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Determination of Poisson's Ratio and Elastic Modulus of Compression Textile Materials

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***Abstract*—**Compression textiles such as compression stockings (CSs) have been extensively applied for the prevention and treatment of chronic venous insufficiency of lower extremities. The involvement of multiple mechanical factors such as interface pressure, frictional force, and elastic materials make the interactions between lower limb and CSs to be complex. Determination of Poisson's ratio and elastic moduli of CS materials are critical for constructing finite element (FE) modeling to numerically simulate a complex interactive system of CS and lower limb. In this study, a mixed approach, including an analytic model based on the orthotropic Hooke's Law and experimental study (uniaxial tension testing and pure shear testing), has been proposed to determine Young's modulus, Poisson's ratio, and shear modulus of CS fabrics. The results indicated a linear relationship existing between the stress and strain properties of the studied CS samples under controlled stretch ratios (< 100%). The proposed method and the determined key mechanical properties of elastic orthotropic CS fabrics facilitate FE modeling for analyzing in-depth the effects of compression material design on their resultant biomechanical function in compression therapy.

***Keywords*—**Elastic compression stockings, Young's modulus, Poisson's ratio, shear modulus, mechanical analysis.

I. Introduction

COMPRESSION textiles have been widely applied to prevent and treat venous discords, e.g., deep vein thrombosis, leg ulcers, lymphedema, and superficial phlebitis, through the designed external pressure system [1]. CSs are one of typical types of compression textiles for reducing venous hypertension [2] and promoting venous return [3]. The therapeutic effects of CSs largely depend on the interactions between the applied CS fabrics and lower limbs. FE modeling has been used to analyze interactive relationships between compression textiles and the applied human bodies for predicting pressure functional performance. Elastic moduli are critical mechanical parameters to control tension, shear, compression, bending, and recovery properties of the applied materials [4], [5], which are also used as mostly important mechanical parameters input to FE modeling for numerically simulating interface interactions between the two bodies (CS and lower limb) [6]. Young's modulus, Poisson's ratio, and shear modulus can reflect mechanical relationships between the stress and the strain of the applied compressive elastic materials (Fig. 1).

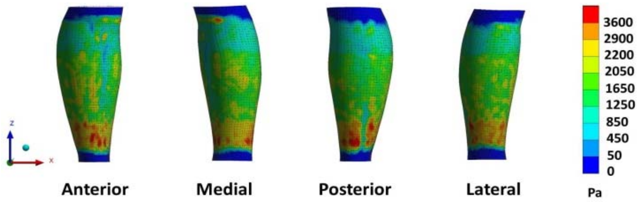

Fig. 1 Distribution of contact interface pressure between compression fabric and lower body [6]

In the CS studies, Young's modulus is commonly applied to assess tensile stiffness of the compression fabrics undergoing extension or compression deformation [4], which is defined as the ratio of normal stress to normal strain, describing a linear relationship between the normal stress and the normal strain in the elastic deformation. Elastic deformation of material presenting the deformation (strain) properties follow the Hooke's Law, indicating that the studied elastic material can return to its original shape after removing extension or compression loading forces [7]. Poisson's ratio is a measure of Poisson's effect, which indicates that the elastic material tends to contract in the direction perpendicular to the direction of tension. The value of Poisson's ratio is a negative ratio of the transverse strain to the longitudinal strain [4]. Shear modulus is used to assess the shear stiffness of material undergoing the shear deformation.

To determine the aforementioned mechanical properties including Young's modulus, Poisson's ratio, and shear modulus, an orthotropic compliance matrix based on the Hooke's Law and experimental testing has been applied. For example, a digital image method [8] and mechanical modeling [9] have been applied to determine correlations between Poisson's ratio of woven fabric and corresponding displacement/deformations along directions of x and y axes in a uniaxial tensile test. Kawabata Evaluation System (KES) and Instron testing systems have been commonly applied to build correlation between digital images and textile materials. KES is an advanced testing solution to determine fabric mechanical properties including tension, compression, and shear [10], while Instron is an apparatus to especially measure tension properties of materials [11]. Our related studies revealed that using three-dimensional (3D) mechanical properties of the CS fabrics can reduce simulation derivation ratios (SDROs) of pressure magnitudes compared with those by using two-dimensional (2D) material mechanical properties [12] and the

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simulated interface pressure by using 2D CSs' mechanical properties appeared comparable magnitudes and similar tendency compared with the measured results. In addition, the determination of 2D material mechanical properties makes the preparation process of FE modeling to be faster. However, to now, a systematic approach to determine Young's modulus, Poisson's ratio, and shear modulus of compressive elastic material in a 2D-scale remain not fully reported.

