

An Intensional Conceptualization Model for Ontology-Based Semantic Integration

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Abstract—Conceptualization is an essential component of semantic ontology-based approaches. There have been several approaches that rely on extensional structure and extensional reduction structure in order to construct conceptualization. In this paper, several limitations are highlighted relating to their applicability to the construction of conceptualizations in dynamic and open environments. These limitations arise from a number of strong assumptions that do not apply to such environments. An intensional structure is strongly argued to be a natural and adequate modeling approach. This paper presents a conceptualization structure based on property, relations, and propositions theory (PRP) to the model ontology that is suitable for open environments. The model extends the First-Order Logic (FOL) notation and defines the formal representation that enables interoperability between software systems and supports semantic integration for software systems in open, dynamic environments.

Keywords—Conceptualization, ontology, extensional structure, intensional structure.

I. INTRODUCTION

ONTOLOGY is becoming a very active topic in the computer science community and recently received considerable impetus and attention following the birth and development of the so called Semantic Web or Web data [1]. Ontology is viewed to gradually be applicable in many contexts in which it can facilitate the sharing of data and information. Additionally, Ontology allows interoperability, enables data integration and aggregation of contents, and empowers the meaningful communication and mutual understanding among humans, as well as between humans and software systems [2].

Various definitions of “ontology” have been proposed. Gruber describes ontology as “an explicit specification of a conceptualization” [3]. In this context, conceptualization is defined as “the objects, concepts, and other entities that are assumed to exist in some area of interest and the relationships that hold among them” [4]. Thus, ontology does not aim to account for, in an objective manner, the whole of reality, but rather only some of its aspects. Furthermore, ontology does not correspond with a conceptualization, but rather with a representation in some language, whose meaning, as far as possible, is made explicit in the ontology. Gruber, however, presented a conceptualization in a different form: “A conceptualization is a simplified, abstract view of the world for some purpose that we wish to represent” [3]. In the notation used here, an intensional structure of conceptualization is

indicated. Other definitions suggest that ontology is correlated to a formal specification of shared conceptualizations by a community of people or by artificial agents [5]. The formal representation enables the interoperability between software systems.

Studer offer a hybrid definition, considering ontology “a formal, explicit specification of a shared conceptualization” [6]. Another definition provided by Guarino refined Gruber’s definition by outlining the differences between an ontology and a conceptualization in such a way that an ontology can be defined as “a logical theory accounting for the intended meaning conceptualization” [7]. Guarino opposed Gruber’s definition and argued that since the term “conceptualization” in this definition refers to ordinary mathematical relations or extensional relationships on the domain. He claimed that “these relations reflect a particular state of affairs, for instance, in the blocks world, they may reflect a particular arrangement of blocks on the table” [7].

It is noteworthy that the aforementioned definitions use conceptualization as a basis for modeling ontology and clearly these definitions indicate that conceptualization structure should be intensional in nature. However, the formal definition given to conceptualization is an extensional and extensional reduction structure.

We argue that the extensional reduction structure presented in [8] and [7] has been adopted in subsequent work, most recently in [9]-[14].

The extensional reduction structure is more appropriate for describing conceptualizations than extensional structures, because they allow the change of relationships between extensions within the possible world and thus can be used to describe closed dynamic systems in which the arrangement of the system may change over time. It must be recognized, however, that adding new extensions will result in new conceptualization. As a result, the extensional reduction model is inappropriate for describing information systems that exist in open environments. In this regard, it is evident that an intensional structure is an adequate and natural choice for conceptualization in open environments. The proposed intensional structure which is based on the theory of PRP, asserts the existence not only of ordinary PRP, but also of abstract individuals and abstract properties and relations [15].

In this paper, we propose a conceptualization structure which is based on PRP theory, to improve the description of

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conceptualization by describing relations as irreducible primitive entities rather than reducing them to extensional functions.

