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Effects of the Mass and Damping Matrix Model in the Nonlinear Seismic Response of Steel Frames

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Abstract**—**Seismic analysis of steel buildings is usually based on the use of the concentrated mass (M_L) matrix and the Rayleigh damping matrix (C). Similarly, the initial stiffness matrix (K_O) and the first two modes associated to lateral vibrations are commonly used to develop the matrix C. The evaluation of the accuracy of these practices for the particular case of steel buildings with moment-resisting steel frames constitutes the main objective of this research. For this, the nonlinear seismic responses of three models of steel frames, representing low-, medium- and high-rise steel buildings, are considered. Results indicate that if the M_L matrix is used, shears and bending moments in columns are underestimated by up to 30% and 65%, respectively, when compared to the corresponding results obtained with the consistent mass matrix (M_C). It is also shown that if K_O is used in C instead the tangent stiffness matrix (K_t), axial loads in columns are underestimated by up to 80%. It is concluded that the consistent mass matrix should be used in the structural modelling of moment resisting steel frames and the tangent stiffness matrix should be used to develop the Rayleigh damping matrix.

Keywords**—**Moment-resisting steel frames, consistent and concentrated mass matrices, nonlinear seismic response, Rayleigh damping.

I. Introduction, Literature Review and Objectives

PROPER modeling of the mass (M), damping (C) and stiffness (K) matrices represents a very important step in estimating the seismic response of any structural system, including moment-resisting frames (MRF). The simplest representation of the mass matrix is the lumped one where the mass of the structure is concentrated at the translational degrees of freedoms (DOF), which is usually determined by statics. It gives a diagonal mass matrix with nonzero values associated to such translational DOF. However, a finite value of rotational inertia can be estimated for the rotational DOF by calculating the moment of inertia of the mass of a portion of the beams with respect to the nodes. Alternatively, it is possible to develop a matrix known as the consistent mass matrix in such a way that inertia forces are associated to both translational and rotational DOF. Because the rotational inertia effects are better represented while using the consistent mass matrix, the responses are expected to be more accurate when compared to those of the lumped mass matrix.

Dissipation of energy has also a significant effect on the structural response. It is expected to be more relevant for steel structures since dissipation of energy is supposed to come from several sources. In modal spectral dynamic analysis procedures specified in many building codes, dissipation of energy is taken into account by using a linear equivalent viscous damper with 5% of critical damping in such a way that energy dissipation sources are approximately included. It is worth noting that there has been a number of investigations where energy dissipation due to plastic deformations is approximately modeled by using an equivalent viscous damping model [1]-[5]. Unfortunately, it is not possible to mathematically represent each of the energy-dissipating mechanisms existing in real buildings. This is the main reason why dissipation of energy is highly idealized for practical purposes in seismic codes.

A reasonable approach, particularly used in the case of steel building structures, consists in modeling dissipation of energy generated by the thermal effects of repeated elastic straining of the material grains, and from the friction among the boundaries of the grains by a linear viscous damper, while that produced by plastic deformations is handled by considering the constitutive relationship between forces and deformations. Viscous energy is traditionally represented by the Rayleigh Damping Model, where the damping matrix C is expressed as a combination of the M and K matrices by using two proportionality coefficients, which in turn are derived by assuming damping ratios, usually at the first and second modes. It is convenient at this state to identify two particular cases of the stiffness matrix: the initial elastic stiffness matrix (K_o) associated to small deformations and the tangent stiffness matrix (K_t) associated to inelastic deformations.

The effects of using different models for the M, C, and K matrices, as discussed below, have been investigated by many researchers [6]-[20]. Nevertheless, there are many issues that have not been considered. The main objectives of this paper are to calculate and compare the seismic responses of steel buildings with MRF considering different ways of modeling the mass and damping matrices. This is discussed further below.

One of the first investigations concerning the mass distribution in a structure was conducted by Archer [6], who analyzed the effects of using the consistent mass matrix on beams. Hayashikawa and Watanabe [7] presented an analytical method to determine eigenvalues of continuous beams by using a general solution for the Bernoulli-Euler differential equation. Stavrinidis et al. [8] proposed a mass matrix formulation based on finite elements, which was improved with respect to the

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consistent mass matrix in terms of computational effort. Hansson and Sandberg [9] presented an approach to calculate the mass matrix for diagonal and mixed mass matrices. Gulkan and Alemdar [10] proposed functions for beam segments supported on elastic foundations to obtain analytic expressions for the coefficients of the element stiffness and the consistent mass matrices. Michaltsos and Konstantakopoulos [11] conducted dynamic analyses of a thin-walled tower considering additional concentrated masses and the effect of the rotational inertia of such masses. Archer and Whalen [12] proposed a mass matrix model, which includes translational and rotational DOF. The resulting matrix is diagonal and, similar to the consistent mass matrix, it maintains the translational and rotational rigid body inertias. Many other important contributions regarding the mass matrix of structures can be found in the literature [13]-[20].