In this study, an integrated analytic model with experimental testing has been proposed. Through applying the designed methods and instrumental settings, the mechanical properties (elastic moduli) of the studied CSs fabrics have been determined in terms of Young's moduli, Poisson's ratios, and shear moduli based on the orthotropic constitutive relationships between stress and strain of the CS fabrics.

II. Methodology

A. Theory Analysis

The CS fabrics were assumed as an orthotropic material with differential mechanical properties along fabric course and wale directions. The elongation of CS fabrics commonly present to be linear with stretching ratios from 15% to 120% [12]. Thus, the mechanical properties of CS fabrics can be determined based on Hooke's Law, which is a principle to describe a linear relationship between stress and strain under a specific deformation including uniaxial tension, compression, shear, or bending. It can be expressed as follows:

$$[\varepsilon] = [S][\sigma] \tag{1}$$

where σ, ε, and S are stress, strain, and compliance matrix [13], respectively.

The CS fabrics present differential mechanical properties along their course and wale directions, therefore they are commonly assumed as an orthotropic lamina [14], which can be expressed based on the generalized Hooke's Law as below:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{21} & S_{22} & S_{26} \\ S_{61} & S_{62} & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{2}$$

where $[S_{ij}]$ is the orthotropic compliance matrix (OCM) of the CS fabrics, ε_i and γ_{ij} are the normal strain and shear strain, respectively, and σ_i and τ_{ij} are the normal stress and shear stress, respectively. The OCM of the CS fabrics indicates the relationship between stress and strain [15], which can be determined based on the orthogonal elastic moduli of the CS fabrics. It can be expressed as:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{v_{yx}}{E_y} & 0 \\ -\frac{v_{xy}}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{3}$$

where E, v, and G are the Young's modulus, Poisson's ratio, and shear modulus of the elastic fabrics, respectively. The x and y are the two principal stress directions of the elastic fabric, corresponding to fabric course and wale directions, respectively.

The Young's modulus of CS fabric is a mechanical property that measures its tensile stiffness, which can be determined by using a uniaxial tension testing to obtain fabric stress-strain curves along both course and wale directions. In the uniaxial tensile testing, $\sigma_y = 0$ when the CS fabrics are stretched along the course direction (x), then the OCM of the CS fabrics can be presented as:

$$E_x = \frac{\sigma_x}{\varepsilon_x} \tag{4}$$

where σ_x and ε_x denote the tensile stress and tensile strain of the CS fabric along its course direction, respectively. Correspondingly, when the CS fabrics are stretched along wale direction (y), the OCM of the CS fabrics can be presented:

$$E_y = \frac{\sigma_y}{\varepsilon_y} \tag{5}$$

where σ_y and ε_y are the tensile stress and strain along the fabric wale direction, respectively.

The Poisson's ratio is also a key parameter reflecting mechanical property of CS fabrics, which describes the deformation (expansion or contraction) of the fabric along a specific direction. The uniaxial tension testing was applied to determine Poisson's ratios of the studied CS fabric. Based on the OCM of the CS, Poisson's ratios can be determined as:

$$v_{xy} = -\frac{\varepsilon_y E_x}{\sigma_x} \tag{6}$$

The shear modulus of the CS fabric is a measure of elastic shear stiffness and is defined as the ratio of shear stress to shear strain [16]. A shear testing was applied to measure shear stress and shear strain within a plane. The OCM of the CS fabric can be presented as:

$$G_{xy} = \frac{\tau_{xy}}{\gamma_{xy}} \tag{7}$$

where τ_{xy} is shear stress in the fabric plane and γ_{xy} is shear strain in the corresponding plane, respectively.

B. Experimental Method

Young's moduli of the CS fabrics along course and wale directions, as well as Poisson's ratios of fabrics along these two directions (x and y) were determined by using Instron 4411 tensile tester (Norwood, MA, USA) with reference to ASTM D4964-96 based on (4) and (5) (Fig. 2 (A)). The sample size was 100 mm ×100 mm and the initial length was 100 mm and the maximum stretched length to be 100 mm. The Poisson's ratios of the fabric samples were calculated by using (6) based on the measured contraction and stretching lengths of the sample fabrics by using Vernier caliper. Shear modulus of CS fabrics within the course-wale (x-y) plane was determined by using pure shear tester (KES-FB1) with CS fabric size of 200 mm × 200 mm based on (7) (Fig. 2 (B)). The stress-strain curves obtained by the uniaxial tension and pure shear testing were recorded to calculate Young's moduli, Poisson's ratios, and shear moduli in a 2D scale, respectively.