II. PROBLEM ANALYSIS

As most of the work cited in the above sections, many approaches adopted an extensional reduction structure for conceptualization. In this section, the limitations of the extensional and extensional reduction structures will be discussed and explained more formally.

It will be shown that the extensional reduction structure can be applicable only for static systems. Yet, for information systems, and dynamic systems in open environments, the extensional reduction structure is not adequate. This section will critically discuss the extensional and extensional reduction models as applied for describing the conceptualization. An intensional structure based on the intensional logic will be proposed as an adequate candidate for describing conceptualization and ontology in open environment.

A. Extensional Structure

This structure is based on the definition provided by [3] which states that the conceptualization is “a body of formally represented knowledge is based on a conceptualization: the objects, concepts, and other entities that are assumed to exist in some area of interest and the relationships that hold among them”. The definition accounts for the extensional notation, which contradicts with the other definition [3] as well which stated that “A conceptualization is an abstract, simplified view of the world that we wish to represent for some purpose”.

Conceptualization is inherent to any knowledge base or information system, either explicitly or implicitly. In formal mathematical terms, it can be expressed as a tuple:

$$C = \langle D, R \rangle$$

where D is a set all entities in a domain that is called the universe of discourse. R is a set of relations on D where each element of R is an extensional structure that reflects a specific world state. To exemplify this notion, let us consider the following example: Let (SO) denote a set of entities are called hereafter Smart Objects¹ which exist in open environment as shown in Fig. 1. A simple conceptualization for these SOs can be denoted by the following tuple $C = \langle R, D \rangle$ such that:

- $D = \{SO_1, SO_2, SO_3, SO_4\}$
- $R = \{r_1, r_2\}$

where the extensions of both relations are:

- $SO_1 = \{SO_1, SO_2, SO_3, SO_4\}$ which holds the elements of universe of discourse which are the Smart objects
- $r_2 = \{r^2(so_1, so_2), r^2(so_1, so_3), r^2(so_2, so_3)\}$ the binary relations between the SOs

This conceptualization accepts specific world state of affairs, which are represented by the world structures. $r(w) = \{\{SO_1, SO_2, SO_3, SO_4\}\}$,

¹ Smart objects are autonomous, self-directed entities with incomplete knowledge that can share capabilities and information with others and make decisions based on self-interest to achieve individual or collective goals.

$$\{r_{12}(so_1, so_2), r_{22}(so_1, so_3), r_{32}(so_2, so_3)\}$$

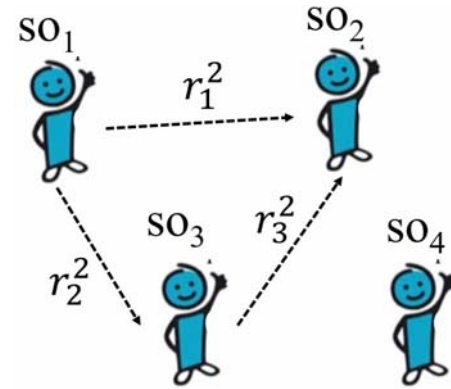


Fig. 1 A specific world with entities (SOs) and the relation between them

Let us consider the following new relationship, illustrated in Fig. 2, in comparison to our previous example:

- $D' = D$
- $R' = SO, r^{2'}$ where $r^{2'} = r^2 \cup r^2(so_1, so_4)$

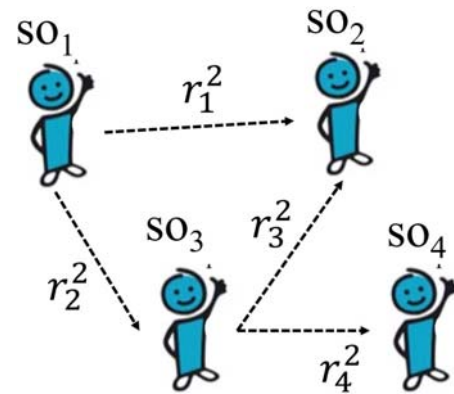


Fig. 2 A specific world with entities and the relation between them where the relation can change