Issues regarding the formulation of the damping matrix have also been addressed. One of the first works was conducted by Rea et al. [21]. Wilson and Penzien [22] proposed two numerical methods for calculating the damping matrix. Crips [23] made a comparative analysis concerning the effects of different damping models on the dynamic response of R/C structures. Léger and Dussault [24] studied the effect of the mathematical modeling of viscous damping on the seismic energy dissipation of multi-degree-of-freedom (MDOF) systems. Kowalsky and Dwairi [25] evaluated the accuracy of using the equivalent viscous damping concept while applied to the direct displacement-based seismic design. Val and Segal [26] studied the differences between responses of structures modeled as SDOF systems with viscous and hysteretic damping. Li and Wu [27] by using SDOF systems proposed relationships between equivalent damping and ductility according to the direct displacement-based seismic design (DBSD) method. Many important contributions to this field were also made by other researchers [28]-[37].

There is no doubt about the important advances in the state of the art of the research mentioned above regarding the effects of modeling mass and damping matrices on the seismic behavior of buildings. Nevertheless, there are many aspects that need additional discussions. Certainly, many investigations have been oriented to compare the responses of structures considering the lumped and consistent mass matrices [6]-[20], or to estimate the accuracy of using the Rayleigh damping matrix. However, many issues for the case of low-, mid- and high-rise steel buildings idealized as complex (MDOF) systems, considering several response parameters at both local and global levels, have not been studied.

The main objective of this paper is the calculate the nonlinear seismic responses of steel MRF, idealized as 2D-MDOF complex systems, subjected to several strong earthquake motions with the aim of studying some issues regarding the modeling of the mass and damping matrices. Global (inter-story displacements, and inter-story shears) and local (axial loads and bending moments) demand parameters are considered. The specific objectives are:

(1) To calculate the seismic demands assuming that the mass matrix in the structural models is lumped (M_L) and compare such demands with those obtained when the consistent mass matrix (M_C) is used.
(2) To estimate the accuracy of considering the initial stiffness matrix in the formulation of the Rayleigh damping matrix by comparing the results to those obtained with the tangent stiffness matrix.

II. Methodology and Procedure

Three steel buildings modeled as complex 2D-MDOF systems and 15 strong ground motions, which are consistent with the seismic hazard of the area where the models are located, are considered in the investigation. The Ruaumoko Software [38] is used to carry out the nonlinear time history analyses required. Large displacement effects are considered in the nonlinear dynamic analysis. The vertical and horizontal structural members are modeled as beam-columns and as beams, respectively. Three DOF per node are used. The hysteretic behavior of the members is modeled as bilinear with 3% of post-elastic stiffness. The interaction between axial loads and bending moments is defined by the interaction surface proposed by Chen and Atsuta [39]. Additional information regarding the models and seismic records, are given below.

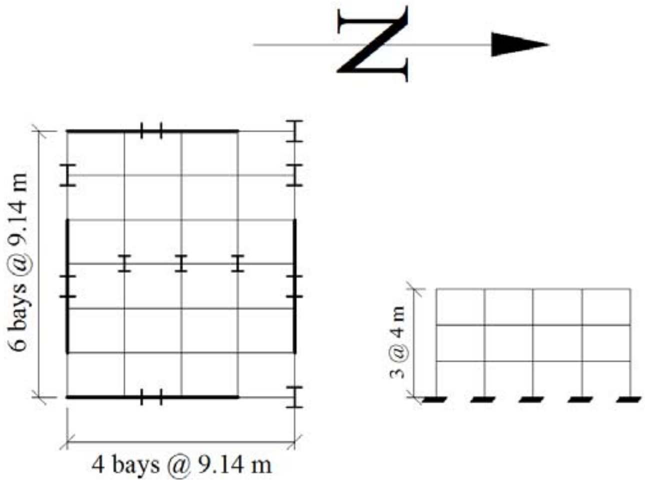

Fig. 1 Plan and elevation: 3-Level Model

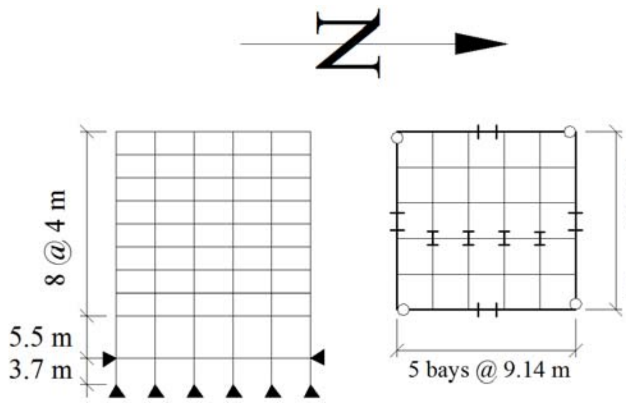

Fig. 2 Plan and elevation: 9-Level Model

The 3-, 9-, and 20-story steel building models, which were specifically designed to be used in a very important research project [40], are adopted in this paper to reach the objectives mentioned above. The plan and elevation (geometry) of the models are shown in Figs. 1-3. The perimeter MRF of such models constitute the bi-dimensional (2D) models used in the study. The lateral vibration fundamental periods of the 3-, 9- and 20-story models, which will be denoted as Models 1, 2 and 3, are 1.03s, 2.38s and 4.07s, respectively. The cross sections of beams and columns of Models 1 and 2 are provided in Table I; the corresponding sections for Model 3 are shown in Table II.