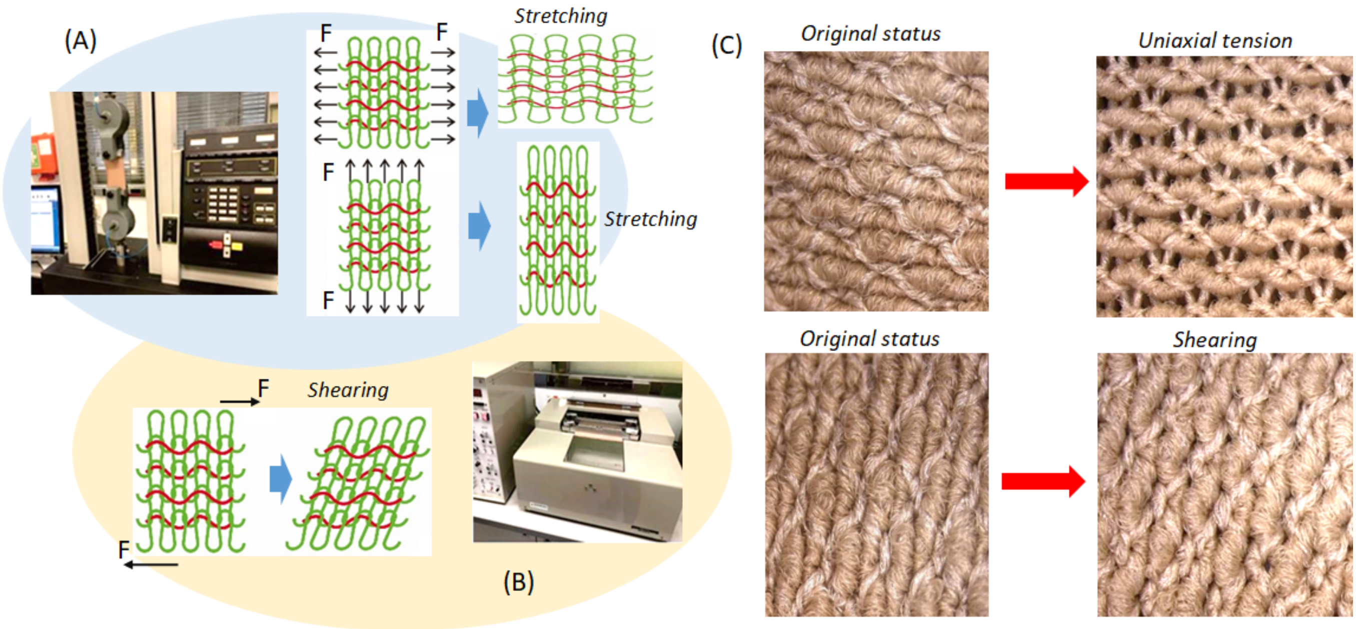

Fig. 2 Experimental testing, (A) uniaxial Instron 4411 tension testing system, (B) shear tester (KES-FB1), and (C) the studied CS fabrics

III. Results

The stress-strain curves were shown in Fig. 3. The linear regression analysis presented a preferable goodness-of-fit with coefficients greater than 0.98 ($R^2 > 0.98$), indicating that the tested CS fabrics have good linear elastic properties under the testing range in practical use. The Young's moduli of the tested three sample fabrics (S1, S2, S3) at the direction of course were calculated to be approximately 0.58 MPa, 0.34 MPa, and 0.55 MPa, respectively. Correspondingly, the Young's moduli of the tested three samples at the wale directions were approximately 0.28 MPa, 0.21 MPa, and 0.24 MPa, respectively. It can be seen that the Young's moduli of the studied CS fabrics at the course direction are generally greater than those along the wale direction. The Poisson's ratios of the CS fabric samples were determined with an aid of uniaxial tension tester. The measured Poisson's ratio was approximately 0.2 for the tested samples. The shear moduli of the studied CS fabrics determined by using pure shear testing (KES-FB1) were about 0.11 MPa, 0.07 MPa, and 0.13 MPa, respectively (Table I). These determined mechanical parameters can be input to the numerical simulation system such as FE modeling to further analyze effects of material mechanical properties on interface pressures and tissue deformation as reported in our previous studies (Fig. 1) [5], [6], [12].

TABLE I
Determined Young's Moduli, Poisson's Ratio, and Shear Moduli of the Studied CS Fabrics

Tested materials	E_x (MPa)	E_y (MPa)	v_{xy}	G_{xy} (MPa)
Sample 1	0.58	0.28	0.19	0.11
Sample 2	0.34	0.21	0.20	0.07
Sample 3	0.55	0.24	0.21	0.13