Although the changes imply the addition of a new relation, it is very clear that,

$$\langle D, R \rangle \neq \langle D, R' \rangle$$

As such, we end up with two different conceptualizations, and so while the structure is acceptable in describing a single situation where the interest is about an isolated snapshot of the world, this notion of conceptualization will become problematic, mainly because it depends too much on a specific state of the world as stated by Guarino in [7]. He argued that a conceptualization is about concepts and the meaning of underlying concepts. Also, he pointed out that the conceptualization should not change when the world changes. Therefore, in order to capture this notion, an extensional structure is not adequate, and as such, the possible world

approach is adopted by [7] to formulate the conceptualization.

B. Extensional Reductions Structure

The definition that Gruber provided does not account for an intensional notion of conceptualization. Guarino explains, “the meaning of the relations lies in the way they refer to certain entity according to their spatial configuration” [7]. In other words, the relations of R , which is a set of relevant relations on D , reflect a specific world when the extension of a conceptualization is presented as $\langle D, R \rangle$. However, the focus has to be on the meaning of those relations, independently of any particular world. Therefore, he suggests in [7] that the meanings of a conceptualization can be represented as conceptual relation which is a function from possible worlds into sets. In this case, a conceptualization C is structured as:

$$C = \langle D, W, R \rangle$$

where $\langle D, W \rangle$ is called a domain space, W a (non-empty) set of possible worlds, D is the domain of individuals and R is the set of n -ary of conceptual relations on the domain space $\langle D, W \rangle$. Hence, each n -ary relation ρ^n is a function from W to n -tuples of extensions in D with signatures as $\rho^n: W \rightarrow {}_2D^n$.

Let us refer back to the previous example in order to illustrate this idea.

$$\langle D, R \rangle \neq \langle D, R' \rangle$$

can be considered as a different possible world (w_1) and (w_2) respectively with the following conceptualization C :

- $D = so_1, so_2, so_3, so_4$ set of individuals
- $W = w_1, w_2, \dots$ the set of possible worlds
- $R = SO, r^2$ the set of conceptual relations where SO are rigid, and, thus, map to the same extensions in every possible world $\forall w \in W$
- $SO(w) = D = so_1, so_2, so_3, so_4$
- $r_2(w_1) = r_1^2(so_1, so_2), r_2^2(so_1, so_3), r_3^2(so_2, so_3)$
- $r_2(w_2) = \{r_1^2(so_1, so_2), r_2^2(so_1, so_3), r_3^2(so_2, so_3), r_4^2(so_2, so_4)\}$
- $r_2(w_3) = \dots$

It is clear that these two distinct intensional relations are possible because they can be defined on a domain space, rather than a domain, which results in the same conceptualization and therefore, it makes it more appropriate in way that describes the conceptualization when it is compared to extensional notation. Nonetheless, there are some disadvantages that have been recognized by [15], even by Guarino himself [7], with the use of sets of possible world approach. It is evident, for instance, that the two relationships “trilateral” and “triangle” are the same, since they have the same extension in all possible worlds, despite the fact that the meaning is different in each case. Besides, the extensional reduction notation might be considered sufficient only for describing closed dynamic systems in which the change occurs in the relations between the extension in the domain space. However, in circumstances such as introducing new extension or reducing existing one, which is the case in open environment, this notation turns out to be inappropriate.

As shown in Fig. 3, let us add some changes to our example

by adding a new extension to clarify this claim. Let $D' = D \cup so_5$, W' the set of possible world on D' .

Although a very tiny change has been made on the conceptualization it turns out that:

- $D \neq D'$
- $W \neq W'$
- because R' is set of $\rho^n: W \rightarrow {}_2D^n$ it is very clear to tell that $R \neq R'$

Therefore, we end up with two different conceptualizations.

