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Decomposition of the Customer-Server Interaction in Grocery Shops

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Abstract—A successful shopping experience without overcrowded shops and long waiting times undoubtedly leads to the release of happiness hormones and is generally considered as the goal of any optimization. Factors influencing the shopping experience can be divided into internal and external ones. External factors are related e. g. to the arrival of the customers to the shop whereas internal factors are linked with the service process itself when checking out (waiting in the queue to the cash register and the scanning of the goods as well as the payment process itself) or any other non-expected delay when changing the status from a visitor to a buyer by choosing goods or items. This paper divides the customer-server interaction in five phases starting with the customer arrival at the shop, the selection of goods, the buyer waiting in the queue to the cash register, the payment process and ending with the customer or buyer departure. Our simulation results show how five phases are intertwined and influence the overall shopping experience. Parameters for measuring the shopping experience based on a burstiness level in each of the five phases of the customer-server interaction are estimated.

Keywords—Customers' burstiness, cash register, customers' waiting time, gap distribution function.

I. Introduction

SHOPPING experience can be marred by many aspects such as long lines and long waiting times. When the number of customers exceeds a given maximum number, delays based on so-called bottlenecks occur and impact the customer shopping experience negatively.

Analysis of the external and internal factors that influence the bottleneck in shop sales [1] has led to the identification of phases in the buying process in shop sales [1]. The buying process was based on the customer-server key interactions: buyers' (in other words, customers') waiting time in the queue to the cash register (in other words, server) and the payment processing time at the cash register. Thereby, the buying process mostly focused on the check out phase. The application of the macroscopic view on customer-server interactions in grocery shops allows shifting the paradigm from only the buying process to the shopping process. The shopping process implies a wider (macroscopic) view on the composition of customer-server relationships.

This work aims at the decomposition of customer-server interactions in the shopping process from the macroscopic view as well as mathematical description of each phase of the shopping process.

The term customer is referred in this work to anything and anyone such as shoppers that arrive at a facility and require service. Customers can be differentiated into visitors (i. e, a shopper who visits a shop and does not buy anything) and buyers (shoppers who visit the shop and buy anything). In turn, a server means a resource that provides the requested service, e. g. a check-out station at a grocery shop. Table I illustrates the relationships between the systems, customers and servers.

This paper investigates how external factors (such as the arrival of the costumers) as well as internal factors such as a number of bought goods, the waiting time in the queue to the cash register as well as the payment processing time at the cash register influence shop sales. The definition of internal and external factors in shop sales was introduced in [1].

Modelling of the customer-server interaction has attracted a lot of research activities [2]. In [3] and [1] the service process consisting of the waiting in the queue to the cash register and the scanning of the goods as well as the payment process have been analysed and parameters for measuring burstiness within the check-out process have been derived. Joint modelling of these two processes by using generation functions is given in [4].

The novelty of the paper is given by joint modelling of the shopping process starting from the customer arrival at the shop and further proceeding through the selection of the goods (leading to the transition from a visitor to a buyer), the payment process consisting of the waiting in the queue to the cash register and the scanning of the goods as well as the payment process and the departure from the shop.

The remaining part of this paper is organized as follows: Section II introduces the theoretical basis for modelling of different phases of the shopping process. The simulation model for analysing the customer-server interaction is introduced in Section III. The associated results of the empirical study analysed by a computer simulation are discussed in Section IV. Finally, some concluding remarks are provided in Section V.

TABLE I
Examples of the Systems, Customers and Servers

System	Customer	Server
grocery shop	shoppers	check-out station
hospital	patients	nurses
reception desk	guest	receptionist

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II. Decomposition of Customer-Server Interactions

By analysing the customer-server relationships with regard to the shopping event, it can be found out that shopping process starts with the arrival of the customer in the shop and ends with the departure from the shop. In the shop, the customer should have bought as many goods as possible and should be served as quickly as possible.