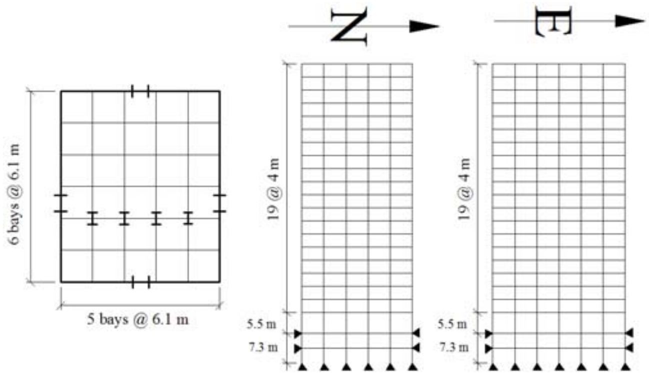

Fig. 3 Plan and elevation: 20-Level Model

TABLE I
BEAM AND COLUMN SECTIONS, PMRF OF MODELS 1 AND 2

Model	Story	Columns		Girders
		Exterior	Interior	
3-Level	1	W14×257	W14×311	W33×118
	2	W14×257	W14×311	W30×116
	3/Roof	W14×257	W14×311	W24×68
9-Level	Basement-1	W14×370	W14×500	W36×160
	1	W14×370	W14×500	W36×160
	2	W14×370	W14×500	W36×160
	3	W14×370	W14×455	W36×135
	4	W14×370	W14×455	W36×135
	5	W14×283	W14×370	W36×135
	6	W14×283	W14×370	W36×135
	7	W14×257	W14×283	W30×99
	8	W14x257	W14x283	W27X84
	9/roof	W14x233	W14x257	W24x68

In order to represent the seismic hazard, the models are excited by 15 seismic records that are representative of the site where the models are located. The main characteristics of the ground motions are summarized in Table III. The deformation of any of the models is elastic under the action of any of the seismic records. To have inelastic responses, the seismic records are scaled up to produce different levels of deformation. They are scaled according to the geometric mean of spectral acceleration (Sa_{avg}), obtained by "averaging" the pseudo-acceleration (S_a) [41] over a range of vibration periods as follow:

$$Sa_{avg}(T_1, \ldots, T_m) = (\prod_{i=1}^{m} S_a(T_i))^{1/m} \tag{1}$$

In (1), the m parameter represents the number of vibration periods of interest. This intensity measure takes into account the elongation of the first lateral vibration period due to nonlinear deformation and the contribution of the higher modes of vibration. The periods used to calculate Sa_{avg} range from 0.2 T_1 to 1.5 T_1, with uniform increments of 0.01 s, where T_1 is the fundamental period of the structure. The values of Sa_{avg} range from 0.2 g up to 1.0 g with uniform increments of 0.2 g for the 3-story building, whereas they range from 0.1 g up to 0.5 g with uniform increments of 0.1 g, for the 9-, and 20-story models.

TABLE II
BEAM AND COLUMN SECTIONS, PMRF OF MODEL 3

Story	Columns		Girders
	Exterior	Interior	
Basement-1	15X15X2.00	W24X335	W14X22
Basement-2	15X15X2.00	W24X335	W30X99
1	15X15X2.00	W24X335	W30X99
2	15X15X2.00	W24X335	W30X99
3	15X15X1.25	W24X335	W30X99
4	15X15X1.25	W24X335	W30X99
5	15X15X1.25	W24X335	W30X108
6	15X15X1.00	W24X229	W30X108
7	15X15X1.00	W24X229	W30X108
8	15X15X1.00	W24X229	W30X108
9	15X15X1.00	W24X229	W30X108
10	15X15X1.00	W24X229	W30X108
11	15X15X1.00	W24X229	W30X99
12	15X15X1.00	W24X192	W30X99
13	15X15X1.00	W24X192	W30X99
14	15X15X1.00	W24X192	W30X99
15	15X15X0.75	W24X131	W30X99
16	15X15X0.75	W24X131	W30X99
17	15X15X0.75	W24X131	W27X84
18	15X15X0.75	W24X117	W27X84
19	15X15X0.75	W24X117	W24X62
20/Roof	15X15X0.50	W24X84	W21X50

