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Fighter Aircraft Selection Using Fuzzy Preference Optimization Programming (POP)

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***Abstract*—**The Turkish Air Force needs to acquire a sixth-generation fighter aircraft in order to maintain its air superiority and dominance against its rivals under the risks posed by global geopolitical opportunities and threats. Accordingly, five evaluation criteria were determined to evaluate the sixth-generation fighter aircraft alternatives and to select the best one. Systematically, a new fuzzy preference optimization programming (POP) method is proposed to select the best sixth generation fighter aircraft in an uncertain environment. The POP technique considers both quantitative and qualitative evaluation criteria. To demonstrate the applicability and effectiveness of the proposed approach, it is applied to a multiple criteria decision-making problem to evaluate and select sixth-generation fighter aircraft. The results of the fuzzy POP method are compared with the results of the fuzzy TOPSIS approach to validate it. According to the comparative analysis, fuzzy POP and fuzzy TOPSIS methods get the same results. This demonstrates the applicability of the fuzzy POP technique to address the sixth-generation fighter selection problem.

***Keywords*—**Fighter aircraft selection, sixth-generation fighter aircraft, fuzzy decision process, multiple criteria decision making, preference optimization programming, POP, TOPSIS, Kizilelma, MIUS, fuzzy set theory.

I. INTRODUCTION

Establishing a strong sustainable defense supply has become one of the critical requirements for nations to survive in an increasingly hostile environment as a result of geopolitical opportunities and threats. The sixth-generation fighter aircraft selection problem falls under the area of multiple criteria decision-making, which considers both numerical and non-numerical evaluation criteria. This study is centered on a method that can consider both numerical and non-numerical decision criteria by taking into consideration the assessment and selection problem for the purchasing process of fighter aircraft [1-20].

Finding the best option out of all the viable options is the process of decision-making problems. The variety of criteria for evaluating the alternatives is ubiquitous in nearly all such challenges. That is, for many of these problems, the decision-maker seeks to resolve a problem involving multiple criteria decision-making analysis (MCDMA) [21-25].

MCDMA is a general framework for supporting complex decision-making systems with multiple and conflicting criteria that decision-makers value differently. MCDMA means the process of determining the best feasible solution according to established criteria and problems that are common occurrences in everyday life [26-41].

Multiple criteria decision-making in general follows six steps including, (1) problem formulation, (2) identify the requirements, (3) set goals, (4) identify various alternatives, (5) develop criteria, and (6) identify and apply decision-making technique [1-20].

In a MCDMA problem, the basic ingredients are the criteria and alternatives. Different alternatives evaluated against set criteria to formulate a comparison of alternatives. The results can be improved further by assigning weights to different criteria, as the importance can vary extremely from one decision-maker to another. Hence, for selected criteria, there can be a different level of importance from the perspective of different decision-makers. It important to evaluate the assign weights to each criterion from different decision-makers to ensure the reliability of results [1-20].

The selection of the MCDMA technique for a solving a particular problem can vary depending on the context, which emphasizes the need to understand decision-making classifications. The MCDMA techniques are categorized based on (1) compensatory and non-compensatory, (2) discrete and continuous, and (3) individual and group decision-making. Classification of MCDMA based on discrete and continuous data is the most applied ones. From the perspective of discrete and continuous data, the MCDMA is divided into multiple attribute decision making (MADM) and multiple objective decision making (MODM). MADM considers the problems under an inherent discrete decision space. MODM is based on mathematical theory and deals with the problems under continuous decision space [1-20].

The ratings and weights of the criteria are exactly known in traditional MCMDA approaches [35-38]. Technique for order performance by similarity to ideal solution (TOPSIS), is suggested to address these challenges as a conventional MCDMA method [35]. It is based on the principle that the preferred option should be closest to the positive ideal solution (PIS) and furthest from the negative ideal solution (NIS). The performance ratings and weights of the criteria are provided as crisp values in the classical TOPSIS procedure [35].