The five phases of the shopping process are highlighted in Fig. 1. By the shopping process chain, the customer arrival in the shop, the selection of goods, the waiting in the queue to the cash register, the payment process as well as the buyer departure is meant. In the first phase (phase 1) the customers arrive with a given intensity to the shop i. e. in some moments more concentrated and in other moments less. In this work the intensity of the customers is statistically modelled by gaps between two neighbouring customers described by the mean value $m_{1,\mathrm{v}}$ and the standard deviation σ_{v}. Here, a customer is a person who visits the shop and requires service. In the second phase the so called good selection phase (phase 2) the customer becomes with a certain probability a buyer and purchases a given number of goods g. Afterwards the buyer moves to the check out and arrives first to the waiting line at the cash register (phase 3) and afterwards s/he will be served at the cash register by scanning the goods and the payment process (phase 4). The shopper departure refers to phase 5.

A. *Customer Arrival in the Shop (Phase 1)*

The customer traffic in grocery stores varies greatly throughout the day. If the time between customers is represented by gaps, then the independence of the gaps between customers can by no means be assumed. Assuming that the time intervals between customers are represented by gaps, then situations with short time intervals will be followed by periods with longer intervals. Often the time intervals between customers undergo an exponential characteristic. However, under a bursty condition of customer arrival non-exponential characteristics such as Weibull [5] or Wilhelm [6], [7] are of high interest. According to [8], a level of burstiness related to the arrival process of the customer can be calculated analytically and is defined as

$$B_{\mathrm{v}} = \frac{\sigma_{\mathrm{v}} - m_{1,\mathrm{v}}}{\sigma_{\mathrm{v}} + m_{1,\mathrm{v}}} \ , \tag{1}$$

by taking the mean value $m_{1,\mathrm{v}}$ (average gap length between two customers entering the shop) as well as the standard deviation σ_{v} into account. The burstiness parameter B ranges between $-1 \leq B \leq 1$. Here, larger values of B indicate a higher level of burstiness. Here, $B = 1$ corresponds to a bursty environment whereas $B = 0$ is referred to a neutral environment. Regular signals are described by negative parameters of B. Therefore a realistic interval for the parameter B, describing the bursty arrival of customers, can be expected to be between zero and one.

Assuming that the customer arrival in the shop is Gaussian distributed, the (cumulative) distribution function is defined as

$$P(\mathrm{gap} < a) = F(a) = \frac{1}{\sqrt{2\,\pi\,\sigma_{\mathrm{v}}^2}} \int_{-\infty}^{a} \mathrm{e}^{-\frac{(x - m_{1,\mathrm{v}})^2}{2\,\sigma_{\mathrm{v}}^2}} \,\mathrm{d}\,x \ . \tag{2}$$

with the mean value $m_{1,\mathrm{v}}$ and the standard deviation σ_{v} of the customer arrival. Here, the mean value $m_{1,\mathrm{v}}$ is expressing the separation of the customer's arrival in the shop (measured in seconds) and the standard deviation how the separation of the customer's arrival is fluctuating with the time. The probability that a gap between two customers is smaller than a given value a is defined by $P(\mathrm{gap} < a)$ as given in (2). This equation can be re-written with the help of the Gaussian error function $\mathrm{erf}(\cdot)$ as

$$P(\mathrm{gap} < a) = F(a) = \Phi\left(\frac{a - m_{1,\mathrm{v}}}{\sigma_{\mathrm{v}}}\right) \tag{3}$$

with

$$\Phi(x) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right) \ . \tag{4}$$

In practical situations the complementary cumulative distribution function (CCDF), defining the probability that a gap is larger or equal to a given threshold a, has to be found beneficial and can be defined as

$$P(\mathrm{gap} > a) = 1 - F(a) = 1 - \Phi\left(\frac{a - m_{1,\mathrm{v}}}{\sigma_{\mathrm{v}}}\right) \ . \tag{5}$$

Using the complementary error function i. e. $\mathrm{erfc}(\cdot)$ instead of the error function $\mathrm{erf}(\cdot)$ the probability $P(\mathrm{gap} > a)$ can be re-written with $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$ as

$$P(\mathrm{gap} > a) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{a - m_{1,\mathrm{v}}}{\sigma_{\mathrm{v}}}\right) \ . \tag{6}$$

The corresponding CCDF is depicted for different combinations of $m_{1,\mathrm{v}}$ and σ_{v} in Fig. 2.

