ad hoc network in [7] and [8] etc. However, WMNs, a modern source of assault inspiration, have not been considered in these works. Sikdar and Ramachandran [9], [10] have researched how malware spreads across phone networks, but the model was unable to identify certain categories when several concurrent entities were present. Also, these research did not analyze the connection between the quantity of nodes and the propagation's behavior.

In order to more accurately depict the characteristics and features of viral propagation in WMNs, we investigate the

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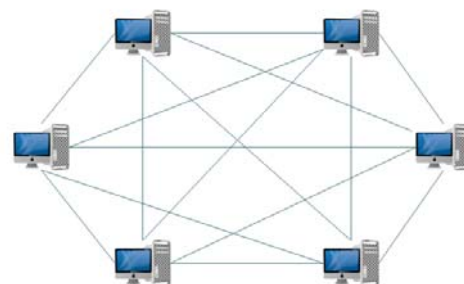


Fig. 1 The topology of a WMN structure

attacking behavior of potential viruses in WMNs by using an enhanced epidemic dynamic systematic model. The dispersed density of nodes in WMNs, the communication radius between nodes, and the energy consumption of each node are taken into account in this model. We examine the dynamic system's equilibrium using this model to analyze the stability of virus prevalence. The paper is set up as follows: We define the formulation in Section II and describe the basic topology of WMNs. The stability of virus spreading at equilibrium is proposed in Section III's analysis of the equilibrium. Numerical simulations are presented in Sections IV and V to demonstrate the accuracy of the theoretical results and conclusion are draw to gain the insight for future prediction.

II. THE PROPOSED MODEL

A. System Description

Suppose that we have a wireless sensor network composed of N nodes. Besides The nodes are uniformly distributed in $L \times L$ area; that is, the density of nodes in the network is $\rho = N/L^2$ and the wireless communication range of every node is r . The topological structure of a WMNs is shown in Fig. 1.

Based on the existing *SIR* epidemic model, the nodes in WMNs could be classified into the flowing states:

- Susceptible state (S): nodes in S have not been infected, and these nodes are vulnerable to malware in WMNs.
- Infected state (I): nodes have been infected by malware and have the ability to infect the other nodes in WMNs.
- Recovered state (R): nodes have been manually installed a detection tool that can identify and remove malware, or nodes have been installed specific software patch to eliminate the node vulnerability exploited by malware.

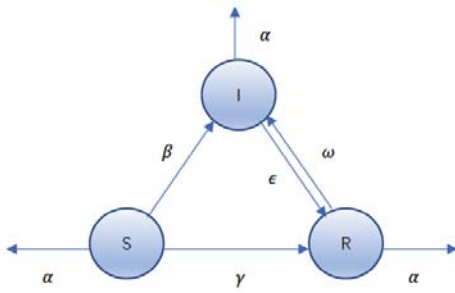


Fig. 2 Transition relationship among different states for a single node

We consider the following state transitions among these three states:

- Clients may immunize their nodes in states S and I with probabilities ω and γ , respectively.
- Due to exhaustion of energy, some nodes become off-line nodes with probability α .
- In the state of I , nodes infect susceptible S with effective infection rate β .
- In the state of R , nodes may become susceptible nodes with probability ε .

Transition relationships among node states are described in Fig. 3.

B. Model Derivation

The communication area of a node is denoted by s and the density of susceptible nodes in a unit area in WMNs is denoted by ρ . Then the following equations hold:

$$\begin{aligned} \rho &= \frac{S}{L^2}, \\ s &= \pi r^2, \\ S'' &= s\rho(t) \end{aligned} \quad (1)$$

From (1), we get

$$S'' = \pi r^2 \frac{S}{L^2} \quad (2)$$

According to state transition relationships in Fig. 2, the mathematical model of malware propagation in WMNs can be derived as follows:

$$\begin{aligned} \frac{dS}{dt} &= \alpha N - \frac{\pi r^2}{L^2} \beta SI - (\alpha + \omega)S + \varepsilon R, \\ \frac{dI}{dt} &= \frac{\pi r^2}{L^2} \beta SI - (\alpha + \gamma)I, \\ \frac{dR}{dt} &= \omega S + \gamma I - (\alpha + \varepsilon)R. \end{aligned} \quad (3)$$

Let

$$\xi = \frac{\pi r^2}{L^2} \beta \quad (4)$$

Then, system (3) can be written as

$$\begin{aligned} \frac{dS}{dt} &= \alpha N - \xi SI - (\alpha + \omega)S + \varepsilon R, \\ \frac{dI}{dt} &= \xi SI - (\alpha + \gamma)I, \\ \frac{dR}{dt} &= \omega S + \gamma I - (\alpha + \varepsilon)R. \end{aligned} \quad (5)$$

