Fuzzy Multiple Criteria Decision Making for Unmanned Combat Aircraft Selection using Proximity Measure Method

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***Abstract*—**Intuitionistic fuzzy sets (IFS), Pythagorean fuzzy sets (PyFS), Picture fuzzy sets (PFS), q-rung orthopair fuzzy sets (q-ROF), Spherical fuzzy sets (SFS), T-spherical FS, and Neutrosophic sets (NS) are reviewed as multidimensional extensions of fuzzy sets in order to more explicitly and informatively describe the opinions of decision-making experts under uncertainty. To handle operations with standard fuzzy sets (SFS), the necessary operators; weighted arithmetic mean (WAM), weighted geometric mean (WGM), and Minkowski distance function are defined. The algorithm of the proposed proximity measure method (PMM) is provided with a multiple criteria group decision making method (MCDM) for use in a standard fuzzy set environment. To demonstrate the feasibility of the proposed method, the problem of selecting the best drone for an Air Force procurement request is used. The proximity measure method (PMM) based multidimensional standard fuzzy sets (SFS) is introduced to demonstrate its use with an issue involving unmanned combat aircraft selection.

***Keywords*—**Standard fuzzy sets (SFS), unmanned combat aircraft selection, multiple criteria decision making (MCDM), proximity measure method (PMM).

I. Introduction

The notion of fuzzy set (FS) , $S=\{<x,\mu_S(x)>|\forall x\in X\}$, characterized by the membership function of interval [0,1], was defined to model complex problems involving uncertainty. In a FS, if the membership degree of an element is μ , then its non-membership is $\nu=1-\mu$. Namely, in an FS, indeterminacy degree of an element is accepted as "0", $\pi=1-\mu-\nu$. However, this perspective has some constraints $\mu+\nu=1$ [1].

The present methodological reviews shed light on the research area that needs to be addressed between the procedural development of studies in the field of fuzzy set theory and the proposed approach.

Therefore, the notion of fuzzy set (FS) can extendedly be structured as a multidimensional fuzzy set $S=\{<x,\mu_S(x),\nu_S(x),\pi_S(x)>|\forall x\in X\}$ with a refusal degree $r=\left(1-(\mu^\gamma+\nu^\gamma+\pi^\gamma)\right)^{1/\delta}$. In sequel, the theoretical transformation of multidimensional fuzzy set structure is gradually presented according to the developmental forms of the standard fuzzy sets.

As a generalization of FSs, the notion of intuitionistic FS (IFS) was developed to overcome these constraints. An IFS is defined by assigning two values from the range [0,1], namely, membership degree μ and non- membership ν, under the condition $0\le\mu+\nu\le 1$ for all elements of the universe of discourse. An IFS with hesitation degree is defined when $0\le\mu+\nu+\pi\le 1$. However, the IFS is not useful when $\mu+\nu>1$ [2].

Therefore, as an extension of IFS, the Pythagorean FS (PyFS) was defined under condition $\mu^2+\nu^2\le 1$. A PyFS with indeterminacy degree is defined under condition $\sqrt{1-(\mu^2+\nu^2)}$ [3-4]. As generalized version of the nonstandard fuzzy sets, q-rung orthopair fuzzy sets (q-ROF) extend the Pythagorean FS (PyFS) under conditions $\mu^q+\nu^q\le 1$ and $\left(1-(\mu^q+\nu^q)\right)^{1/q}$. Therefore, IFS is a q-ROF with $q=1$ and a PyFS is a q-ROF with $q=2$. The q-rung orthopair fuzzy sets (q-ROFs), which outperform intuitionistic fuzzy sets and Pythagorean fuzzy sets, are a crucial method for expressing uncertain information. Their distinguishing feature is that they may depict a larger range of uncertain information because the sum of the *q*th powers of the degrees of membership and non-membership is equal to or less than 1 [5-6].

Another extension of FS and IFS, Picture FS (PFS) was defined as a useful tool for representing human opinion, because a PFS can model judgments about an object or idea using degrees of yes, abstention (neutral), no, and rejection. A PFS can address the issue of refusal degree in a voting system where voters can be divided into four classes: yes, no, abstain, and refuse. Therefore, a PFS has degree of membership, indeterminacy (neutral), non-membership, $\mu+\nu+\pi\le 1$, and refusal $r=1-(\mu+\nu+\pi)$. A PFS is not sufficient in modeling some problems under condition $\mu+\nu+\pi>1$ [7-8].

