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Appraisal of Relativistic Effects on GNSS Receiver Positioning

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***Abstract*—**The Global Navigation Satellite System (GNSS) started with the launch of the United State Department of Defense Global Positioning System (GPS). GNSS systems has grown over the years to include: GLONASS (Russia); Galileo (European Union); BeiDou (China). Any GNSS architecture consists of three major segments: Space, Control and User Segments. Errors such as; multipath, ionospheric and tropospheric effects, satellite clocks, receiver noise and orbit errors (relativity effect) have significant effects on GNSS positioning. To obtain centimeter level accuracy, the impacts of the relative motion of the satellites and earth need to be taken into account. This paper discusses the relevance of the theory of relativity as a source of error for GNSS receivers for position fix based on available relevant literature. Review of relevant literature reveals that due to relativity; Time dilation, Gravitational frequency shift and Sagnac effect cause significant influence on the use of GNSS receivers for positioning by an error range of ± 2.5 m based on pseudo-range computation.

***Keywords*—**GNSS, relativistic effects, pseudo-range, accuracy.

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I. INTRODUCTION

SINCE the introduction of the first GNSS, various advancement and more recent systems have been installed to improve previous ones and also to meet the standards and needs of current applications [35]. Artificial satellites rapidly continue to affect civilian applications ranging from telecommunication, television, transport systems, cars, google, surveying to other navigation-based services [57]. The level and nature of impact of GNSS in daily life activities of mankind makes it significant to study the routes and motion of satellites, which remains an important factor for accurate navigation and other purposes particularly for satellites intended for position applications, deformation monitoring and remote sensing. Basically, the core function of every GNSS operation is to accurately fix the position of the satellite and the factors relating to the transmitted signal and how the signal flows onto the earth's surface [32]. Determining positions of GNSS is basically dependent on the stability and precision of the satellite clocks synchronization on board the satellite vehicles [28]. However, GNSS performance is affected with various challenges which makes these clocks inaccurate and unstable [34]. The relative motion between satellites and earth to obtain positional parameters are distant away from each other orbiting with high speeds [7], it is therefore necessary to support position determination with the theories of special and general relativity [6].

II. GLOBAL NAVIGATION SATELLITE SYSTEMS

GNSS has been used for civilian purpose for determining locations of points and navigation [20]. It is centered on the use of artificial satellite vehicles and their associated augmentation components [43]. GNSS is built on satellites arranged and grouped in space [38] covering the global world to transmit signals to calculate the location (longitude, latitude and height) of a receiver located on the surface or near the earth [39]. Currently GNSS encompass: United States' GPS, Russian Global Navigation Satellite System (GLONASS), European Union Galileo and the Chinese Beidou Navigation System [50], [23] and other augmentation systems [8].

Augmentation systems comprise of satellite based (Fig. 1) and ground based system which include: Wide-Area Augmentation System (WAAS), Local Area Augmentation System (LAAS), European Geostationary Navigation Overlay System (EGNOS), India Regional Navigation Satellite System (IRNSS), and Japanese Quasi-Zenith Satellite System (QZSS) [62], [46].

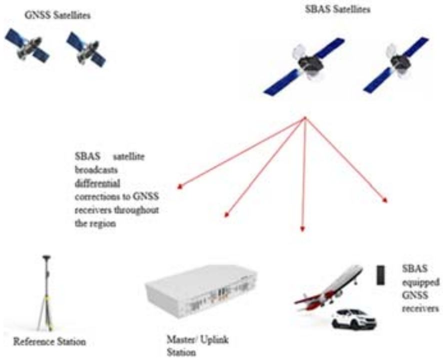

Fig. 1 Satellite-Based Augmentation System [49]

A. Basic Segments of GNSS

GNSS encompasses three main segments namely (Fig. 2); space, control and user segments.

The space segment is made up of space vehicles (SVs) operating on a constellation of several satellites orbiting the Earth in 12 sidereal hours. Each of the satellites is fixed with atomic clocks rapidly synchronizing to each other within nanoseconds [12], [40].