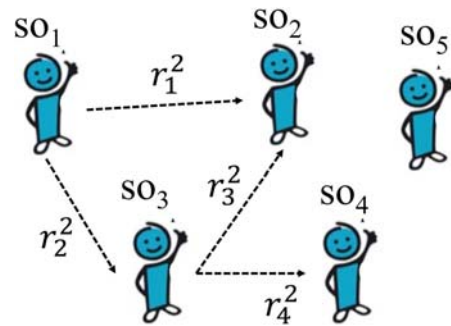


Fig. 3 A specific world with entities and the relation between them where both the entities and the relation can change

The preceding problem arises from the following:

- The conversion of entities are defined by their properties and characteristics (intensional) to entities that are defined by their behavior and actions (extensional), either as functions or sets.
- The set of individuals in the domain are rigid which are the same things from possible world to possible world and mapped to the same extensions in every possible world.
- The problem, according to Guarino, is that Gruber’s definition does not provide an intensional account of the notion of conceptualization. We found out that even Guarino’s definition does not provide an intensional account of the notion of conceptualization as well. It simply provides all possible relations between the entities in all possible worlds.

In general, many researchers have deemed the use of extensional reduction structures based on possible worlds theory to be illogical, as cited in [15]-[17]. The alternative is to take properties and relations, as well as propositions, at face value, i.e., as real, irreducible entities [15]. Therefore, and based on the above observation, an intensional notation for modeling the conceptualization is required. In the following sections, we propose an intensional model and the relevant notation which is an extension of the FOL utilizing [15]. However, before proposing the intensional model, it is worth to shed some light on the distinction between extensions and intensions as well as the conception of intensional entities.

C. Distinction between Extensions and Intensions

Traditionally, the distinction between extensions and intensions has been made regarding expressions. This distinction is based on the fact that the expression has both an extension and intention meaning [15], [18]. On the one hand,

two different expressions refer to the same entity. Therefore, they have the same extension.

We consider the example of Frege: both the “morning star” and “evening star” expressions indicate Venus. On the other hand, each of these expressions brings to mind a different perspective of Venus; “evening star” brings to mind the first to appear in the evening, whereas “the morning star” brings to mind the last luminous body to disappear in the morning. Thus, these expressions have two distinct intensions [19]. Two expressions may refer to the same object, but in a different way. The terms “morning star” and “evening star” refer to the same object as a result of an astronomical discovery of the same star; it follows that the two expressions have the same reference or extension. However, they refer to the same object in a different way; and this way of referring to them is their sense or intension.

Two expressions can have the same extension (Denotation, reference), but intension (Connotation, sense) might be totally different [16].

Identity: The difference in cognitive significance between “evening star is morning star” and “evening star is evening star” is that the first statement tells us that two different entities, the evening star and the morning star, are actually the same object (Venus) and thus adds new information. The second statement, however, simply confirms that the evening star is indeed the evening star and does not add any new information. This is known as Frege’s Puzzle and highlights the importance of understanding the distinction between sense and reference in language and thought [15].

Substitutivity: in intensional contexts, the reference of some term is not what it normally denotes, but rather its sense [15]. “John believes that ‘evening star’ is beautiful” does not imply that “John believes that ‘morning star’ is beautiful” just because they have the same references. Similar examples can be created using several techniques, such as quotations, indirect speech, propositional attitudes (beliefs, knowledge, ...), etc. In these situations, the replacement of an expression to another that refers to the same individual usually produces alterations in meaning due to the ambiguous assignment of a contact person (referential opacity). For example, “Lex Luthor discovered that Clark Kent was Superman”; using Clark Kent to replace Superman in “Lex Luthor discovered that Clark Kent was Clark Kent”, produces a change in the meaning of the sentence, although the two expressions have the same referent.