TABLE III
STRONG MOTION RECORDS

Designation	Station	Magnitude (Mw)	PGA (m/s^2)		Dominant Period (s)	
			N-S	E-W	N-S	E-W
LA1	Imperial Valley, 1940	6.9	4.52	6.63	0.53	0.46
LA2	Imperial Valley, 1979	6.5	3.86	4.80	0.16	0.34
LA3	Landers, 1992	7.3	4.14	4.17	0.73	0.33
LA4	Kern, 1952	7.3	5.11	3.53	0.25	0.23
LA5	Loma Prieta, 1989	7	6.53	9.50	0.21	0.20
LA6	Northridge, 1994, Newhall	6.7	6.65	6.45	0.31	0.31
LA7	Northridge, 1994, Rinaldi	6.7	5.23	5.69	0.39	0.29
LA8	Northridge, 1994, Sylmar	6.7	5.59	8.03	0.31	0.36
LA9	North Palm Springs, 1986	6	10.01	9.68	0.17	0.21
LA10	Coyote Lake, 1979	5.7	5.79	3.28	0.15	0.21
LA11	Morgan Hill, 1984	6.2	3.12	5.36	0.18	0.16
LA12	Parkfield, 1966, Cholame 5W	6.1	7.65	6.20	0.37	0.30
LA13	Parkfield, 1966, Cholame 8W	6.1	6.81	7.75	0.17	0.21
LA14	North Palm Springs, 1986	6	5.08	3.71	0.13	0.21
LA15	Whittier, 1987	6	7.54	4.70	0.70	0.28

As mentioned before, the simplest way to obtain the mass matrix of a structure is by concentrating the mass at the translational DOFs. With this procedure, the mass matrix at an element (M_{LE}) level is obtained and is given by (2). The consistent mass matrix for an element (M_{CE}) is expressed by (3). The symbols M_L and M_C will be used in the subsequent discussions to denote the lumped and consistent mass matrices at a global structural level.

$$M_{LE} = \frac{ml}{2}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

$$M_{CE} = \frac{ml}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22l & 0 & 54 & -3l \\ 0 & 22l & 4l^2 & 0 & 13l & -3l^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13l & 0 & 156 & -22l \\ 0 & -13l & -3l^2 & 0 & -22l & 4l^2 \end{bmatrix} \quad (3)$$

In (2) and (3), m and l are the uniformly distributed mass and the length of the element, respectively.

Since the consistent mass matrix takes into account the rotational inertial effects [38], [42], [43], it results in a better representation of the mass distribution through the structure than the concentrated one.

The Rayleigh Damping Model is represented by (4), where the α and β parameters are proportionality constants that are calculated by defining modal damping ratios at two modes (ζ_i and ζ_j). Several alternatives associated to Rayleigh Damping, obtained from different combinations of M and K in (4), have been studied [19], [30], [32], [34].

$$C = \alpha M + \beta K \quad (4)$$

Excepting what is presented in Section IV of the paper (Objective 2), the tangent stiffness matrix (K_t) is used in (4). It seems to be more reasonable than using the elastic stiffness matrix (K_o) since if K_o is used, damping will not change as the structure reduces its stiffness while experimenting inelastic behavior, resulting in an increment of the fractions of critical damping [44]. This aspect is explained further in Section IV. The use of K_t in the matrix C has been criticized in the sense that when the structure deforms in the inelastic range, a reduction of damping is not expected since additional damping will occur due to the inelastic behavior. However, although extra damping is expected, it is taken into account in the hysteretic behavior of the members.

III. Concentrated vs. Consistent Mass Matrix

A. Global Parameters

The seismic demands in terms of inter-story shears and inter-story displacements (drifts) are calculated by assuming first that the mass matrix is lumped (M_L) and then consistent (M_C). Before starting the discussion, it is important to say that some differences are observed between the lateral periods of vibration of the models with M_L and M_C. For the 3-story building, the first two periods of the model with M_C are essentially the same as those of the model with M_L. However, the period of the third mode is 28% greater for the model with M_C. For the case of the 9-story model, the periods of the first five modes are quite similar for M_L and M_C, but they are greater for M_C for modes 6 to 9, with the differences ranging from 5% to 16%. In the same manner, for the 20-story model, the periods are quite similar for modes 1 through 11 for the two types of matrices; however, for modes 12 to 20 they are greater for M_C with the differences ranging from 7% to 17%.

1. Inter-Story Shears

The R_{V1} parameter, defined by (5), is used to compare the inter-story shear demands. In such an equation, V_{ML} and V_{MC} represent the inter-story shears for the models with the M_L and M_C matrices, respectively. Since the consistent mass matrix represents better the rotational inertial effects a value of R_{V1} larger than unity will indicate that the inter-story shears are overestimated if the concentrated mass is assumed.

$$R_{V1} = \frac{V_{ML}}{V_{MC}} \quad (5)$$

The mean values of R_{V1}, averaged over all the strong motions, are presented in Fig. 4. Since the results are quite similar for the two horizontal directions, only the results for the *NS* direction are shown. The results can be seen in Fig. 4. It is observed that for the 3-story model (Fig. 4 (a)), on average, the inter-story shears are slightly underestimated if the concentrated mass model is used. The maximum underestimation is about 4%, which occurs for the upper story. The results are essentially the same for all seismic intensities.