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The Control segment consists of monitor stations. These monitor stations measure signals from the SVs which are incorporated into orbital models for each satellite. The models compute precise orbital data (ephemeris) and clock corrections of every satellite [2]. The Master Control station uploads ephemeris and clock data to the SVs. The SVs then send subsets of the orbital ephemeris data to GNSS receivers over radio signals.

The User Segment consists of the receivers and the user community. GNSS receivers convert signals into position, velocity, and time using their local quartz clocks [19] for navigation, positioning and timing.

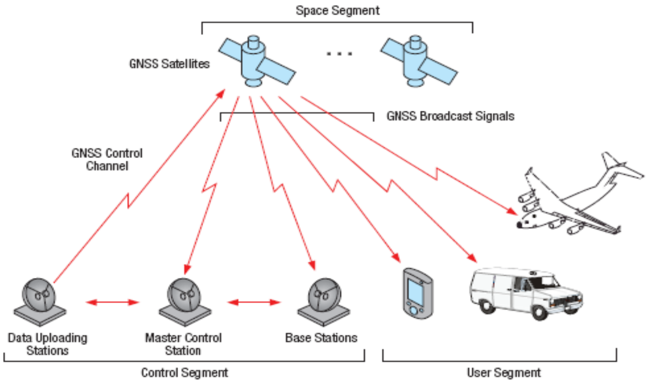

Fig. 2 Basic Segments of GNSS [49]

B. Determining Positions with GNSS Receivers

The method of operation for fixing position in satellite-based systems works on the speed of electromagnetic signals traveling at the speed of light [25]. Signals are transmitted from four or more satellites to determine three spatial locations and a time related component by a receiver far away in a different frame. Trilateration is the measuring principle used by the satellite to measure distances between the receiver and the different satellites based on their geometrical properties [2], [11]. Distance measurements are calculated from the signals propagated when the clocks of the satellites and the users are synchronized in a particular reference frame which are mutual to one another [35], [29]. Broadcast satellites send signals in an encrypted message form to receivers [58]. The message is embedded with information about the emission time of signal, satellite identity and coded position of the satellite. By comparing the time of the satellite vehicle and receiver, the distance is calculated [47]. Fig. 3 illustrates the determination of distance based on four satellite vehicles.

C. Differential GNSS Positioning

Differential positioning techniques (DGNSS) are based on one or more reference GNSS receivers located at known positions [61]. Differential GNSS positioning techniques correct bias errors at one location with bias errors measured at a known position [63]. A reference receiver, or base station, computes and adjusts corrections for each satellite signal to fix the position of the unknown point [21]. This principle of relative positioning contains more precise information to support centimeter level accuracy in positioning, navigation and timing [54].

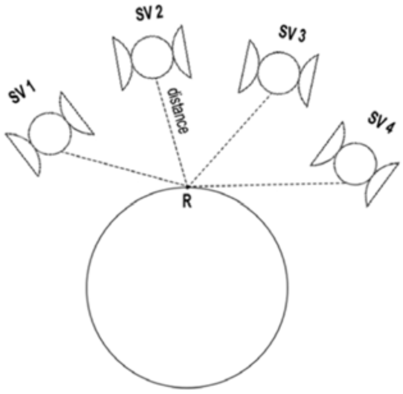

Fig. 3 Determining distances between satellites and receivers [47], [35]

III. GNSS Error Sources and Mitigation

The path followed by a GNSS satellite enables a receiver to receive a clear line of sight to obtain signals for navigation and positioning. These signals are affected by both geometrical and physical parameters from transmission to reception introducing errors in the distance measured. These errors span from various sources [17] and consequently disturb positional accuracy. Reference [33] classifies these errors according to the nature of the error which constitutes clock related errors from both the satellites and the receivers. Errors affecting signal transmission, which encompasses multipath and the movement of the satellites and receivers relative to each other. Temporary and intentional errors include satellite orbit parameters, signal jamming and spoofing.