Crisp data are frequently insufficient to model actual circumstances. Since human judgments, including preferences, are frequently ambiguous, it is impossible to quantify a person's choice precisely. Instead of using numerical values, a more realistic method might be to employ linguistic assessments, or to assume that the ratings and weights of the criteria in the problem are determined using linguistic variables [42-50].

The concept of preference optimization programming (POP) was further extended to solve the fuzzy decision-

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making problem, to create a methodology for group decision making process used in multi-criteria decision analysis problems in fuzzy environments. The weights of all criteria and the ratings of each alternative in relation to each criterion are evaluated using linguistic variables considering the fuzziness in the decision data and group decision-making process.

Once the fuzzy ratings of the decision makers are combined, the decision matrix can be transformed into a fuzzy decision matrix and used to build a weighted normalized fuzzy decision matrix. The fuzzy ideal solution (z_j^*) is defined by the POP methodology.

The distance between two triangular fuzzy ratings is then calculated using a vertex approach. The vertex approach can be used to determine the distance of each alternative from (z_j^*) and (z_{ij}). In order to determine the ranking order of all alternatives, a preference optimization value (π_i) of each option is defined. The lower value of (π_i) indicates that an alternative is closer to (z_j^*). Finally, the results of the fuzzy POP approach are validated using the fuzzy TOPSIS technique. According to the comparative analysis, fuzzy POP and fuzzy TOPSIS methods get the same ranking order results.

Sixth-generation fighter aircraft systems can provide Air Force a wide range of advantages, capability, capacity, and efficiency improvements. Increasing global geopolitical risks force countries to seek new effective defense systems to strengthen their defense processes and shape their defense policy better, leading to increased demand for sixth-generation fighter aircraft solutions.

Considering various needs of Air Force and diverse features of available sixth-generation fighter aircraft alternatives, choosing the most suitable sixth-generation fighter aircraft is an important decision for strategic, tactical, and operational requirements. The contribution is a sixth-generation fighter aircraft evaluation framework for decision makers to compare available sixth-generation fighter aircraft alternatives of different vendors by first identifying relevant evaluation criteria and then proposing a group decision making framework using the fuzzy POP technique. This method has more flexibility in handling uncertainties compared to the other MCDMA approaches and enables decision makers to effectively analyze, compare and select the most appropriate sixth-generation fighter aircraft. The framework is also used in a case study for the sake of demonstrating its potential in Air Force.

Preliminaries. The fuzzy POP technique is suggested to assess the sixth-generation fighter aircraft candidates. Since the evaluation and selection of the fighter aircraft problem is a fuzzy decision-making process, some properties of fuzzy sets are briefly reviewed as a preliminary information [42-50]. Unless otherwise stated, the basic definitions and representations of fuzzy number and linguistic variable below will be used.

Definition 1. A fuzzy set *A* in a universe of discourse *X* is characterized by a membership function $\mu_A(x)$ which associates with each element *x* in *X* a real number in the interval [0, 1]. The function value $\mu_A(x)$ is termed the grade of membership of *x* in *A*.

$$A = \{x, \mu_A(x) : x \in [0,1]\} \tag{1}$$

Definition 2. The support of fuzzy set A is the set of all points *x* in *X* such that $\mu_A(x) > 0$.

$$\text{Support } (A) = \{x \mid \mu_A(x) > 0\} \tag{2}$$

Definition 3. The α- cut of α- level set of fuzzy set A is a set consisting of those elements of the universe *X* whose membership values exceed the threshold level α.

$$A_\alpha = \{x \mid \mu_A(x) > \alpha\} \tag{3}$$

where A_α is a non-empty bounded closed interval contained in X and it can be denoted $A_\alpha = [A_l^\alpha, A_u^\alpha]$, A_l^α and A_u^α are the lower and upper bounds of the closed interval, respectively.

Definition 4. A fuzzy set *A* of the universe of discourse *X* is convex if and only if for all x_1, x_2 in X ,

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq Min(\mu_A(\lambda x_1, +\mu_A(x_2)) \tag{4}$$

where $\lambda \in [0,1]$.