Propositions and properties are examples of intensional entities because they violate the extensionality principle [15]. There are two conceptions into which an intensional entity is built. The first is based on reductionist approaches such as the possible-worlds reduction treatment of PRP. This conception as it was shown above is intuitively implausible. In this conception, intensional entities are reduced to extensional entities, such as extensional functions or sets, which has some shortcomings [15]. A non-reductionist approach is adopted in the second conception, based on algebraic traditions in extensional logic. Therefore, in our proposal, an extension of the FOL by Bealer’s intensional abstraction operator with the Intensional Algebraic structure according to the theory of PRP

for intensional logic will be considered in modelling the conceptualization.

D. An Intensional Algebraic Structure

The FOL (which includes the quantification of variables but not on Sets) reveals a highly extensional tool, incapable to formally capture the meaning of such expressions. It can be shown as the following theorem, extensionality formulation of the principle:

$$\forall x(Ax \leftrightarrow Bx) \models \theta \leftrightarrow [B/A]\theta$$

Intuitively, if two predicates (A, B) are valid for the same set of entities (indicated by the x variable on which varies the quantifier), they can replace each other without changing the truth values (i.e., $\theta \leftrightarrow [B/A]\theta$) then the semantics of the sentence θ where the substitution takes place B/A .

Extensional logic, however, uses techniques from algebraic tradition, an algebraic way for definition of complex intensional entities from atomic entities can be offered by the so-called Intensional FOL language with intensional abstraction and an Intensional Algebraic which is a structure that contains a domain of primitive entities such as particulars, properties, relations and propositions [16].

The idea is to take properties and relations as well as propositions at face value, i.e., as real, irreducible entities [15], instead of identifying them with function that lies at the very heart of the possible-worlds semantic method. The new denotation is determined by the application of the relevant fundamental logical operation to the denotation(s) of the relevant syntactically simpler term (s) [15].

III. PROPOSED MODEL

Having clarified the distinction between extensions and intensions, let us now turn our attention to the proposed description of the conceptualization by recalling Guarino’s argument and the definition of conceptualization and, as it has been shown that historically there are two conceptions of which intensional entity is built into, we are in favor of the second conceptions of PRP theory for the reasons mentioned above. We strongly believe that the conceptualization should be formalized intensionally using intensional structure.

$$C = \langle D, K, t \rangle$$

where D is divided into subdomains: such that D_0 is the subdomain of propositions and D_1 the subdomain of properties, while D_{-1} is the subdomain of particulars (extensional entities). Additionally, K is a set of extensionalization functions.

The semantics of the extensionalization functions $h \in K$ is such that they assign the following possible extensions to individuals, propositions and properties:

- $x \in D_{-1} \rightarrow h(x) = x$
- $x \in D_0 \rightarrow h(x) = n$ for $n \in T, F$
- $x \in D_1 \rightarrow h(x) \subseteq D$
- $x \in D^n \rightarrow h(x) \subseteq D^n$

where: t is a set of logical operations on D . The set includes the

operation *preds* of singular predication, which is, defined as $D_1 \times D_1 \rightarrow D_0$. The semantics of *preds* is the following, if the quantificational range of *y* is restricted to D_1 .

$$\forall x \in D_1 \forall y \in D_1 \forall h \in K(h(\text{preds} \langle x, y \rangle) = T \leftrightarrow y \in h(x))$$

where *preds* (the operation of singular predication) maps pairs of elements in the domain of discourse onto the relevant singular proposition.