The maximum average underestimations are observed to be 12% and 14%, for the 9- and 20-story models, respectively. It is worth to mention that underestimations larger than 28% occur for some individual strong motions for Models 1 and 2 (not presented). The graphs in Fig. 4 also indicate that for a given model, the magnitude of the underestimation tends to increase through the building height. One of the reasons for this is that higher-mode contribution in terms of inter-story shears for the upper stories is more significant when the M_C matrix is used in the seismic analyses. The level of underestimation is observed to increase as the height of the building increases. Inter-story displacements were also compared, but the graphs are not presented. It is worth to mention, however, that the underestimation is smaller than that of inter-story shears; the maximum values of average and individual underestimations are observed to be 4% and 9%, respectively.

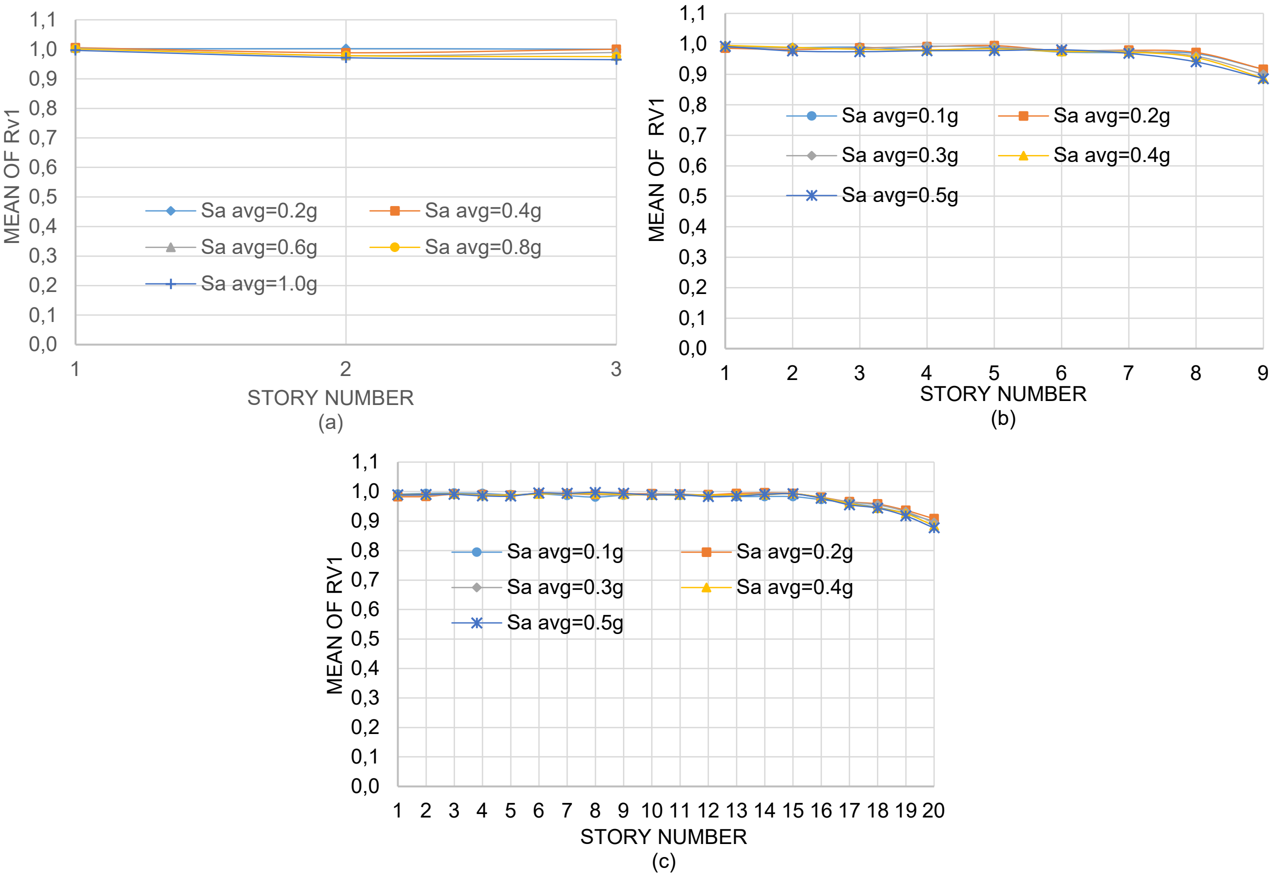

Fig. 4 Mean values of R_{V1}, *NS* direction; (a) 3-story model, (b) 9-story model, and (c) 20-story model

B. Local Individual Response Parameters

1. Axial Loads

The axial loads on columns of the models with the M_L and M_C matrices are now compared. The comparison is made for all columns: from the interior up to the exterior and from the base up to the top of the models. The comparison is made through the R_{A1} parameter expressed by (6). In such an equation, A_{ML} and A_{MC} represent the axial loads on columns of the frames with the lumped and consistent mass matrices, respectively.