A. Errors Affecting Satellite/Receiver Clocks

GNSS errors can be recognized in both satellites and receiver clocks. Satellite have onboard very accurate and expensive atomic clocks which make it very stable to about 1 to 2 parts in 10^{13} per day [59]. However, GNSS receivers are made of inexpensive crystal clocks of low accuracy [15]. This introduces errors in the receivers to a higher degree than the satellite. Reference [42] proposed that cesium and rubidium clocks can be used instead of the crystal clocks to minimize clock errors in the receivers. A minimum of four satellites for position fix which solves four position parameters provides an alternative solution for the receiver and using signals from all available satellites can also help improve the accuracy of the clocks by rectifying clock biases [30].

B. Errors Affecting Signal Propagation

Signals are propagated through a long travel distance between the satellite and the receiver. These signals, in the course of transmission through the atmosphere, are disturbed by the ionosphere and the troposphere [36]. The original line of sight of the signal can also be reflected on surfaces around the receiver to generate multiple paths.

Table I presents the various GNSS system errors based on pseudo-range computations. The degree of the errors (Table I) affect positioning accuracy depends on the satellite geometry used.

Ionospheric Errors

The ionosphere forms part of the atmosphere covering approximately 50 to 2000 km above the earth surface. This part of the atmosphere contains ions that affect signal that travels through it causing plasma bubbles [24] and ionospheric refraction of the signal [43]. The errors form an important part of the GNSS positioning errors [56] reaching about 300 ns [45] in certain situations.

The error is corrected by using dual frequency receivers [13], [44] and satellite-based augmentation systems are used for correction in single frequency receivers [60].

From (1) and (2), the combined code and phase observations have the sense that they are code and phase observables at a special frequency. The standard (first order) ionosphere-free phase and code combinations can be represented as:

$$\frac{f_1^2\delta_p(f_1) - f_2^2\delta_p(f_2)}{f_1^2 - f_2^2} = 0 \tag{1}$$

$$\frac{f_1^2\delta_g(f_1) - f_2^2\delta_g(f_2)}{f_1^2 - f_2^2} = 0 \tag{2}$$

A dual-frequency combination can only eliminate the first order ionospheric effects. A triple-frequency combination can eliminate the ionospheric effects up to the second order.

The GNSS broadcast message includes the parameters of a predicted ionospheric model. Using the model parameters, the ionospheric effects can be computed and corrected.

The input parameters of the broadcast ionospheric model are the eight model coefficients of α i, β i, i = 1, 2, 3, 4, geodetic latitude ϕ and longitude λ of the GNSS antenna, GNSS observing time T in seconds, as well as the azimuth A and elevation E of the observed satellite. All four angular arguments ϕ, λ, A and E have the units of semicircles (SC), and 1 SC equals 180 degrees.

Tropospheric Delay

The troposphere is a non-dispersive medium at GNSS carrier frequencies. That is, the tropospheric effects on the GNSS signal transmission are independent from the working frequency. The electromagnetic signals are affected by the neutral atoms and molecules in the troposphere. The amount of tropospheric delay in the zenith direction is about 2 m. It increases with the increase of the zenith angle of the sight line to the satellite. In the case of a lower satellite elevation of a few degrees, the tropospheric delay of the GNSS signal can reach up to more than a few meters. Generally, the tropospheric delay depends on temperature, pressure, humidity as well as the location of the GNSS antenna. Therefore, the tropospheric effect is an important error source in precise GNSS applications. The troposphere extends about 20 km above sea level [16]. It forms the lowest part of the atmosphere made up of dry gases and water vapor. These conditions affect signal transmission by reducing the velocity and curvature of its trajectory. The delays, which are frequency dependent, are grouped under wet and dry errors and can be compensated by using Sanso and Hopfield models.