For this reason, an extension of PFS satisfying the condition $0\le\mu^2+\nu^2+\pi^2\le 1$, the notion of spherical FS (SFS) was introduced to handle fuzzy information modeling with a refusal degree $r=\sqrt{1-(\mu^2+\nu^2+\pi^2)}$ [9-10]. As an extension of the SFS with condition $0\le\mu^q+\nu^q+\pi^q\le 1$, the

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T-spherical FS (T-SFS) was defined with a refusal degree $r=\sqrt{1-(\mu^q+\nu^q+\pi^q)}$ [11].

As a generalization of FS and IFS, the notion of Neutrosophic set (NS) was developed to overcome these constraints under condition $^{-}0\le\mu+\nu+\pi\le 3^{+}$, and the functions real or nonstandard subsets $]^{-}0,1^{+}[$ [xx]. Due to the nonstandard subsets of the Neutrosophic set, it is obviously challenging to employ in actual scientific and technical fields. As a result, Single-Valued Neutrosophic Set (SVNS) was presented with the condition $0\le\mu+\nu+\pi\le 3$, and each NS function value from the range [0,1] [12-13].

Theoretically, motivated from the fundamental structure of FS with four parameters; membership degree μ, indeterminacy degree ν, falsity degree π, and refusal degree r, as a generalization of FS, a standard fuzzy set (SFS), $S=\{<x,\mu_S(x),\nu_S(x),\pi_S(x)>|\,\forall x\in X\}$ is defined, and the SFS is characterized by set function values in the range [0,1]. The refusal degree is defined by $r=\sqrt{1-(\mu^\gamma+\nu^\gamma+\pi^\gamma)}$.

Then, the SFS structure $\zeta\le\mu^\gamma+\nu^\gamma+\pi^\gamma\le\xi$ can be set to according to the nature and requirements of the problem in question. The mathematical formulations of the SFSs are classified by setting parameter values $\{\zeta,\ \gamma,\ \xi\}$ of $\zeta\le\mu^\gamma+\nu^\gamma+\pi^\gamma\le\xi$.

Definition 1. Let *X* be a universe of discourse (a non-empty fixed set), then, a standard fuzzy set is defined as follows.

$$S=\{<x,\mu_S(x),\nu_S(x),\pi_S(x)>|\,\forall x\in X\} \quad (1)$$

where $\mu_S(x):\ X\ \rightarrow\ [0,1]$ is called the truth degree of *x* in *S*, $\nu_S(x):\ X\ \rightarrow\ [0,1]$ is called the indeterminacy degree of *x* in *S* and $\pi_S(x):\ X\ \rightarrow\ [0,1]$ is called the falsity degree of *x* in *S*, and where $\mu_S(x)$, $\nu_S(x)$ and $\pi_S(x)$ satisfy the following conditions when all three components are independent.

$$\zeta\le\mu_S(x)^\gamma+\nu_S(x)^\gamma+\pi_S(x)^\gamma\le\xi \quad (2)$$

A generalized refusal degree is defined as follows:

$$r=\left(1-(\mu^\gamma+\nu^\gamma+\pi^\gamma)\right)^{1/\delta} \quad (3)$$

a) when $\zeta=0$, $\xi=3$, $\gamma=1$, the standard fuzzy set gets the following form:

$$0\le\mu_S(x)+\nu_S(x)+\pi_S(x)\le 3 \quad (4)$$

b) when $\zeta=0$, $\xi=1$, $\gamma=3$, the standard fuzzy set gets the following form:

$$0\le\mu_S(x)^3+\nu_S(x)^3+\pi_S(x)^3\le 1 \quad (5)$$

c) when $\zeta=0$, $\xi=1$, $\gamma=2$, the standard fuzzy set gets the following form:

$$0\le\mu_S(x)^2+\nu_S(x)^2+\pi_S(x)^2\le 1 \quad (6)$$

d) when $\zeta=0$, $\xi=1$, $\gamma=1$, the standard fuzzy set gets the following form:

$$0\le\mu_S(x)+\nu_S(x)+\pi_S(x)\le 1 \quad (7)$$

In the sequel, the mathematical representation is simplified by using the triplet components of standard fuzzy sets $p=<\mu,\nu,\pi>$.

Definition 3. Let $\alpha_i=<\mu_i,\nu_i,\pi_i>$ be a standard fuzzy set, then the score function $S(\alpha_i)$ of α_i is defined as

$$S(\alpha_i)=\frac{1}{2}(1+\mu_i-\nu_i-\pi_i) \quad (8)$$

$$S(\alpha_i)\in[0,1]$$

Definition 4. Let $\alpha_i=<\mu_i,\nu_i,\pi_i>$ be a standard fuzzy set, then the accuracy function $G(\alpha_i)$ of α_i is defined as

$$G(\alpha_i)=(\mu_i-\pi_i) \quad (9)$$

$$G(\alpha_i)\in[-1,1]$$

Definition 5. Let $\alpha_1=<\mu_1,\nu_1,\pi_1>$ and $\alpha_2=<\mu_2,\nu_2,\pi_2>$ be two standard fuzzy sets. Then, the following rules can be used to compare them:

1. If $S(\alpha_1)>S(\alpha_2)$, then $\alpha_1>\alpha_2$;
2. If $S(\alpha_1)=S(\alpha_2)$, and
   $G(\alpha_1)>G(\alpha_2)$, then $\alpha_1>\alpha_2$;
   $G(\alpha_1)=G(\alpha_2)$, then $\alpha_1\sim\alpha_2$.