Definition 5. A fuzzy set *A* of the universe of discourse *X* is called a normal fuzzy set implying that

$$\exists x_i \in X, \mu_A(x_i) = 1 \tag{5}$$

Definition 6. A fuzzy set *A* on *R* must possess at least the following three properties to qualify as a fuzzy number,

(i) *A* must be a normal fuzzy set,

(ii) A_α must be closed interval for every $\alpha \in [0,1]$,

(iii) the support of *A*, A_{0+} , must be bounded.

Definition 7. A fuzzy number is a fuzzy subset in the universe of discourse *X* that is both convex and normal.

Definition 8. A triangular fuzzy number *n* can be defined by a triplet (*l*, *m*, *u*) shown in Fig. 1. This representation is interpreted as membership functions and holds the following conditions,

(i) (*l* and *m*) is increasing function,

(ii) (m and u) is decreasing function,

(iii) $l \le m \le u$.

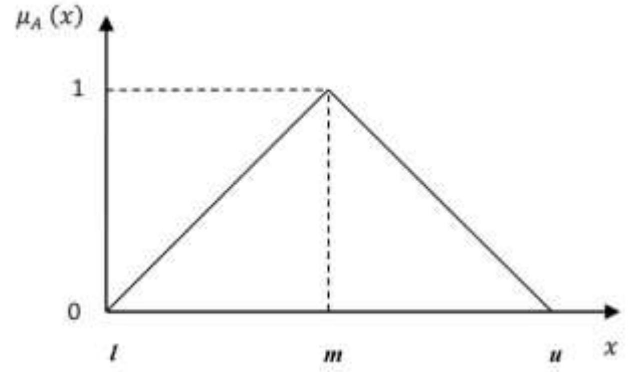

Fig. 1 A triangular fuzzy number *A*.

$$\mu_n(x)\begin{cases} 0, & x < l \\ \dfrac{x-l}{m-l}, & l \le x \le m \\ \dfrac{x-u}{m-u}, & m \le x \le u \\ 0, & x > u \end{cases} \tag{6}$$

Definition 9. α- cut of a triangular fuzzy number, a crisp interval by α-cut operation is defined; interval A_α will be obtained as follows $\forall \alpha \in [0,1]$. Thus,

$$A_\alpha = [A_l^\alpha, A_u^\alpha] = [(m-l)\alpha + l, -(u-m)\alpha + u] \tag{7}$$

Definition 10. A positive triangular fuzzy number *A*, is denoted as $A = (a_1, a_2, a_3)$, where all a_i 's >0 for all $i = 1,2,3$.

Definition 11. A negative triangular fuzzy number *A*, is denoted as $A = (a_1, a_2, a_3)$, where all a_i 's <0 for all $i = 1,2,3$.

Definition 12. A partial negative triangular fuzzy number *A*, is denoted as $A = (a_1, a_2, a_3)$, where at least one a_i negative for all $i = 1,2,3$.

Definition 13. Some important properties of operations on Triangular Fuzzy Numbers are summarized in Table 1.

Table 1. Standard approximate arithmetic operations on triangular fuzzy numbers and their definition for two fuzzy numbers $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$

Arithmetic operations	Definition
$A + B$ (addition)	$(a_1 + a_2, b_1 + b_{2,}, c_1 + c_2)$
$A - B$ (subtraction)	$(a_1 - b_3, a_2 - b_{2,}, a_3 - b_1)$
$A \, x \, B$ (multiplication)	(a_1b_1, a_2b_2, a_3b_3)
A / B (division)	$(a_1 / b_3, a_2 / b_{2,}, a_3 / b_1)$
kA (multiplication by a real number)	(ka_1, ka_2, ka_3)
A^{-1} (reciprocal of a fuzzy number)	$(1/a_3, 1/a_2, 1/a_1)$

Definition 14. M is called a fuzzy matrix, if at least an entry in M is a fuzzy number.

Definition 15. A linguistic variable is a variable whose values are linguistic terms.

Definition 16. Let $m = (m_1, m_2, m_3)$ and $n = (n_1, n_2, n_3)$ be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them as

$$d(m,n) = \sqrt{\frac{1}{3}[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2]} \tag{8}$$

Definition 17. Let *m* and *n* be two triangular fuzzy numbers. The fuzzy number *m* is closer to fuzzy number *n* as $d(m,n)$ approaches 0.