Tropospheric models (3) and (4) are used to remove the effects of tropospheric delay. The following model is used:

- The modified Saastamoinen tropospheric model [3] for calculating the tropospheric path delay:

$$\delta = \frac{0.002277}{\cos z}\left[P + \left(\frac{1255}{T} + 0.05\right)\mathbf{e} - B\tan^2 z\right] + \delta R \tag{3}$$

where z is the zenith angle of the satellite, T is the temperature at the station (in units of Kelvin (K)), P is the atmospheric pressure (in units of millibars (mb)), e is the partial pressure of water vapour (in mb). B and δR are the correction terms that depend on H and z, respectively. H is the height of the station. δ is the tropospheric path delay (in meters):

$$\mathbf{e} = R_h \cdot \exp(-37.2465 + 0.213166T - 0.000256908T^2) \tag{4}$$

where Rh is the relative humidity (in %), and T is the temperature.

Multipath

Multipath occurs when direct GNSS signals are reflected on different surfaces during its transmission [31], [53] producing multiple signals spanning from several directions. These signals, which are superimposed on the original line of sight of the signal are caused by structures/ground near the antennas [10]. The detection of multipath requires a model for the characteristics of a multipath affected received signal [5]. Multipath affects observation and limits position accuracy even when most errors have been corrected [37] for since it is dependent on the surrounding environment of the receiver and the relative motion between the satellites and the receiver [26]. Typical errors of this situations are modelled by methods such as antenna ray [55] and choke ring [27] methods. The error can also be corrected by simply weighing measurement according to the elevation angle since most of these errors occurs at lower elevations.

IV. Theory of Relativity

Italian mathematician and physicist, Galilei Galileo introduced the principle of relativity in the 16300s. His findings were based on the fact that, the motion between heavenly bodies is significant relative to another object, different from earlier proposals that assume absolute space and time. Relativistic effect is made significant by the development of satellite measurement techniques that must be accounted for by the theories of general and special relativity [2]. Receivers measure pulses from arrival times of transmitted signal to calculate position [41]. Relativity introduces distortions in these pulse measurements ranges as a result of the rotation between the earth and the satellites [2], and the gravity field of the earth. When the Earth's satellites are used as the signal transmission sources, relativistic effects can be up to 12 kilometers after one day.

TABLE I
GNSS SYSTEM ERROR BASED ON PSEUDORANGE COMPUTATIONS [49]

SN	Error Source	Error Range
1	Ionosphere	± 5.0 m
2	Obit errors	± 2.5 m
3	Satellite Clocks	± 2.0 m
4	Multipath	± 1.0 m
5	Troposphere	± 0.50 m
6	Receiver Noise	± 0.30 m

A. Special Theory of Relativity

Albert Einstein proposed the special theory of relativity (STR or SR) in 1905 [64], to explain experimental results related to the propagation of electromagnetic waves (e.g. light). The findings of special relativity are based on two fundamental postulates:

i. The laws of physics are the same for observers in uniform motion to another.
ii. The speed of light in vacuum is the same, regardless of the relative motion between the observer and the light source.

B. General Theory of Relativity

The general theory of relativity is the opposite of the special theory which takes the effects of mass and gravity field into consideration [14]. The laws of GR are expressed by methods of differential geometry. These methods shift perspective relating absolute positioning to curvilinear coordinate systems. The core statement of GR is that all forms of energy curve the 4-dimensional space time. Light travels with velocity along geodesic lines in this curved manner [4] explaining gravity as geometric feature. Fig. 4 illustrates the principle of general relativity in which the speed of light is represented in a curvilinear surface.

C. GNSS and Relativistic Effects

GNSS would work perfectly in an ideal condition in the absolute positioning systems with the assumption that satellites and the receiver were at motionless in an inertial reference frame [9]. Relativistic effects and curvature of the earth existing in space-time are considered since they limit the level of precision of the GNSS when neglected and not taken into consideration [51]. The theory of special and general relativity proposes that space and time are constantly in motion relative to other bodies [64]. Satellite clocks in GNSS constellation keep time to an accuracy of about 4 nanoseconds per day, which amounts to a fractional time stability of 5 parts in 10^{13}. Therefore, the errors introduced by the relativistic effects at order c^{-2} and rejection of these effects would account for huge errors in GNSS mainly due to [52], [22]:

i. Time dilation;
ii. Gravitational redshift effect; and
iii. Sagnac effect.