Definition 6. Let $\alpha_1=<\mu_1,\nu_1,\pi_1>$ and $\alpha_2=<\mu_2,\nu_2,\pi_2>$ be two standard fuzzy sets numbers, and $\lambda>0$, and their relations $<\mu_i,\nu_i,\pi_i>$ among are contained as follows.

(1) $\alpha_1\oplus\alpha_2=<(1-(1-\mu_{\alpha_1})(1-\mu_{\alpha_2})),\nu_{\alpha_1}\nu_{\alpha_2},\pi_{\alpha_1}\pi_{\alpha_2}>$

(2) $\alpha_1\otimes\alpha_2=<\mu_{\alpha_1}\mu_{\alpha_2},(1-(1-\nu_{\alpha_1})(1-\nu_{\alpha_2})),(1-(1-\pi_{\alpha_1})(1-\pi_{\alpha_2}))>$

(3) $\lambda\alpha_i=<(1-(1-\mu_{\alpha_i})^\lambda),{\nu_{\alpha_i}}^\lambda,{\pi_{\alpha_i}}^\lambda>$

(4) ${\alpha_i}^\lambda=<{\mu_{\alpha_i}}^\lambda,(1-(1-\nu_{\alpha_i})^\lambda),(1-(1-\pi_{\alpha_i})^\lambda)>$

(5) $\alpha_i^C = <\pi_{\alpha_i}, \nu_{\alpha_i}, \mu_{\alpha_i}>$

Suppose that there is a group of standard fuzzy sets $\alpha_i = <\mu_i, \nu_i, \pi_i> (i = 1, 2, ..., n)$ with their related weights $\omega_k \in [0,1]$ for $\sum_{i=1}^{n} \omega_i = 1$. Then, the standard fuzzy set weighted arithmetic mean (WAM) operator and the standard set weighted geometric mean (WGM) are introduced, respectively as follows.

Definition 7. Let $\alpha_i (i = 1, 2, ..., n) \in SFS(X)$, then, the standard fuzzy set weighted arithmetic mean WAM operator is defined as follows.

$$WAM = (\alpha_1, \alpha_2, ..., \alpha_n) = \sum_{i=1}^{n} \omega_i \alpha_i \tag{10}$$

$$= \left\langle \left(1 - \prod_{i=1}^{n} (1 - \mu_{\alpha_i})^{\omega_i}\right), \prod_{i=1}^{n} \nu_{\alpha_i}^{\omega_i}, \prod_{i=1}^{n} \pi_{\alpha_i}^{\omega_i} \right\rangle$$

where ω_i is the weight of $\alpha_i (i = 1, 2, ..., n)$, $\omega_i \in [0,1]$ and $\sum_{i=1}^{n} \omega_i = 1$. Especially, assume that $\omega_i = 1/n$, $(i = 1, 2, ..., n)$, then, WAM is called an arithmetic mean operator for SFSs and the aggregation value is still a standard fuzzy set.

Definition 8. Let $\alpha_i (i = 1, 2, ..., n) \in SFS(X)$, then the standard fuzzy set weighted geometric mean WGM operator is defined as follows.

$$WGM = (\alpha_1, \alpha_2, ..., \alpha_n) = \prod_{i=1}^{n} \alpha_i^{\omega_i} \tag{11}$$

$$= \left(\prod_{i=1}^{n} \mu_{\alpha_i}^{\omega_i}, \left(1 - \prod_{i=1}^{n} \left(1 - \nu_{\alpha_i}\right)^{\omega_i}\right), \left(1 - \prod_{i=1}^{n} \left(1 - \pi_{\alpha_i}\right)^{\omega_i}\right) \right)$$

where ω_k is the weight of $\alpha_i (i = 1, 2, ..., n)$, $\omega_i \in [0,1]$ and $\sum_{i=1}^{n} \omega_i = 1$. Especially, assume that $\omega_i = 1/n$, $(i = 1, 2, ..., n)$, then WGM is called a geometric mean operator for SFSs and the aggregation value is still a standard fuzzy set.