In order to develop the linguistic POP method, this paper is organized as follows: section two presents the linguistic POP method in group decision making and the choice process. The third section involves using the suggested model, and the last section analyzes the study's findings.

II. METHODOLOGY

A. The POP method

The POP method is performed according to the following procedural steps:

Step 1. Constructing the decision matrix $X = [x_{ij}]_{ixj}$

$$X = \begin{pmatrix} \overset{g_1}{x_{11}} & \cdots & \overset{g_j}{x_{1j}} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} \end{pmatrix}_{ixj} \tag{9}$$

Suppose that multiple criteria decision making analysis (MCDMA) problem has I alternatives $x_i = (x_1, ..., x_i)$, $i \in$ { $i = 1, ..., I$ }, and J criteria $g_j = (g_1, ..., g_j)$, $j \in$ { $j = 1, ..., J$ }, and the importance weight of each criterion (ω_j, $j \in$

{ $j = 1,..., J$ }) is defined. If x_{ij} is negative, then do the calculation $x_{ij}^t = x_{ij} - \min_j x_{ij}$, then, x_{ij}^t is used to calculate the next steps.

Step 2. Normalizing the decision matrix $N = [n_{ij}]_{ixj}$. The decision matrix of the alternatives is normalized using the linear normalization scale.

If g_j is the criterion, the bigger the better ($j \in B$)

$$n_{ij} = \frac{x_{ij}}{x_j^{\max}} \tag{10}$$

If g_j is the criterion, the smaller the better ($j \in C$)

$$n_{ij} = \frac{x_j^{\min}}{x_{ij}} \tag{11}$$

where B represents a criterion as large as possible, C represents a criterion as small as possible. n_{ij} is an element of the normalized matrix $N = [n_{ij}]_{ixj}$.

Step 3. Calculating the weighted normalized value $Z = [z_{ij}]_{ixj}$

$$z_{ij} = \omega_j n_{ij} \tag{12}$$

Step 4. Determining the preference optimization value (μ_i)

$$\mu_i = \sum_{j=1}^{J} z_{ij} \tag{13}$$

Step 5. Ranking the options in accordance with the principle that the choice with the highest value (μ_i) is the best option.

Step 6. Determining the elements of ideal solution (z_j^*)

$$z_j^* = (z_i^*, z_2^*, ..., z_j^*) \tag{14}$$

where $z_j^* = (1)$ or the actual maximum value of (z_j^*).

Step 7. Computing the distance of each alternative from (z_j^*) and (z_{ij})

$$\pi_i = \sum_{j=1}^{J} (z_j^* - z_{ij}) \tag{15}$$

where $d_i = (z_j^* - z_{ij})$ is the distance measurement between two crisp numbers, and (π_i) is the preference optimization value.

Step 8. Ranking the options in accordance with the principle that the choice with the lowest value (π_i) is the best option.

B. Fuzzy POP method

In this section, a methodical approach for extending Preference Optimization Programming (POP) to a fuzzy environment is suggested. This approach is ideal for handling group decision-making problems in uncertain environment. The ratings of qualitative criteria and the important weights of various criteria are both considered as linguistic variables. These linguistic variables can be expressed as positive triangular fuzzy numbers as shown in Tables 2 and 3.

Table 2. Linguistic variables for the importance weight of each criterion

Linguistic variables	**Fuzzy numbers**
Very low (VL)	0,0,0.1
Low (L)	0,0.1,0.3
Medium low (ML)	0.1,0.3,0.5
Medium (M)	0.3,0.5,0.7
Medium high (MH)	0.5,0.7,0.9
High (H)	0.7,0.9,1.0
Very high (VH)	0.9,1.0,1.0

The importance weight of each criterion can be obtained by either directly assign or indirectly using pairwise comparisons [26-28]. Here, it is suggested that the decision-makers assess the importance of the criteria and the evaluations of alternatives with respect to various criteria using the linguistic variables (shown as Tables 2 and 3).