III. STABILITY ANALYSIS OF EQUILIBRIUM

In this section, we will find the equilibrium of system (5) and investigate its stability. The equilibrium of system (5) is given by solution of

$$\begin{aligned} \alpha N - \xi SI - (\alpha + \omega)S + \varepsilon R &= 0, \\ \xi SI - (\alpha + \gamma)I &= 0, \\ \omega S + \gamma I - (\alpha + \varepsilon)R &= 0. \end{aligned} \quad (6)$$

Let $\xi SI - (\alpha + \gamma)I = 0$. Then, we have $\bar{I} = 0$ or $\bar{I} > 0$ and $\bar{S} = (\alpha + \gamma)/\xi$.

If $\bar{I} = 0$, then we have malware-free equilibrium

$$E_0 = (\bar{S}_0, \bar{I}_0, \bar{R}) = \left(\frac{(\alpha + \varepsilon)}{\alpha + \varepsilon + \omega} N, 0, \frac{\omega S}{\alpha + \varepsilon} \right) \quad (7)$$

If $\bar{I} > 0$, we have endemic equilibrium

$$\begin{aligned} \bar{E} = (\bar{S}_1, \bar{I}_1, \bar{R}_1) &= \left(\frac{\alpha + \gamma}{\varepsilon}, \right. \\ &\left. \frac{\xi(\alpha + \varepsilon)N - (\alpha + \gamma)(\alpha + \varepsilon + \omega)}{\xi(\alpha + \varepsilon + \omega)}, \frac{\omega \bar{S}_1 + \gamma \bar{I}_1}{\alpha + \varepsilon} \right) \end{aligned} \quad (8)$$

Let

$$\tau = \frac{\xi(\alpha + \varepsilon)}{(\alpha + \gamma)(\alpha + \varepsilon + \omega)} N \quad (9)$$

Note that, the endemic equilibrium is valid only if $\tau > 1$

A. Malware-Free Equilibrium and Its Stability

Lemma 1. lemma 1 The malware-free equilibrium is locally asymptotically stable if $\tau < 1$ and unstable if $\tau > 1$

Proof: since $E_0 = (\bar{S}_0, \bar{I}_0, \bar{R}) = \left(\frac{(\alpha + \varepsilon)}{\alpha + \varepsilon + \omega} N, 0, \frac{\omega S}{\alpha + \varepsilon} \right)$, the characteristic equation of system (5) at malware-free equilibrium E_0 is

$$\det \begin{pmatrix} -(\alpha + \omega) - \lambda & -\xi \bar{S}_0 & \varepsilon \\ 0 & \xi \bar{S}_0 - (\alpha + \gamma) - \lambda & 0 \\ \omega & \gamma & (\alpha + \varepsilon) - \lambda \end{pmatrix} = 0 \quad (10)$$

and it is equivalent to

$$[\lambda^2 + (2\alpha + \varepsilon + \omega)\lambda + \alpha^2 + \varepsilon\alpha + \alpha\omega][\xi \bar{S}_0(\alpha + \gamma)\lambda] = 0 \quad (11)$$

Equation (11) has a characteristic root $\lambda_1 = \xi \bar{S}_0 - (\alpha + \gamma) = (\alpha + \gamma)(\tau - 1)$ and also roots of

$$\lambda^2 + (2\alpha + \varepsilon + \omega)\lambda + \alpha^2 + \varepsilon\alpha + \alpha\omega = 0 \quad (12)$$

Obviously, according to the relationship between roots and coefficients of quadratic equation, there is no positive real part characteristic root of (12). Hence, when $\tau < 1$, (11) has no

positive real root, and malware-free equilibrium E_0 is locally asymptotically stable. When $\tau > 1$, (11) has a positive root, thus, malware-free equilibrium E_0 is an unstable saddle-point. ■

Furthermore, the following theorem holds.

Theorem 1. *The malware-free equilibrium is globally asymptotically stable if $\tau \leq 1$*

Proof: From the first equation of system (5)

$$\frac{dS}{dt} \leq (\alpha + \varepsilon)N - (\alpha + \varepsilon + \omega)S \quad (13)$$

Thus

$$S \leq \frac{(\alpha + \varepsilon)N}{\alpha + \varepsilon + \omega} + \frac{S(0) - (\alpha + \varepsilon + \omega)}{\alpha + \varepsilon + \omega} e^{-(\alpha + \varepsilon + \omega)t} \quad (14)$$

as $t \rightarrow \infty$ we obtain $S \leq \frac{\alpha + \varepsilon}{\alpha + \varepsilon + \omega} N$ we consider a Lyapunov function, see [17].