Time Dilation

The relative movement of clocks accounts for time dilation also known as second-order or transverse Doppler effect [22]. This occurs when clocks in motion beat slower than the clocks that are fixed [48]. According to [9], the movement of satellites and receivers with different speeds through space-time delays satellite clocks by approximately 2130 m distance unit error, a daily effect of 7100 nanoseconds. The Lorentz transformation invariance applies to processes in inertial reference frames as:

$$t_s = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

where; c is the speed of light and t_0 time in referent frame fixed with observed process.

Gravitational Redshift Effect

The theories of relativity (General and Special) have proven that clocks run faster when they are far away from a center of gravitational attraction compared to clocks on the surface of the earth. This effect, also called gravitational frequency shift, has opposite effect compared to time dilation. This relative movement of the produces an equivalent cumulative error of about 11500 m. Clock errors of this nature are compensated for by introducing a proper offset in satellite clocks before taking off into space.

Slight eccentricity in each noncircular orbits also poses a residual effect which is corrected at the user side according to the relation:

$$\gamma = -\frac{2}{c^2} \cdot p^s \cdot q^s$$

where p^s and q^s are the instantaneous satellite position and velocity vectors, respectively.

Sagnac Effect

Signal propagation through the atmosphere causes a relativistic error called Sagnac effects. The Sagnac effect is caused by rotation of the of the earth during signal propagation between satellites and the receivers [36]. Broadcast signals present ephemeris parameters consisting of coded information are expressed in a particular frame related to the earth (Earth Centered Earth Frame (ECEF)) at the time of signal transmission [1]. The transmission travelled by the signal between the satellite and the receiver does not reflect the correct range which requires a correction to express the position of the satellite in in the earths frame. These errors are usually inherent in the geometrical range calculation and subsequently affect satellite position [58]. The model of sagnac correction faction can be added to the geometrical range as:

$$\rho_r^s = \left\| r_r(t_r) - R_z(w_e(t_r - t_t)) \cdot r_s(t_t) \right\| \tag{5}$$

where r_r is the receiver position vector and r_s is the satellite position vector, both in ECEF frame.

Sagnac effect, if not taken into account, presents a cumulative positional error of 30 m daily in distance units [18]. This error is equivalent to 207 nanoseconds influencing transmitted signals significantly in GNSS. Fig 4 shows the rotation of the earth in relation to signal transmission.

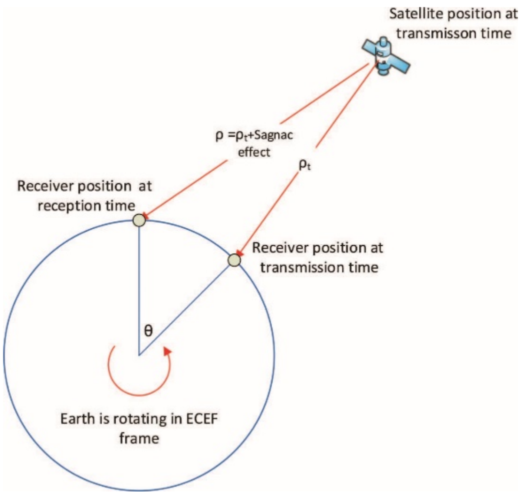

Fig. 4 Sagnac Effect [36]

V. SUMMARY

The study reviewed errors in GNSS positioning with emphasis on relativistic effects. The study confirms that, relativity in GNSS satellite positioning and navigation is significant and should be rigorously considered and modelled. The future of GNSS should consider relativistic positioning systems where autonomous and proper time signals are emitted from satellites to receivers for positional and navigational purposes, where corrections for relativistic effects are already calculated.

Satellite and ground augmentation systems should consider relativistic corrections to meet accuracy standards. State of the art relativistic and clock synchronization missions should be undertaken to address issues of such kind on civilian applications, positioning and timing.

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