Definition 9. Let $\alpha_1 = <\mu_1, \nu_1, \pi_1>$ and $\alpha_2 = <\mu_2, \nu_2, \pi_2>$ be two standard fuzzy sets (SFS). Then, the Minkowski distance between them is defined as follows:

$$d_\delta(\alpha_1, \alpha_2) = \left(|\mu_1^\gamma - \mu_2^\gamma|^\delta + |\nu_1^\gamma - \nu_2^\gamma|^\delta + |\pi_1^\gamma - \pi_2^\gamma|^\delta \right)^{1/\delta} \tag{12}$$

when $\delta = 1$, the $d_1(\alpha_1, \alpha_2)$ is called the Hamming distance between α_1 and α_2; when $\delta = 2$, the $d_2(\alpha_1, \alpha_2)$ is called the Euclidean distance between α_1 and α_2; and when $\delta = \infty$, the $d_\infty(\alpha_1, \alpha_2)$ is called the Chebyshev distance between α_1 and α_2. However, the normalized Minkowski distance can be used by standard fuzzy sets (SFS) as follows:

$$d_\delta(\alpha_1, \alpha_2) = \left(\frac{1}{n} \sum_{j=i}^{n} \left(\begin{array}{l} |\mu_1^\gamma - \mu_2^\gamma|^\delta + |\nu_1^\gamma - \nu_2^\gamma|^\delta \\ + |\pi_1^\gamma - \pi_2^\gamma|^\delta \end{array} \right) \right)^{1/\delta} \tag{13}$$

The rest of the paper is organized as follows. Standard fuzzy sets (SFS) and proximity measure method (PMM) are concisely reviewed in Section 2. In Section 3, a numerical example for the unmanned combat aircraft selection application is presented. Concluding remarks and future research directions are given in Section 4.

II. Methodology

The current research that elucidates the fusion of standard fuzzy sets and the proximity measure method is briefly presented. Multiple criteria decision making (MCDM) is a crucial component of decision sciences since it may produce ranking results for finite choices based on the attributes of various alternatives. Multiple criteria group decision making (MCGDM), is an important tool for expressing the value of evaluation in MCGDM problems.

Classical MCDM approaches such as AHP [14-17], ELECTRE [18-19], TOPSIS [20-21], PROMETHEE [22-25] and VIKOR [26-27] paved the way for the development of the extensions of fuzzy set concept due to the limitations and difficulties in representing and modeling the uncertainty. Fuzzy proximity measure method (PMM) is proposed to evaluate the alternatives of unmanned combat aircraft selection problem using multiple criteria decision making (MCDM) technique. Using traditional MCDM techniques, fuzzy sets, and their extensions, aircraft evaluation and selection problems have been taken into consideration [28-49].

A. Proximity measure method (PMM)

The proximity measure method (PMM), which has the advantage of reducing rank reversal phenomena and computational complexity, is proposed as an effective MCDM method. In this method, the distance between each alternative and the optimum option is measured using a proximity measure value. Therefore, this method compares alternatives based on a proximity measure value that represents the deviation from the best alternative.

The value of the proximity measure, which is used to rank the options, represents the minimum deviation from the best option. The ranking of the alternatives declines with increasing proximity measure value, starting with the alternative with the lowest proximity measure value.

The procedural steps of this proposed MCDM method are given as follows:

Step 1. Decision matrix is structured.

MCDM Model	ω_1	ω_2	$\cdots$	ω_j	$\cdots$	ω_n
	c_1	c_2	$\cdots$	c_j	$\cdots$	c_n
a_1	x_{11}	x_{12}	$\cdots$	x_{1j}	$\cdots$	x_{1n}
a_2	x_{21}	x_{22}	$\cdots$	x_{2j}	$\cdots$	x_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

a_i	x_{i1}	x_{i2}	$\cdots$	x_{ij}	$\cdots$	x_{in}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_m	x_{m1}	x_{m2}	$\cdots$	x_{mj}	$\cdots$	x_{mn}

Step 2. Decision matrix is normalized.

$$r_{ij} = <\mu_{ij}, \nu_{ij}, \pi_{ij}> = \begin{cases} <\mu_{ij}, \nu_{ij}, \pi_{ij}>, \text{ for } c_j \in \Omega_b \\ <\pi_{ij}, \nu_{ij}, \mu_{ij}>, \text{ for } c_j \in \Omega_c \end{cases} \quad (14)$$

where Ω_b and Ω_c are the sets of benefit criteria and cost criteria, respectively.