$$\begin{aligned} \frac{dL(t)}{dt} &= \xi SI - (\alpha + \gamma)I \\ &\leq [\xi \frac{\alpha + \omega}{\alpha + \omega + \varepsilon} N - (\alpha + \gamma)]I \\ &= \frac{1}{\alpha + \gamma} (\tau - 1)I \leq 0 \end{aligned} \quad (15)$$

Hence the malware-free equilibrium E_0 is globally asymptotically stable. ■

B. Endemic Equilibrium and Its Stability

Now, we consider the local stability of endemic equilibrium \bar{E} . The characteristic equation of system (5) at endemic equilibrium \bar{E} is:

$$\det \begin{pmatrix} -\xi \bar{I}_1 - (\alpha + \omega) - \lambda & -\xi \bar{S}_1 & \varepsilon \\ \xi \bar{I}_1 & \xi \bar{S}_1 - (\alpha + \gamma) - \lambda & 0 \\ \omega & \gamma & (\alpha + \varepsilon) - \lambda \end{pmatrix} = 0 \quad (16)$$

and it is equivalent to

$$\lambda^3 + p_0 \lambda^2 + p_1 \lambda + p_2 = 0 \quad (17)$$

where $p_0 = 3\alpha + \varepsilon + \omega + \frac{\alpha + \varepsilon}{\alpha + \varepsilon + \gamma} N$; $p_1 = (\alpha + \varepsilon)[2\alpha + \gamma + \frac{\alpha + \varepsilon}{\alpha + \varepsilon + \gamma} N] + \alpha\omega + \xi^2 \bar{S}_1 \bar{I}_1$; $p_2 = \alpha \xi (\alpha + \varepsilon)(\bar{I}_1 - \bar{S}_1) + \alpha(\alpha + \gamma)(\alpha + \gamma + \omega) + \alpha \xi (\gamma \bar{I}_1 - \omega \bar{S}_1)$.

Obviously, p_0, p_1, p_2 are all positive and $p_0 p_1 - p_2 > 0$. According to the theorem of Routh-Hurwitz [18], it follows that the roots of (17) have negative real parts and therefore, the endemic equilibrium is locally asymptotically stable. From the above discussion, we can draw the following conclusion.

Lemma 2. *If $\tau > 1$, then the endemic equilibrium is locally asymptotically stable.*

Note the total number of the nodes in WMNs is relatively stable; that is, at time t , the number of nodes S , I , and R in states S , I and R , respectively satisfy:

$$S + I + R = N \quad (18)$$

then, the dynamic system (5) is equivalent to the following system:

$$\begin{aligned} \frac{dS}{dt} &= (\alpha + \omega)N - \xi SI - (\alpha + \omega + \varepsilon)S \\ &\quad - \varepsilon I \end{aligned} \quad (19)$$

$$\frac{dI}{dt} = \xi SI - (\alpha + \gamma)I(t)$$

clearly, when $\tau > 1$, system (19) has a unique positive equilibrium $\bar{E}(\bar{S}_1, \bar{I}_1)$, where

$$(\bar{S}_1, \bar{I}_1) = \left(\frac{\alpha + \gamma}{\xi}, \frac{\xi(\alpha + \varepsilon)N - (\alpha + \gamma)(\alpha + \varepsilon + \omega)}{\xi(\alpha + \varepsilon + \gamma)} \right) \quad (20)$$

Theorem 2. *When $\tau > 1$, the endemic equilibrium (\bar{S}_1, \bar{I}_1) of system (19) is globally asymptotically stable.*

Proof: Consider the following Lyapunov function

$$L(t) = \int_{\bar{S}_1}^S \frac{x - \bar{S}_1}{x} + \int_{\bar{I}_1}^I \frac{x - \bar{I}_1}{x} dx \quad (21)$$

Then

$$\begin{aligned} \frac{dL(t)}{dt} &= \frac{S - \bar{S}_1}{S} S'' + \frac{I - \bar{I}_1}{I} I'' \\ &= (1 - \frac{\bar{S}_1}{S}) [(\alpha + \varepsilon)N - \xi SI - (\alpha + \varepsilon + \omega)S - \varepsilon I] + \\ &\quad (1 - \frac{\bar{I}_1}{I}) [\xi SI - (\alpha + \gamma)I] \\ &\leq (1 - \frac{\bar{S}_1}{S}) [(\alpha + \varepsilon)N - \xi SI - (\alpha + \varepsilon + \omega)S] + \\ &\quad (1 - \frac{\bar{I}_1}{I}) [\xi = -(\alpha + \varepsilon)N \frac{S}{\bar{S}_1} (\frac{\bar{S}_1^2}{S} - 1)^2] \\ &\leq 0 \end{aligned} \quad (22)$$

The proof is completed. ■

Remark: By theorem 4, we obtain that $\tau > 1$ then endemic equilibrium \bar{E} of system (5) is globally asymptotically stable.