Table 3. Linguistic variables for the ratings

Linguistic variables	**Fuzzy numbers**
Very poor (VP)	0,0,1
Poor (P)	0,1,3
Medium poor (MP)	1,3,5
Fair (F)	3,5,7
Medium good (MG)	5,7,9
Good (G)	7,9,10
Very good (VG)	9,10,10

The fuzzy POP method is performed according to the following procedural steps:

Step 1. For the fighter aircraft selection process, k decision makers are determined. Fighter aircraft selection criteria are determined by the determined k decision makers.

Step 2. The importance levels of the selection criteria and the evaluation of the alternatives for each criterion by k decision makers are calculated as follows:

$$\omega_{ij} = \frac{1}{k}\left[\omega_j^1 + \omega_j^2 + ... + \omega_j^k\right] \tag{16}$$

$$x_{ij} = \frac{1}{k}\left[x_{ij}^1 + x_{ij}^2 + ... + x_{ij}^k\right] \tag{17}$$

Step 3. A fuzzy decision matrix is created for the criteria and alternatives $X = [x_{ij}]_{ixj}$.

$$X = \begin{pmatrix} g_1 & & g_j \\ x_{11} & \cdots & x_{1j} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} \end{pmatrix}_{ixj} \tag{18}$$

Suppose that multiple criteria decision making analysis (MCDMA) problem has I alternatives $x_i = (x_1, ..., x_i)$, $i \in \{ i = 1,..., I \}$, and J criteria $g_j = (g_1, ..., g_j)$, $j \in \{ j = 1,..., J \}$, and the importance weight of each criterion (ω_j, $j \in \{ j = 1,..., J \}$) is defined.

Step 4. The fuzzy decision matrix of the alternatives is normalized using the linear normalization scale.

$$n_{ij} = \left(\frac{l_{ij}}{u_j^*}, \frac{m_{ij}}{u_j^*}, \frac{u_{ij}}{u_j^*} \right), j \in B \tag{19}$$

$$n_{ij} = \left(\frac{l_j^-}{u_{ij}}, \frac{l_j^-}{m_{ij}}, \frac{l_j^-}{l_{ij}} \right), j \in C \tag{20}$$

$$u_j^* = \max_i u_{ij} \quad if \quad j \in B$$
$$l_j^- = \min_i l_{ij} \quad if \quad j \in C$$

where B represents a criterion as large as possible, C represents a criterion as small as possible. n_{ij} is an element of the normalized matrix, $N = [n_{ij}]_{ixj}$.

Step 5. The weighted normalized fuzzy decision matrix is calculated.

$$z_{ij} = \omega_j n_{ij} \tag{21}$$

Step 6. Determining the preference optimization value (μ_i)

$$\mu_i = \sum_{j=1}^{J} z_{ij} \tag{22}$$

Step 7. Determining the elements of ideal solution (z_j^*)

$$z_j^* = (z_i^*, z_2^*, ..., z_j^*) \tag{23}$$

where $z_j^* = (1,1,1)$.

Step 8.Computing the distance of each alternative from (z_j^*) and (z_{ij})

$$\pi_i = \sum_{j=1}^{J} d(z_j^*, z_{ij}) \tag{24}$$

where $d(.,.)$ is the distance measurement between two fuzzy numbers, and (π_i) is the preference optimization value.

Step 9. Ranking the options in accordance with the principle that the choice with the lowest value (π_i) is the best option.

In sum, the following contains an algorithm for multiple criteria decision making using a fuzzy set approach for the POP technique.

Step 1. Form a committee of decision-makers, then identify the evaluation criteria.

Step 2. Select the appropriate linguistic variables that best reflect the importance weight of the criteria and the linguistic ratings of alternatives to the criteria.

Step 3. Aggregate the weight of criteria to get the aggregated fuzzy weight (ω_j) of criterion g_j and combine the ratings of the decision-makers to get aggregated fuzzy rating (x_{ij}) alternative x_i under criterion g_j.