In comparison, the probability density function (pdf) of a Gaussian distribution depending on the real valued random gap parameter a is given as

$$p_{\mathrm{gap}}(a) = \frac{1}{\sqrt{2\,\pi\,\sigma_{\mathrm{v}}^2}} \mathrm{e}^{-\frac{(x - m_{1,\mathrm{v}})^2}{2\,\sigma_{\mathrm{v}}^2}} \ . \tag{7}$$

Fig. 3 highlights the visitor-specific gap-based probability density function (PDF) for different combinations of $m_{1,\mathrm{v}}$ and σ_{v}.

It is worth noting that for a given value of $m_{1,\mathrm{v}}$ lower fluctuations within the gap length appear for low values of σ_{v}. A value of σ_{v} close to zero will lead to a deterministic gap process.

B. *Good Selection Process (Phase 2)*

The distribution of the numbers of bought goods g can be modelled by different probability density functions depending on the shops characteristic. Next to the exponential characteristic, Weibull and Rayleigh have to be found beneficial. Table II highlights several probability distributions when describing the good selection process. The exponential distribution frequently appears in many applications, including queuing models and reliability theory [9]. Analysing the

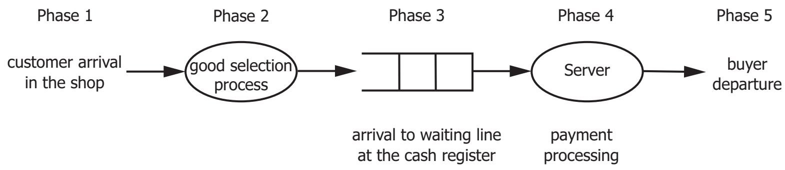

Fig. 1 Phases of the shopping process

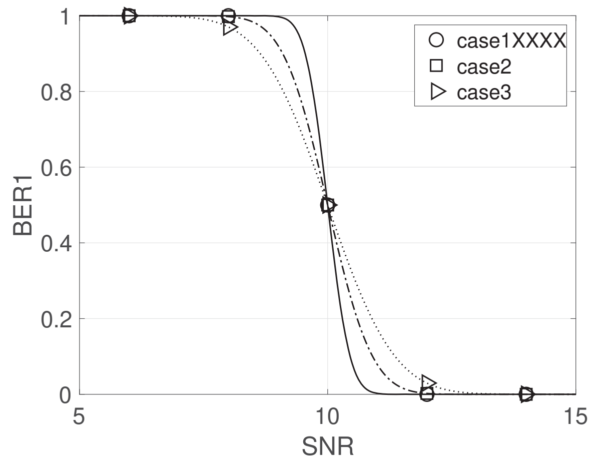

Fig. 2 Visitor gap-based CCDF assuming an average gap length of $m_{1,\mathrm{v}} = 10$ seconds and different parameters for the standard deviation σ_v

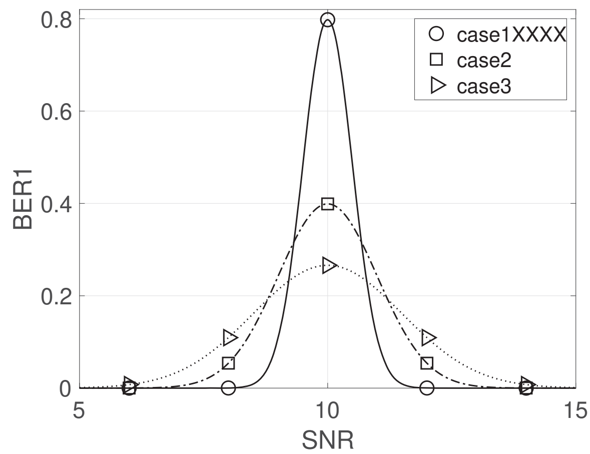

Fig. 3 Visitor gap-based probability density function (PDF) assuming an average gap length of $m_{1,\mathrm{v}} = 10$ seconds and different parameters for the standard deviation σ_v