IV. NUMERICAL SIMULATION

We have demonstrated that the threshold τ influences whether worms are eliminated above the theory. The equations of the system we established in (5) will attain a malware-free equilibrium state and be stable in the situation of $\tau \leq 1$, which allows for the cleaning of malware in WMNs. In the event that *tau geq1* malware is consistently found in WMNs, the system will enter a pandemic state. Two sides of a numerical simulation should be run to support the theoretical results that have been shown.

A. Impact of Physical Communication

We are concerned in how physical interaction among the web nodes affects it. As shown in Figs. 3 (a)-(d), when r takes different values between 0.1 and 1.0, we perform the simulation with a threshold of 1. We discovered that the radius threshold is 0.8 dollars, and the outcome of the simulation validated the accuracy of our theoretical demonstration.

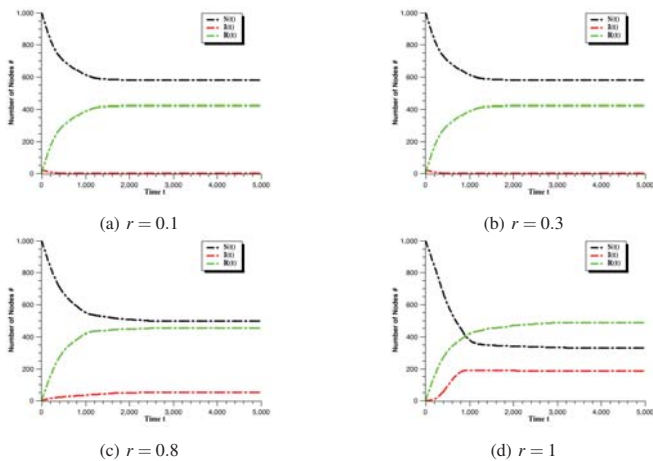


Fig. 3 Simulation results of different communication radius

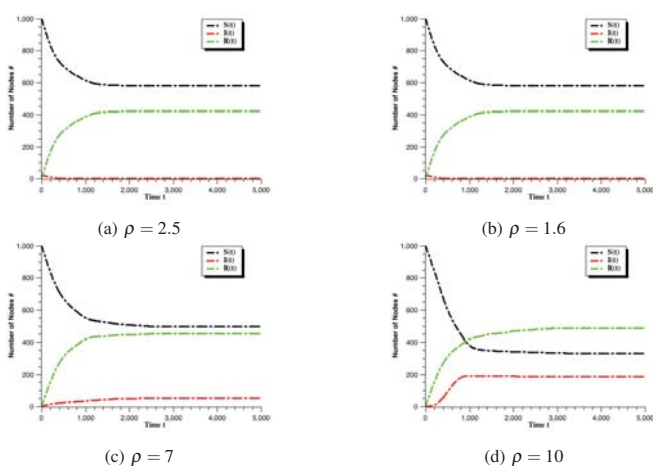


Fig. 4 Simulation results of different density

B. Impact of Physically Distributed Nodes

We are curious in how physical interaction among the web nodes affects it. As illustrated in Figs. 4 (a)-(d), when ρ has values ranging from 2.5 to 10, we conduct the simulation with a threshold of 1. Our research revealed that the density threshold is roughly 8, and the simulation's findings validated the accuracy of our theoretical justification.

V. CONCLUSIONS

We have developed a better model for understanding the dynamics of malware spread in WMNs in this paper. The suggested model provides an insight into how viral propagation with energy exhaustion and various dispersed node densities might function. Based on a theoretical examination of the suggested model, we conclude that the threshold parameter τ may be used to identify the dynamics of viral spread globally. When the threshold is less than 1, the virus may be contained, but if it is greater than 1, a pandemic may result. Lastly, we discuss the various viral propagation strategies in relation to the distributed node densities and communication radii in WMNs. The aforementioned numerical simulation findings could serve as a guarantee of the theoretical analyses' correctness.

According to research, lowering the communication radius or the distributed density of nodes is a good way to stop malware from spreading in WMNs. This paper's contribution offers a theoretical framework for forecasting and limiting malware transmission in WMNs. Last but not least, this work has to be expanded to include more instances involving several virus competitors. Also, it would be beneficial to apply this paper's research to the spreading processes utilizing alternative methodologies.

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