$$R_{A1} = \frac{A_{ML}}{A_{MC}} \tag{6}$$

The mean values of R_{A1} for exterior columns associated to the *NS* direction are presented in Figs. 5 (a)-(c), for the 3-, 9- and 20-story frames, respectively. It can be observed that, unlike inter-story shears and displacements, the axial loads may be considerably overestimated if the concentrated mass matrix is used. Average overestimation of up to 60% can be observed. Even if the structure remains elastic (Sa_{avg} = 0.2 g), average overestimations of about 30% are observed. The magnitude of the maximum overestimations clearly tends to decrease as the height of the building increases; the values are 21% and 5% for Models 2 and 3, respectively. Although they are not shown, overestimations larger than 90% occurred for some individual strong motions, particularly for the 3-story model.

The mean values of R_{A1} for interior columns were also calculated, but the results are not presented. It is important, however, to mention that significant differences are observed between the results of interior and exterior columns. For the 3-story frame for example, as for exterior columns, the interior axial loads are overestimated, the individual and average overestimations can be up to 44% and 30%, respectively. For the 9- and 20-story models, on the other hand, the interior axial loads are underestimated by up to about 8% and 18%, on average and individually, respectively.

2. Bending Moments

The bending moments at interior and exterior columns, as well as for exterior and interior beams, at all structural locations, are now compared. To this aim, the R_{B1} parameter given by (7) is used. B_{ML} and B_{MC} in (7) have a similar meaning as A_{ML} and A_{MC} in (6), but bending moments are now being compared.