exponential distribution, the parameter β_e is called good rate, and the averaged number of bought goods g per buyer, i. e. the mean value $m_{\mathrm{e},1}$, results in $m_{\mathrm{e},1} = 1/\beta_\mathrm{e}$. Fig. 4 illustrates these functions with exemplary parameters ($\beta_\mathrm{e} = 1/2$, $\alpha_\mathrm{w} = 1$, $\beta_\mathrm{w} = 1$, $\beta_\mathrm{r} = 2/3$). Here it is worth noting that the Weibull distribution coincides with the exponential one for $\alpha_\mathrm{w} = 1$.

C. Arrival to Waiting Line at the Cash Register (Phase 3)

Phase 3 describes the arrival of the buyers to the waiting line at the cash register.

TABLE II
SEVERAL PROBABILITY DISTRIBUTIONS

Type	Probability density $p(g)$	Distribution $F(g)$
Exponential	$\beta_\mathrm{e}\,\mathrm{e}^{-\beta_\mathrm{e} g}$	$\mathrm{e}^{-\beta_\mathrm{e} g}$
Weibull	$\beta_\mathrm{w}\alpha_\mathrm{w}(\beta_\mathrm{w} g)^{\alpha_\mathrm{w}-1}\,\mathrm{e}^{-(\beta_\mathrm{w} g)^{\alpha_\mathrm{w}}}$	$\mathrm{e}^{-(\beta_\mathrm{w} g)^{\alpha_\mathrm{w}}}$
Rayleigh	$\beta_\mathrm{r}^2\, g\,\mathrm{e}^{-\beta_\mathrm{r}^2 \frac{g^2}{2}}$	$\mathrm{e}^{-\beta_\mathrm{r}^2 \frac{g^2}{2}}$

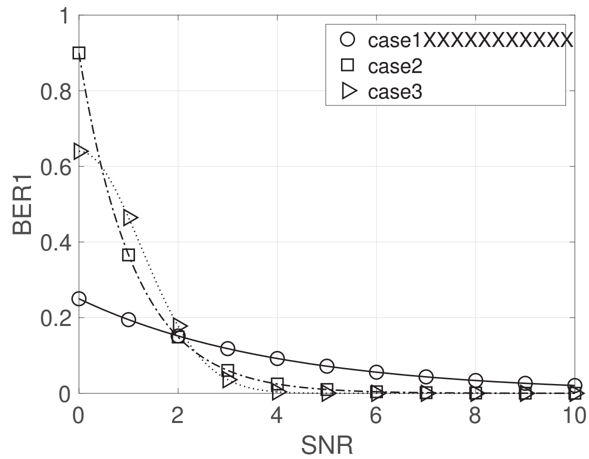

Fig. 4 Different probability density distributions of the number of bought goods with exemplary parameters

D. Payment Processing at the Cash Register (Phase 4)

Based on the experimental results in different shops in Lithuania (see e. g. [2], [1]), the payment processing time t_s can be assumed as

$$t_\mathrm{s} = 1{,}9\,g + 22{,}8 \tag{8}$$

and depends on the number of bought goods g. Here it was found out that for one good about $1,9$ seconds and additionally about $22,8$ seconds for each buyer are required. The data were collected in different grocery shops and it is expected that other check out stations work in a similar way as they work with similar equipment. Thereby it was not differentiated between different kinds of payment (e. g. cash, credit card or via mobile apps) [10].

E. Buyer's Departure (Phase 5)

The buyers' departure is not further studied in the work as this phase describes the end of the shopping.

III. Modelling of Customer-Server Interaction

In situations, when the mathematics is intractable, agent-based simulation provides an efficient solution to simulate the customer-server interactions by taking decisions of individual buyers, so-called agents, into account [11]. By agents, relatively autonomous computational objects are understood.