An intensional interpretation of the intensional FOL is a mapping between the set L_w of formulae of the logic language and intensional entities in D , $I: L_w \in D$. Thus, the interpretation function I will assign terms obtained by abstraction operator of L_w to the subdomains respectively, for instance, assigning a value to the individual constant '*a*', $I('a') = a \in D_1$, and to the predicate '*P*', $I('P') = P \in D_1c$. Singular predication of unary predicates satisfies the following truth-condition: $h(\text{Preds} \langle P, a \rangle) = T \leftrightarrow a \in h(P)$. The operation *preds* (or singular predication) takes a pair consisting of a property and an entity (i.e., element in the domain of discourse) and maps them onto a proposition that is true if and only if (iff) the entity is in the extension of the property. The extension or the denotation of the proposition $\text{Preds} \langle P, a \rangle$ is identical to the truth value T iff the individual a is an element of the extension of the property P . That is, the result of predicating a property of an individual is a proposition that is true iff the individual is in the extension of the property. Let us assume that the interpretation function I assigned the property $\text{Human} \in D_1$ to '*H*' and $\text{John} \in D_1$ to '*j*'. Then the formal semantics of the predication of John of being human is a straightforward. $h(\text{Preds} \langle H, j \rangle) = T \leftrightarrow j \in h(H)$. The proposition that *John is human* is true iff John is in the extension of the property of human that the semantic interpretation is assigned to '*H*'.

In order to verify that the proposed model of intensional conceptualization helps to solve the problem of extensional reduction approach we have illustrated earlier. Let the interpretation function I assign the property $\text{Smart Object} \in D_1$ to '*SO*' and, $so \in D_{-1}$ to so_5 . Thus, the formal semantics of the predication of so_5 of being Smart Object is now straight as well. $h(\text{Preds} \langle SO, so_5 \rangle) = T \leftrightarrow so_5 \in h(SO)$ That is, the proposition that so_5 is Smart Object is true iff so_5 is in the extension of the property of Smart Object that the semantic interpretation has assigned to '*SO*'.

It is evident that when introducing any new extension relevant to the conceptualization, these new concepts and relationships will be captured accordingly because of the fact that the conceptualization is defined as an abstract model that consists of the relevant intensional that exists in a certain domain. Applying this idea to the binary relation between two individuals (smart objects), the interpretation function I assigned the property relation $\in D_1$ to '*R*', $(so, so) \in D_2$ to so_1, so_4 . Then the formal semantics of the predication of so_1, so_4 as binary relation is as following: $h(\text{Preds} \langle R, (so_1, so_4) \rangle) = T \leftrightarrow (so_1, so_4) \in h(R)$

Consequently, the result of predicating a property relation between two individuals is a proposition that is true iff the

individuals are in the extension of the property.

IV. DISCUSSION

The concepts by which ordinary individuals, properties, and relations are perceived are considered abstract. It takes the denotation of a predicate to be a property or a relation (intensionally perceived), and it analyzes the sense of that predicate as a concept by which the perceived property or relation denoted by the predicate and therefore, properties and concepts can be notably distinguished. Moreover, the concepts of individuals might be combined with other concepts of properties and relations so as to form new aggregate complex concepts. These complex concepts need to be viewed as entities by which ordinary propositions (or states of affairs) can be conceived. It is also worth mentioning that the use of the singular term in the intensional logic avoids higher order syntax for intensional logic [15]. The proposed intensional description of conceptualization is extensible and can be expanded to describe and capture more details about the world where entities and their volatile relationships are in continuous change.

V. CONCLUSION AND FUTURE WORK

The work presented in this paper has reviewed some of the prominent approaches of modeling of conceptualization using the extensional model and the extensional reduction model. We have illustrated the limitations of the extensional and extensional reduction models when applied to an open environment where entities are of a dynamic nature and can join/disjoin unpredictably, and thus software systems should have means to handle joining/removal of entities and their corresponding relationships at the run-time without losing the system's integrity. The proposed intensional-based conceptualization structure captures concepts and their relevant relationships and provides suitable representation means to define entities' PRP that satisfies the intrinsic requirements inherent for semantic integration approaches. The current and future work is geared towards validating the proposed model with special focus on semantic integration in healthcare application domain. The focus will be on providing a semantic view for patient various concepts that are differently represented in various healthcare information systems that act independently to synergize medical data according to mutual interests. The proposed model provides semantic integration means for distributed, autonomous, and possibly heterogeneous information sources of different healthcare service providers and requesters.

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