$$R_{B1} = \frac{B_{ML}}{B_{MC}} \tag{7}$$

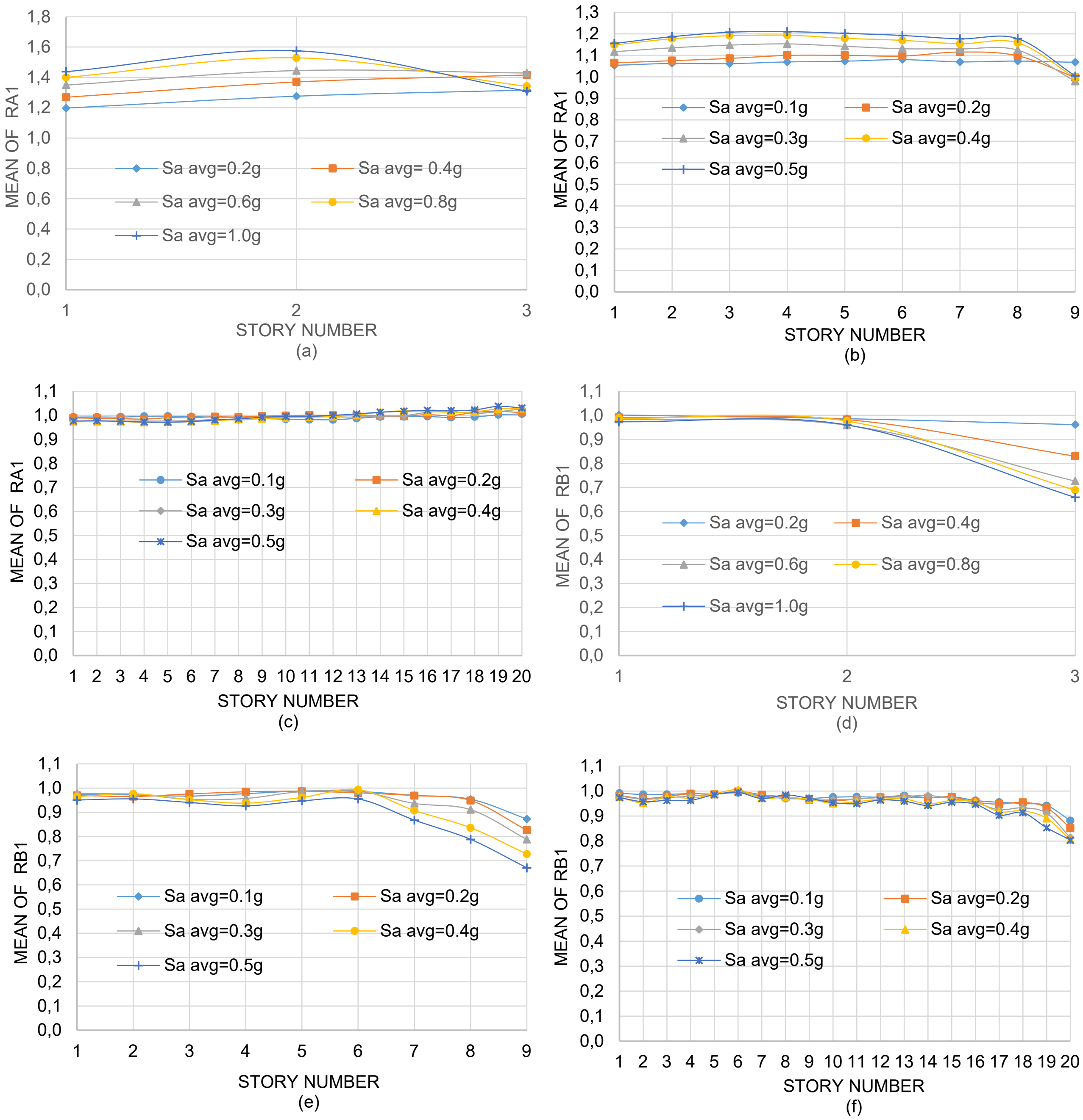

Fig. 5 Mean values of R_{A1} and R_{B1}, exterior columns, *NS* direction: (a), (b) and (c) R_{A1}, Models 1, 2 and 3; (d), (e) and (f) R_{B1}, Models 1, 2 and 3

The mean values of R_{B1} for exterior columns corresponding to the *NS* direction are presented in Figs. 5 (d)-(f), for the 3-, 9-, and 20-story frames, respectively. Results indicate that, unlike the case of axial loads, the bending moments at exterior columns can be considerably underestimated if the lumped mass model is used. Such underestimation tends to increase through the height of the models and with the seismic intensity, but tends to decrease as the building becomes taller. The maximum levels of underestimation are about 36%, 31% and 20%, for the 3-, 9-, and 20-story models, respectively. The corresponding maximum underestimations for individual strong motions are about 68%, 59% and 41%. The level of underestimation in bending moments for interior columns was also calculated but are not presented. However, it is much smaller than that of exterior columns; the maximum individual and average underestimations are about 20% and 10%, respectively, for both the 3- and 9-story buildings. For the 20-story building, on the other hand, the bending moments are practically the same for the lumped or consistent mass matrix. The mean values of R_{B1} for exterior and exterior beams were also calculated, but the results are not given. It can be said, however, that bending moments at beams are accurately estimated when the lumped mass matrix is used.

Commonly, structural members of steel buildings are designed for resultant stresses with a final revision in displacements. The results of this study show that there is not introduced errors in terms of lateral displacements if the mass matrix is assumed to be concentrated type. However, for axial loads and bending moments, significant errors can be introduced. Therefore, it is strongly recommended to use the consistent mass matrix in the numerical modelling of the structural system under consideration.

IV. Initial vs. Tangent Stiffness Matrix

The tangent stiffness matrix (K_t) was used in the earlier section in (2) to calculate the Rayleigh damping matrix. The main reasons for this are:

(a) If K_o is used instead of K_t, damping (elements C_{ij} of the C matrix) will not change as the structure reduces its stiffness due to inelastic behavior.

(b) The values of ζ_n in all the vibrating modes of the structure are approximately the same. The use of the Rayleigh damping model results in very large damping values in the higher modes. The reduction in damping due to the use of K_t with respect to K_o partially compensates for these very large damping values in the higher modes.

(c) The dissipation of energy produced by thermal effects of repeated elastic straining of the material grains and from the friction among the boundaries of the grains mentioned in Section I, which is modeled by viscous damping, also occurs after yielding of the material. In this sense, it is more reasonable to use K_t than K_o in (2) since a reduction of the elements of the C matrix is expected due to the *structural softening* (reduction of stiffness) produced by inelastic behavior.

Therefore, in the context of this investigation, it is assumed that the responses that result when using the matrix K_t in the seismic analysis are more accurate than those obtained when using the K_o matrix. In practice, however, it is common to use the initial stiffness matrix (K_o) to construct the Rayleigh damping matrix. In this section of the paper, the accuracy of this approach is evaluated by comparing the seismic responses obtained when using the tangent stiffness matrix (K_t) to those obtained when using K_o. Only the axial forces in columns are discussed since they are the only parameter that, on average, presented significant sensitivity to the modeling of the stiffness matrix. To make the comparison, the R_{A3} ratio given by (8) is used. In such an equation, A_{Ko} and A_{Kt} represent the axial loads on columns when the K_o and K_t matrices are used in the formation of the C matrix.