The input parameters of the customer-server interaction are the customer arrival in phase 1 described by the mean value and the standard deviation of an underlying Gaussian distribution. These two parameters determine the distances between neighbouring customers entering the shop. The corresponding level of B, defined in (1), gives the expected level of burstiness.

Phase 2 is dedicated to the process of the choice of goods. Here it will be decided with a given probability p_e if the customer becomes a buyer and how many goods the buyer purchases. This process is divided into two parts: First with the probability p_e (also called buyer probability) the visitors are splitted into non-buyers and buyers. Second, only for buyers the average number of goods $m_{\mathrm{e},1}$ with the condition $m_{\mathrm{e},1} \geq 1$ is chosen (as a visitor has to buy at least one item in order to become a buyer). When using the beforehand introduced exponential distribution for the random variable g, describing the number of bought goods, the exponential distribution defined in Table II moves to the shifted exponential distribution given as

$$p(g) = \beta_\mathrm{e}\,\mathrm{e}^{-\beta_\mathrm{e}(g-1)} \tag{9}$$

with the mean value

$$m_{\mathrm{e},1} = \frac{1}{\beta_\mathrm{e}} + 1 \tag{10}$$

and the standard deviation

$$\sigma_\mathrm{e} = \frac{1}{\beta_\mathrm{e}} \ . \tag{11}$$

Phase 3 and 4 describe the service process consisting of the waiting in the queue at the cash register and the payment process. In this work the payment processing is based on experimental results obtained from different shops in Lithuania as given in (8). With the buyers' departure in phase 5 the shopping process ends.

IV. Simulation Results

Tables III-V show different investigated configurations with regard to the selected parameter in phase 1 and 2. The

TABLE III
Customer-Server Configuration I (CSC-I)

Phase	Parameter	Burstiness B
1	$m_{1,\mathrm{v}} = 14$ s, $\sigma_\mathrm{v} = 2$ s	$-0,75$
2	$p_\mathrm{e} = 0,5$, $m_{\mathrm{e},1} = 2$	not applicable

calculated level of burstiness shows more equally distributed customers in phase 1 than bursty ones. For describing bursty buyers' distribution, positive values of B should be expected.

TABLE IV
Customer-Server Configuration II (CSC-II)

Phase	Parameter	Burstiness B
1	$m_{1,\mathrm{v}} = 16$ s, $\sigma_\mathrm{v} = 2$ s	$-0,77$
2	$p_\mathrm{e} = 0,5$, $m_{\mathrm{e},1} = 2$	not applicable

TABLE V
Customer-Server Configuration III (CSC-III)

Phase	Parameter	Burstiness B
1	$m_{1,\mathrm{v}} = 18$ s, $\sigma_\mathrm{v} = 2$ s	$-0,89$
2	$p_\mathrm{e} = 0,5$, $m_{\mathrm{e},1} = 2$	not applicable

For this, the Gaussian characteristic has to be moved to the Weibull characteristic.

Fig. 5 shows the obtained values of the probability density function (pdf) with regard to the waiting time in queue when using the CSC-I setup. The payment processing time t_s in

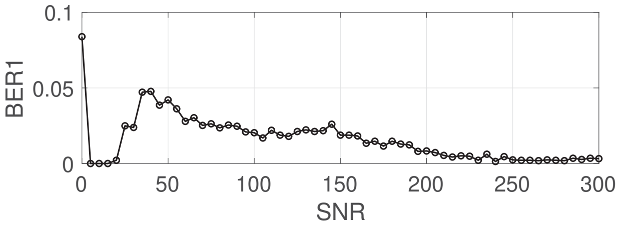

Fig. 5 Histogram of waiting time in queue when using CSC-I setup

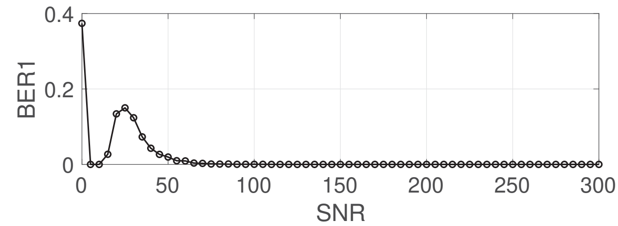

Fig. 6 Histogram of waiting time in queue using CSC-I setup with shortening of the payment processing time in phase 3 ($t_\mathrm{s} = 1{,}9\,g + 15{,}0$ instead of $t_\mathrm{s} = 1{,}9\,g + 22{,}8$