Step 3. The normalized values r_{ij} are multiplied by the weights of criteria ω_j.

$$\eta_{ij} = \omega_j r_{ij} \quad (15)$$

Step 4. The weighted proximity measure ε_{ij} is obtained.

$$\varepsilon_{ij} = \eta_j^{\max} - \eta_{ij} \quad (16)$$

Step 5. The overall proximity measure z_i is calculated.

$$z_{ij} = \sum_{j=1}^{J} \varepsilon_{ij} \quad (17)$$

The alternative having the least value of z_i is identified as the best alternative.

III. Application

In this section, the application of the proposed fuzzy MCDM method for deriving the most desirable unmanned fighter aircraft is presented.

A. Determine the weights of criteria

In the MCDM problem, suppose that a set of k experts $E = \{E_i, ..., E_k\}$ evaluate the decision criteria to determine the weights of criteria. The WAM or WGM operator is used to aggregate the weights of the criteria and get the overall standard fuzzy set (SFS) decision matrix.

B. Problem description

Let $a_i = (a_1, ..., a_i)$ $i = 1, ..., I$ be a collection of i alternatives, $c_j = (c_1, ..., c_j)$, $j = 1, ..., J$ be a collection of j attributes, $\omega_j = (\omega_1, ..., \omega_j)^T$, $j = 1, ..., J$ be the weight vector of criteria, $c_j \, (j = 1, ..., J)$, with $\omega_j \in [0,1]$ and $\sum_{j=1}^{J} \omega_j = 1$.

Let $E = \{E_i, ..., E_k\}$ be a set of k experts, where E_l denotes the lth expert who takes part in the decision making process, $l = 1, ..., k$, and each expert E_l is assigned a weight $\xi_l \geq 0 \, (l = 1, ..., k)$ satisfying $\sum_{l=1}^{k} \xi_l = 1$ to reflect his/her importance in the analysis process.

Suppose that $R^l = (r_{ij}^l)_{ixj}$ is the standard fuzzy set decision matrix, where $r_{ij}^l = <\mu_{ij}^l, \nu_{ij}^l, \pi_{ij}^l>$ is evaluation value provided by the expert E_l for alternative $a_i = (i = 1, ..., I)$ with respect to the criterion $c_j \, (j = 1, ..., J)$, $\mu_{ij}^l \in [0,1]$, $\nu_{ij}^l \in [0,1]$, $\pi_{ij}^l \in [0,1]$ and $0 \leq \mu_{ij}^l + \nu_{ij}^l + \pi_{ij}^l) \leq 3$. To obtain the optimal alternative, the proposed PMM approach, the WAM and WGM operators are used to aggregate the individual standard fuzzy set matrices and obtain the collective standard fuzzy set matrix.

Assume that an air force wants to acquire unmanned combat aircraft under the MCDM problem. Three bidders submit applications for the acquisition procurement after it is announced. To undertake the evaluation and selection process, namely, to examine options in accordance with five decision criteria, the Air Force assigns three decision-making experts, $E = \{E_i, ..., E_k\}$. Following an initial decision-making interview, experts establish a weight vector $\xi_l = (0{,}3, 0.4, 0.3)^T$ for their evaluations. This identified weight vector is used by experts to calculate the weights of the criteria and assess the alternatives.

Through communication with the three experts, five qualitative criteria related to unmanned combat aircraft are considered, which are listed as follows: priceability (C1), payloadability (C2), stealthability (C3), speedability (C4), and survivability (C5). According to the characteristic of the criteria, C2, C3, C4, C5 are regarded as benefit criteria and C1 is regarded as the cost criterion.

The experts are allowed to express their thoughts by utilizing a standard fuzzy set number in order to more accurately reflect their criteria weight evaluations. The definitions of the truth, indeterminacy, and falsity degrees in the standard fuzzy set number are also taught to the experts to guarantee that they give unbiased and accurate answers.

From Table 1, taking the evaluation value $r_{31}^3 = <0.5, 0.4, 0.3>$ for an example, the expert E_3 provides the evaluation value for a_3 with respect to c_1 in which 0.5 represents the degree of truth, 0.4 represents the degree of indeterminacy and 0.3 represents the degree of falsity. Similarly, one can determine the evaluation values for all decision criteria.

According to the expertise of the three experts, the fuzzy information about the weights of criteria is given in Table 1.

Table 1. Decision-making matrix for weighing the criteria

	c_1	c_2	c_3	c_4	c_5
E_1	0,6 0,4 0,2	0,6 0,1 0,2	0,8 0,1 0,2	0,6 0,4 0,2	0,7 0,1 0,1
E_2	0,4 0,3 0,1	0,5 0,6 0,1	0,7 0,2 0,1	0,6 0,3 0,1	0,6 0,3 0,2
E_3	0,5	0,6	0,9	0,7	0,7

	0,4	0,4	0,1	0,4	0,3
	0,3	0,3	0,1	0,1	0,4

Using the WAM operator, the calculated criteria weights are shown in Table 2.