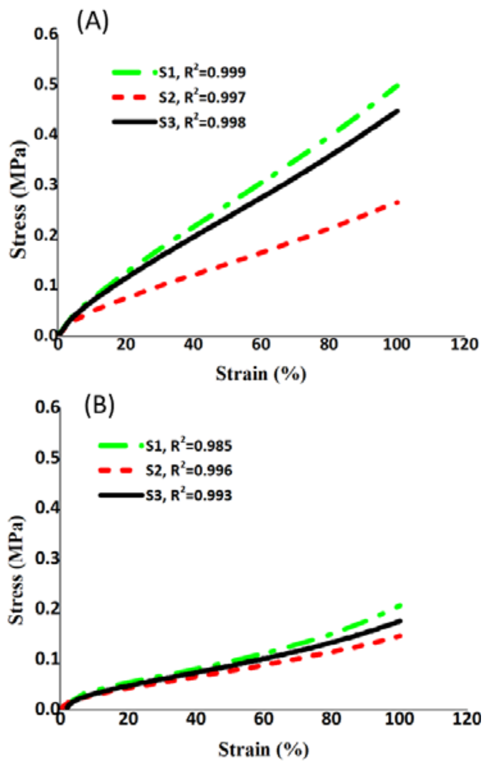

Fig. 3 Obtained stress-strain curves of the tested three CS fabric samples by applying uniaxial tension tester along fabric (A) course and (B) wale directions

IV. Conclusion

In this study, an approach to determine Poisson's ratio and elastic modulus (Young's modulus and shear modulus) of compressive textile materials has been proposed based on the orthotropic theory and experimental studies. The performed regression analysis has demonstrated the linear relationships between stress and strain properties of the studied CS fabrics under the controlled stretching ratios (< 100%). The proposed study methods and outcomes not only offer a faster solution to determine key mechanical properties of elastic compression materials, but also facilitate the understanding of effects of mechanical properties of CS fabrics on pressure performances through input of the determined mechanical parameters to numerical simulation system as to reveal complex interactions between compression textiles/fabrics and the applied human bodies.

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REFERENCES

[1] Liu, R., Guo, X., Lao, T. T., & Little, T. (2017). A critical review on compression textiles for compression therapy: Textile-based compression interventions for chronic venous insufficiency. *Textile Research Journal*, *87*(9), 1121-1141.

[2] Partsch, H. (2012). Compression therapy: clinical and experimental evidence. *Annals of Vascular Diseases*, 5(4), 416-422, 2012.

[3] Mosti G., Partsch, H. (2010). Duplex scanning to evaluate the effect of compression on venous reflux. *International angiology: a journal of the International Union of Angiology*, 29(5), 416-420.

[4] Silva, C.W.D. (2014). Mechanics of materials. *Boca Raton: CRC/Taylor & Francis Group* (Book style), 2014.

[5] Liu, R., Kwok, Y.L., Li, Y., Lao, T.T., Zhang, X., Dai, X.Q. (2006). A three-dimensional biomechanical model for numerical simulation of dynamic pressure functional performances of graduated compression stocking (GCS). *Fibers Polym*, 7(4), 389-397.

[6] Ye, C. Y., Liu, R. (2020). Biomechanical prediction of veins and soft tissues beneath compression stockings using fluid-solid interaction model. *International Journal of Biological and Medical Research*. 14(10), 285-290.

[7] Beer, F.P. (2020). Mechanics of materials. New York, NY: McGraw-Hill Education. Chen, W.K. *Linear Networks and Systems* (Book style). Belmont, CA: Wadsworth, 1993, pp. 123–135.

[8] Hursa, A. Rolich, T., Ražić,S.E. (2009). Determining pseudo-Poisson's ratio of woven fabric with a digital image correlation method. *Text. Res. J*, 79(17), 1588-1598.

[9] Sun, H., Pan, H., Postle, R. (2005). On the Poisson's ratios of a woven fabric. *Compos Struct*, 68(4), 505-510.

[10] Carr, C.M. (1988). Investigation of the effect of water repellent finishes on the mechanical properties of textiles using the Kawabata Evaluation System for fabrics, *Journal of Coated Fabrics*, 18(2), 106-113.

[11] Turl, L.H. (1956). The measurement of tearing strength of textile fabrics," *Text. Res. J*, 26(3), 169-176.

[12] Ye, C.Y., Liu, R., Wu, X.B., Liang, F.Y., Ying,T.C., Lv,J. (2022). New analytical model and 3D finite element simulation for improved pressure prediction of elastic compression stockings. *Mater Design*. 217, 110634.

[13] Sitharam, T.G., Govindaraju, T. (2021). Theory of Elasticity," first ed., *Springer Singapore.*

[14] Zhou, J.Y., Li, Y. Lam, K.C., Cao, X.Y. (2010). The Poisson ratio and modulus of elastic knitted fabrics. *Text. Res. J*, Vol 80 (18): 1965–1969.

[15] Yang, J.P., Chen, W.Z., Dai, Y.H., Yu, H.D. (2014). Numerical determination of elastic compliance tensor of fractured rock masses by finite element modeling. *Int. J. Rock. Mech*. Min, 70, 474-482.

[16] McNaught, A.D., Andrew, W. (1997). Compendium of chemical terminology: IUPAC Recommendations. *Oxford: Blackwell Science.*

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