seconds (i. e. the time needed for processing the goods as well as the payment process) is assumed as

$$t_\mathrm{s} = 1{,}9\,g + 22{,}8 \ . \tag{12}$$

Here, for the processing of one item about $1,9$ seconds and additionally about $22,8$ seconds for each buyer are required. When shortening the payment processing time to $t_\mathrm{s} = 1{,}9\,g + 15{,}0$ the processing of the buyer at the check out becomes faster and leads to shorter waiting times as highlighted in Fig. 6. It is assumed that this can be reached by cardless payment e. g. by smartwatches or phones.

The CSC-II and CSC-III setup assumes different parameters for the customer arrival in phase 1 compared with CSC-I setup. Here, the average time separation between two customers is increased from 14 seconds to 16 seconds and 18 seconds.

As the standard deviation remains unchanged, the level of burstiness remains nearly unchanged. The results depicted in Figs. 7 and 8 show that a further separation between the customer arrival is highly beneficial for shortening the buyer waiting at the checkout, when keeping the buying behaviour in phase 2 unchanged.

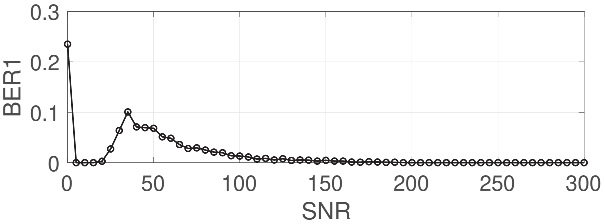

Fig. 7 Histogram of waiting time in queue when using CSC-II setup

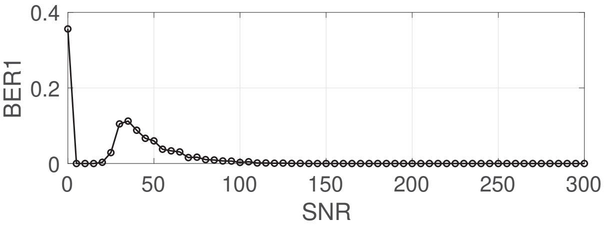

Fig. 8 Histogram of waiting time in queue when using CSC-III setup

In comparison to Fig. 8, Fig. 9 shows the obtained values with an increased σ_{v} in phase 1. Here again, a higher value of σ_{v} seems to be a good choice for a fast and efficient service at the checkout. So far, the expected value of goods

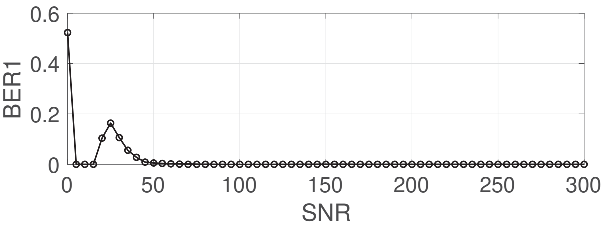

Fig. 9 Histogram of waiting time in queue using CSC-III setup with increased fluctuations regarding the customer arrival in phase 1 ($\sigma_{\mathrm{v}} = 4$ instead of $\sigma_{\mathrm{v}} = 2$

purchased was $m_{\mathrm{e},1} = 2$. Fig. 10 in combination with the CSC-III setup shows the obtained results when increasing the average number of goods in basket to $m_{\mathrm{e},1} = 5$. In comparison to Fig. 8, as expected longer waiting times will appear.