Table 2. Decision criteria weights

	c_1	c_2	c_3	c_4	c_5
ω_i	0,13	0,18	0,54	0,26	0,29
	0,36	0,31	0,13	0,36	0,24
	0,17	0,17	0,12	0,12	0,18

Using the Step 1, similarly, one can determine the evaluation values for all alternatives regarding different criteria. Then, one can construct individual standard fuzzy set decision matrices $R^l = (r_{ij}^l)_{ixj}$ based on judgments of the experts, which are listed in Table 3.

Table 3. Initial standard fuzzy decision matrices provided by three experts

E_i	a_i	c_1	c_2	c_3	c_4	c_5
E_1	a_1	0,5	0,6	0,7	0,7	0,7
		0,3	0,4	0,3	0,5	0,5
		0,4	0,4	0,1	0,2	0,1
	a_2	0,6	0,2	0,6	0,6	0,4
		0,4	0,4	0,5	0,3	0,3
		0,3	0,7	0,3	0,4	0,4
	a_3	0,5	0,6	0,4	0,8	0,7
		0,5	0,1	0,3	0,5	0,4
		0,3	0,4	0,1	0,2	0,1
E_2	a_1	0,6	0,4	0,5	0,7	0,6
		0,1	0,5	0,4	0,3	0,3
		0,2	0,6	0,3	0,2	0,4
	a_2	0,6	0,7	0,4	0,3	0,7
		0,4	0,2	0,1	0,5	0,4
		0,5	0,4	0,6	0,2	0,1
	a_3	0,4	0,5	0,7	0,6	0,6
		0,2	0,3	0,2	0,4	0,3
		0,1	0,1	0,1	0,1	0,2
E_3	a_1	0,5	0,6	0,7	0,7	0,7
		0,6	0,4	0,5	0,3	0,5
		0,4	0,4	0,1	0,2	0,3
	a_2	0,6	0,2	0,6	0,6	0,4
		0,4	0,4	0,3	0,2	0,3
		0,1	0,7	0,3	0,4	0,4
	a_3	0,5	0,6	0,4	0,8	0,7
		0,1	0,1	0,6	0,4	0,4
		0,3	0,4	0,1	0,2	0,1

Using the Step 2, Initial standard fuzzy decision matrices are normalized as shown in Table 4.

Table 4. Normalized standard fuzzy decision matrices provided by three experts

E_i	a_i	c_1	c_2	c_3	c_4	c_5
E_1	a_1	0,4	0,6	0,7	0,7	0,7
		0,3	0,4	0,3	0,5	0,5
		0,5	0,4	0,1	0,2	0,1
	a_2	0,3	0,2	0,6	0,6	0,4
		0,4	0,4	0,5	0,3	0,3
		0,6	0,7	0,3	0,4	0,4
	a_3	0,3	0,6	0,4	0,8	0,7
		0,4	0,1	0,3	0,5	0,4
		0,5	0,4	0,1	0,2	0,1
E_2	a_1	0,2	0,4	0,5	0,7	0,6
		0,1	0,5	0,4	0,3	0,3
		0,6	0,6	0,3	0,2	0,4
	a_2	0,5	0,7	0,4	0,3	0,7
		0,4	0,2	0,1	0,5	0,4
		0,6	0,4	0,6	0,2	0,1
	a_3	0,1	0,5	0,7	0,6	0,6
		0,2	0,3	0,2	0,4	0,3
		0,4	0,1	0,1	0,1	0,2
E_3	a_1	0,4	0,6	0,7	0,7	0,7
		0,6	0,4	0,5	0,3	0,5
		0,5	0,4	0,1	0,2	0,3
	a_2	0,1	0,2	0,6	0,6	0,4
		0,4	0,4	0,3	0,2	0,3
		0,6	0,7	0,3	0,4	0,4
	a_3	0,3	0,6	0,4	0,8	0,7
		0,1	0,1	0,6	0,4	0,4
		0,5	0,4	0,1	0,2	0,1

Using the Step 3, weighted normalized standard fuzzy decision matrices are calculated as shown in Table 5.