$$R_{A3} = \frac{A_{Ko}}{A_{kt}} \quad (8)$$

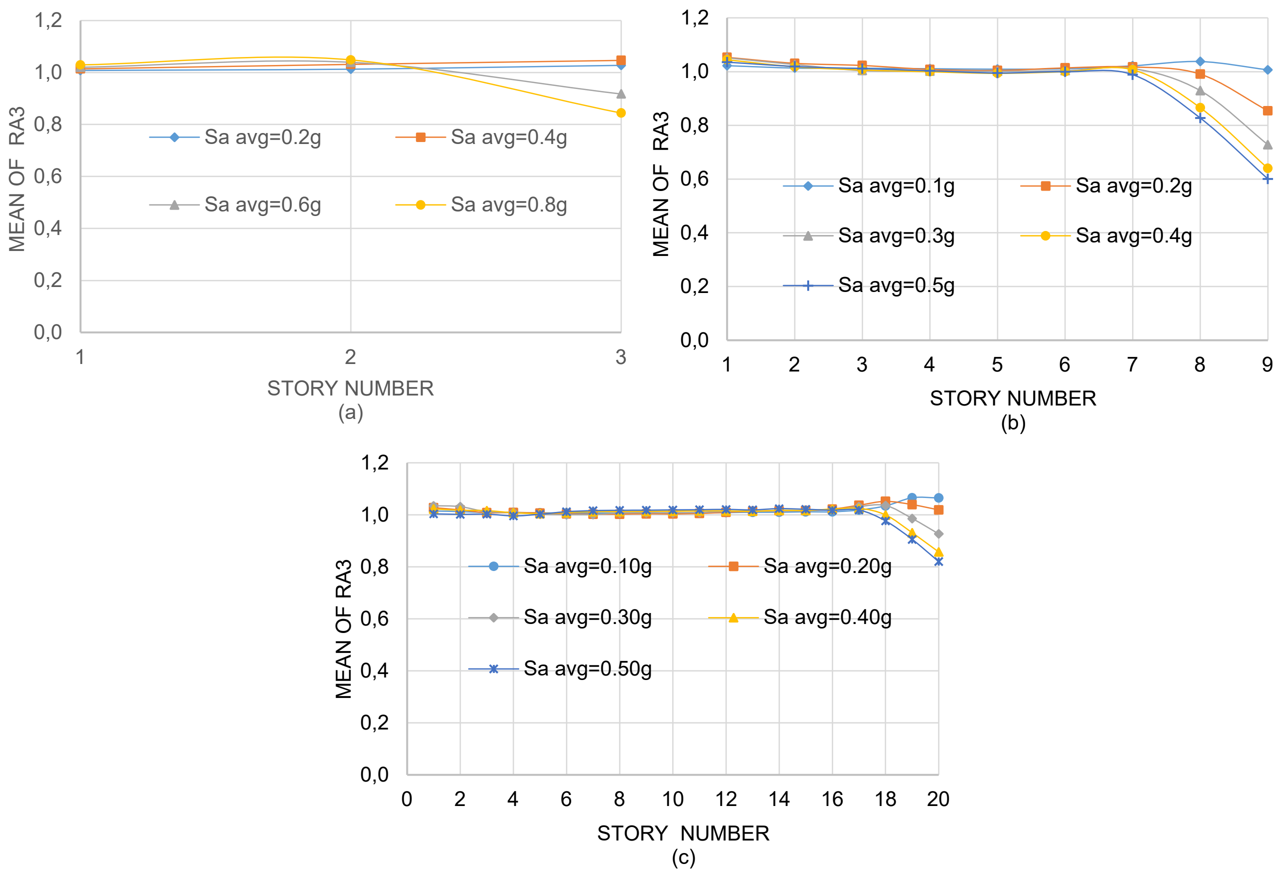

Fig. 6 Mean values of R_{A3}, exterior columns, *NS* direction: (a) 3-story model, (b) 9-story model, (c) 20-story model

The mean values of the R_{A3} parameter for exterior columns of the *NS* direction are shown in Fig. 6. It can be seen that the axial loads can be slightly overestimated for the smallest values of the seismic intensities, if the K_o matrix is used in the damping matrix. For the largest seismic intensities, on the other hand, the axial loads are significantly underestimated; the maximum average underestimations are 16%, 40% and 18% for Models 1, 2 and 3, respectively. The corresponding underestimations can be up to 31%, 80% and 41% for the case of individual strong motions.

V. Conclusions

Proper modeling of the mass and damping matrices represents a key step in estimating the responses of buildings. Software users concerning seismic analysis of buildings commonly use the concentrated mass (M_L) matrix and the Rayleigh damping matrix (C). Similarly, often the initial stiffness matrix (K_O) and the first two modes are used in the construction of the Rayleigh damping matrix. The evaluation of the accuracy of these practices constitutes the main objective of this research for the case of steel buildings. To this aim the nonlinear seismic responses of three steel MRF models, representing steel buildings of low-, mid- and high-rise are considered. The main conclusions of the study are:

1. If the matrix M_L is used, the seismic responses may be underestimated, overestimated or accurately estimated, depending mainly on the parameter considered. The inter-story shears are underestimated by up to 31% while the inter-story displacements are accurately estimated. The axial loads and bending moments in columns are overestimated and underestimated by up to 95% and 65%, respectively, but the bending moments at beams are accurately estimated.
2. It is observed that the axial loads in columns can be significantly underestimated if the initial stiffness matrix (K_o) is used to develop the C matrix. The maximum underestimation is observed to be up to 80% for exterior columns.
3. The results of this study give rise to state that, in order to minimize the errors in the seismic response calculation, the consistent mass matrix should be used in the structural modelling of the structural system under consideration. Similarly, the tangent stiffness matrix should be used to calculate the coefficients of the Rayleigh damping matrix.

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