V. Conclusions

This work is dedicated to the decomposition of the customer-server interaction in grocery shops. The application of the macroscopic view to the shop processes have contributed to the paradigm change from only the buying process to the wider shopping process. The theoretical analysis allows defining that customer-server interactions are the ground for building the shopping process. Theoretical

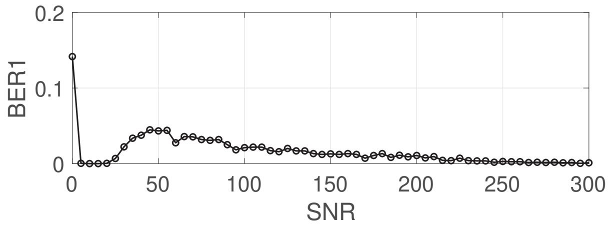

Fig. 10 Histogram of waiting time in queue using CSC-III setup with increasing number of goods in basket ($m_{\mathrm{e},1} = 4$ instead of $m_{\mathrm{e},1} = 2$)

modelling resulted in the decomposition of the shopping process. The macroscopic view allowed building the shopping process in five phases: customer arrival in the shop in phase 1, goods selection process in phase 2, arrival to the waiting line at the cash register in phase 3, server or, in other words, payment processing in phase 4, and buyer departure in phase 5. Each phase of the shopping process has been mathematically described.

References

[1] D. Hartleb, A. Ahrens, O. Purvinis, J. Zaščerinska, and D. Micevičiene, "Internal and External Factor Analysis in Bottleneck Detection in Shop Sales: The Case of Grocery Shops in Lithuania," in *International Conference on Pervasive and Embedded Computing and Communication Systems (PECCS)*, Online Streaming, 03.–05. November 2020.

[2] D. Miceviciene, O. Purvinis, R. Glinskiene, and A. Tautkus, "Alternative Solution for Client Service Management," *Applied Research in Studies and Practice*, vol. 14, no. 1, pp. 47–51, 2018.

[3] A. Ahrens, O. Purvinis, D. Hartleb, J. Zaščerinska, and D. Micevičiene, " Analysis of a Business Environment using Burstiness Parameter: The Case of a Grocery Shop." in *International Conference on Pervasive and Embedded Computing and Communication Systems (PECCS)*, Vienna (Austria), 19.–20. September 2019.

[4] A. Ahrens, D. Hartleb, and J. Zaščerinska, "Modelling the Service Process at the Cash Register using Generating Functions," in *5th International Conference on Signal Processing and Information Communications (ICSPIC 2022)*, Paris (France), 14.–16. March 2022.

[5] E. W. Weisstein, *The CRC Concise Encyclopedia of Mathematics*. Boca Raton and London: CRC Press, 1999.

[6] H. Wilhelm, *Calculation of Error Structures in Binary Channels with Memory*. Norderstedt: Books on Demand, 2018.

[7] ——, *Datenübertragung (in German)*. Berlin: Militärverlag, 1976.

[8] K.-I. Goh and A.-L. Barabási, "Burstiness and Memory in Complex Systems," *Exploring the Frontiers of Physics (EPL)*, vol. 81, no. 4, p. 48002, 2008. [Online]. Available: https://doi.org/10.1209/0295-5075/81/48002

[9] H. Kobayashi, B. L. Mark, and W. Turin, *Probability, Random Processes, and Statistical Analysis: Applications to Communications, Signal Processing, Queueing Theory and Mathematical Finance*. Cambridge University Press, 2011.

[10] D. Hartleb, A. Ahrens, O. Purvinis, and J. Zaščerinska, "Analysis of Free Time Intervals between Buyers at Cash Register using Generating Functions," in *International Conference on Pervasive and Embedded Computing and Communication Systems (PECCS)*, Online Streaming, 03.–05. November 2020.

[11] R. Axelrod, "Agent-based Modeling as a Bridge Between Disciplines." in *Handbook of Computational Economics*, 1st ed., L. Tesfatsion and K. L. Judd, Eds. Elsevier, 2006, vol. 2, ch. 33, pp. 1565–1584. [Online]. Available: https://EconPapers.repec.org/RePEc:eee:hecchp:2-33