Table 5. Weighted normalized standard fuzzy decision matrices provided by three experts

E_i	a_i	c_1	c_2	c_3	c_4	c_5
E_1	a_1	0,48	0,67	0,86	0,78	0,79
		0,11	0,12	0,04	0,18	0,12
		0,09	0,07	0,01	0,02	0,02
	a_2	0,39	0,34	0,82	0,70	0,58
		0,14	0,12	0,07	0,11	0,07
		0,10	0,12	0,04	0,05	0,07
	a_3	0,39	0,67	0,72	0,85	0,79
		0,18	0,03	0,04	0,18	0,09
		0,09	0,07	0,01	0,02	0,02
E_2	a_1	0,30	0,51	0,77	0,78	0,72
		0,04	0,16	0,05	0,11	0,07
		0,10	0,10	0,04	0,02	0,07
	a_2	0,57	0,75	0,72	0,48	0,79
		0,14	0,06	0,01	0,18	0,09
		0,10	0,07	0,07	0,02	0,02
	a_3	0,22	0,59	0,86	0,70	0,72
		0,07	0,09	0,03	0,14	0,07
		0,07	0,02	0,01	0,01	0,04
E_3	a_1	0,48	0,67	0,86	0,78	0,79
		0,21	0,12	0,07	0,11	0,12
		0,09	0,07	0,01	0,02	0,06
	a_2	0,22	0,34	0,82	0,70	0,58
		0,14	0,12	0,04	0,07	0,07
		0,10	0,12	0,04	0,05	0,07
	a_3	0,39	0,67	0,72	0,85	0,79
		0,04	0,03	0,08	0,14	0,09
		0,09	0,07	0,01	0,02	0,02

Using the Definition 7, weighted normalized standard fuzzy decision matrix by the WAM operator is calculated as shown in Table 6.

Table 6. Weighted normalized standard fuzzy decision matrix by the WAM operator

a_i	c_1	c_2	c_3	c_4	c_5
a_1	0,41 0,08 0,09	0,61 0,14 0,08	0,83 0,05 0,02	0,78 0,12 0,02	0,76 0,10 0,04
a_2	0,43 0,14 0,10	0,56 0,09 0,10	0,78 0,03 0,05	0,63 0,12 0,04	0,68 0,08 0,04
a_3	0,33 0,08 0,08	0,64 0,05 0,04	0,79 0,04 0,01	0,80 0,15 0,02	0,76 0,08 0,02

Using the Definition 8, weighted normalized standard fuzzy decision matrix by the WGM operator is calculated as shown in Table 7.

Table 7. Weighted normalized standard fuzzy decision matrix by the WGM operator

a_i	c_1	c_2	c_3	c_4	c_5
a_1	0,40 0,11 0,09	0,60 0,14 0,08	0,82 0,05 0,02	0,78 0,13 0,02	0,76 0,10 0,05
a_2	0,38 0,14 0,10	0,47 0,10 0,10	0,78 0,04 0,05	0,60 0,13 0,04	0,65 0,08 0,05
a_3	0,31 0,09 0,08	0,64 0,06 0,05	0,78 0,05 0,01	0,79 0,15 0,02	0,76 0,08 0,03

Using the Definition 3, score values $S(a_i)$ and ranking orders R_i of alternatives a_i according to WAM and WGM values are shown in Table 8.

Table 8. Score values $S(a_i)$ and ranking orders R_i of alternatives a_i according to WAM and WGM values

	WAM	R_i	WGM	R_i
$S(a_1)$	3,82	2	3,82	2
$S(a_2)$	3,64	3	3,64	3
$S(a_3)$	3,88	1	3,88	1

Naturally, the WAM and WGM operators obtain the identical ranking order patterns, which are as follows:

$$S(a_2) \prec S(a_1) \prec S(a_3)$$

As a result, the standard fuzzy sets (SFS) based WAM and WGM operators select the unmanned combat aircraft alternative a_3 as the best choice.

Using the Step 4, first, PMM vector of ideal values $\eta_j^{\max}$ is identified as shown in Table 9. Then, weighted proximity measure values ε_{ij} are obtained as shown in Table 10.

Table 9. PMM vector of ideal values $\eta_j^{\max}$

	c_1	c_2	c_3	c_4	c_5
$\eta_j^{\max}$	0,43 0,08 0,08	0,64 0,05 0,04	0,83 0,03 0,01	0,80 0,12 0,02	0,76 0,08 0,02

Table 10. PMM weighted proximity measure values ε_{ij}

a_i	c_1	c_2	c_3	c_4	c_5
a_1	0,02	0,10	0,02	0,03	0,03
a_2	0,07	0,11	0,06	0,18	0,09
a_3	0,10	0,00	0,04	0,04	0,00

Using the Step 4 and Definition 9, the Minkowski distance $\varepsilon_{ij} = \eta_j^{\max} - \eta_{ij}$ is applied as follows:

$$d_\delta(\alpha_1, \alpha_2) = \left(|\mu_1^\gamma - \mu_2^\gamma|^\delta + |\nu_1^\gamma - \nu_2^\gamma|^\delta + |\pi_1^\gamma - \pi_2^\gamma|^\delta \right)^{1/\delta}$$

when $\delta = 2$, $\gamma = 1$ the $d_1(\alpha_1, \alpha_2)$ is called the Euclidean distance between α_1 and α_2.

Using the Step 5, finally, PMM overall proximity measure values z_i and ranking orders R_i of alternatives a_i are obtained as shown in Table 11.

Table 11. PMM overall proximity measure values z_i and ranking orders R_i of alternatives a_i

a_i	z_i	R_i
a_1	0,20	2
a_2	0,50	3
a_3	0,18	1

Naturally, the proposed PMM approach (Table 11) achieves the same sort order patterns as the WAM and WGM operators (Table 8), as follows:

$$S(a_2) \prec S(a_1) \prec S(a_3)$$

As a result, the fuzzy MCDM analysis selects the unmanned combat aircraft alternative a_3 as the best choice.

One can get the conclusion that the proposed PMM technique is validated with the same ranking order patterns of WAM and WGM operators based on the MCDM analysis of the decision-making problem utilizing the standard fuzzy sets (SFS).

The Air Force may employ the MCDM assessments of the decision-making experts as a decision-making aid system for the procurement management in military decision-making processes since the three ranking order patterns produce consistent MCDM evaluations.

As can be seen from the example, the proposed fuzzy decision-making method is more suited for actual scientific and engineering applications since it can handle not only incomplete information but also inconsistent information and uncertain information that exist in real life. The method that has been proposed extends on current decision-making processes and provides a fresh perspective on multi-criteria group decision making.

The proposed PMM technique is an essential technical component of the multiple criteria decision-making process. There are numerous MCDM methods for tackling multiple criteria decision making problems using fuzzy information and its extensions. The present classical MCDM approaches are not adequate for managing the fuzzy information containing unknown decision maker weights and the criteria values for alternatives since the standard fuzzy sets broaden the ideas of fuzzy sets and its extensions. As a result, the MCDM method needs to be extended to the standard fuzzy set context. In the process of multiple criteria decision making based on the developed decision-making approach can use the proposed score function of standard fuzzy set numbers to rank alternatives.

Decision makers' performance ratings and criteria for alternatives are characterized by standard fuzzy set numbers. The weights of the decision makers are predetermined, and the weights of the criteria are obtained by aggregating the criteria values provided by the decision makers and the decision makers' weight values regarding the importance of each criterion. The method proposed minimizes computational complexity and is more flexible than existing decision-making methods.

IV. Conclusion

Much progress has been achieved in the study of fuzzy sets and its numerous extensions, which are useful in tackling real-life multiple criteria decision-making (MCDM).

The extended PMM approach is a novel solution for the problem of multiple criteria group decision making based on standard fuzzy sets, and it was proposed for the assessment of the choice of unmanned combat aircraft. Standard fuzzy sets are more helpful in overcoming fuzzy circumstances since it is more difficult to address a decision-making challenge stated by crisp data in an uncertain environment. To carry out the assessment process properly, it is crucial to consider the decision makers' relative weights, the criteria's aggregate values, and the effect of criteria on the alternatives.

Standard fuzzy sets numbers were used to provide the ratings of each alternative in accordance with each criterion and the weights of each criterion in MCDM problem. Additionally, the WAM or WGM operator is used to combine the opinions of each individual decision maker in order to assess the significance of the criteria and the alternatives. First, the ideal solution for the standard fuzzy set was found. Next, the minimum deviation of the alternatives from the ideal solution was computed using the Euclidean distance.

Because it takes into account the relevance of each decision maker, the PMM method based on a standard fuzzy set is more beneficial for handling multiple criteria decision making problems. So, when classification, evaluation, ranking, and selection are involved, the standard fuzzy set PMM may be preferred for handling partial, indeterminate, and inconsistent information in MCDM problems.

By making use of the FS and its multidimensional expansions, the standard fuzzy set (SFS) concept, as well as its theoretical formations, functions, and operations, are gradually established. For applicability and effectiveness of the standard fuzzy set (SFS), the MCDM problem of unmanned combat aircraft selection is given related to the defined SFS functions and operations.

Based on WAM and WGM operators, an MCGDM method PMM has been developed and an application is presented to the MCGDM problem, which involves selecting an unmanned fighter aircraft for supply in any air force.

Also, the proposed set structure has been applied to the MCGDM problem. As a result, the fuzzy MCDM analysis selects the unmanned combat aircraft alternative a_3 as the best choice. The obtained results are compared to rank the alternatives.

The proposed PMM approach can be applied to decision making problems in other research fields. Further research is encouraged on the development of the standard fuzzy set (SFS). In the future, research objectives are to study on other aggregation operators, similarity measures, distance measures and decision-making methods based on traditional AHP , TOPSIS, and VIKOR techniques.

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