Step 4. Construct the fuzzy decision matrix and the normalized fuzzy decision matrix.

Step 5. Create the weighted normalized fuzzy decision matrix (z_{ij}).

Step 6. Determine the preference optimization value (μ_i).

Step 7. Rank the options in accordance with the principle that the choice with the highest value (μ_i) is the best option.

Step 8. Determine the fuzzy ideal solution (z_j^*).

Step 9.Determine the distance of each alternative from (z_j^*) and (z_{ij}).

Step 10.Determine the preference optimization value (π_i) of each alternative.

Step 11. Determine the ranking order of all options in accordance with the principle that the choice with the lowest value (π_i) is the best option.

C. Fuzzy TOPSIS method

The fuzzy TOPSIS method is performed according to the following procedural steps:

Step 1. For the fighter aircraft selection process, *k* decision makers are determined. Fighter aircraft selection criteria are determined by the determined *k* decision makers.

Step 2. The importance levels of the selection criteria and the evaluation of the alternatives for each criterion by *k* decision makers are calculated as follows:

$$\omega_{ij} = \frac{1}{k}\left[\omega_j^1 + \omega_j^2 + ... + \omega_j^k\right] \quad (25)$$

$$x_{ij} = \frac{1}{k}\left[x_{ij}^1 + x_{ij}^2 + ... + x_{ij}^k\right] \quad (26)$$

Step 3. A fuzzy decision matrix is created for the criteria and alternatives $X = [x_{ij}]_{ixj}$.

$$X = \begin{pmatrix} \overset{g_1}{x_{11}} & \cdots & \overset{g_j}{x_{1j}} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} \end{pmatrix}_{ixj} \quad (27)$$

Suppose that multiple criteria decision making analysis (MCDMA) problem has *I* alternatives $x_i = (x_1, ..., x_i)$, $i \in \{ i = 1, ..., I \}$, and *J* criteria $g_j = (g_1, ..., g_j)$, $j \in \{ j = 1, ..., J \}$, and the importance weight of each criterion (ω_j, $j \in \{ j = 1, ..., J \}$) is defined.

Step 4. The fuzzy decision matrix of the alternatives is normalized using the linear normalization scale.

$$n_{ij} = \left(\frac{l_{ij}}{u_j^*}, \frac{m_{ij}}{u_j^*}, \frac{u_{ij}}{u_j^*}\right), j \in B \quad (28)$$

$$n_{ij} = \left(\frac{l_j^-}{u_{ij}}, \frac{l_j^-}{m_{ij}}, \frac{l_j^-}{l_{ij}}\right), j \in C \quad (29)$$

$$u_j^* = \max_i u_{ij} \quad if \quad j \in B$$

$$l_j^- = \min_i l_{ij} \quad if \quad j \in C$$

where *B* represents a criterion as large as possible, *C* represents a criterion as small as possible. n_{ij} is an element of the normalized matrix, $N = [n_{ij}]_{ixj}$.

Step 5. The weighted normalized fuzzy decision matrix is calculated.

$$z_{ij} = \omega_j n_{ij} \quad (30)$$

Step 6. Determining the preference optimization value (μ_i)

$$\mu_i = \sum_{j=1}^{J} z_{ij} \quad (31)$$

Step 7. Determining the elements of fuzzy positive-ideal solution (FPIS, A^*) and fuzzy negative-ideal solution (FNIS, A^-)

$$A^* = (max_i\, z_{ij} \mid j \in B) = z_j^* = (1,1,1) \quad (32)$$

$$A^- = (\min_i z_{ij} \mid j \in C) = z_j^- = (0,0,0)$$

Step 8.Computing the distance of each alternative from A^* and A^-

$$d_i^* = \sum_{j=1}^{J} d(z_{ij}, z_j^*) \quad (33)$$

$$d_i^- = \sum_{j=1}^{J} d(z_{ij}, z_j^-) \quad (34)$$

where $d(.,.)$ is the distance measurement between two fuzzy numbers.

Step 9. Defining a closeness coefficient to determine the ranking order of all alternatives

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \quad (35)$$

Naturally, as $CC_i \in [0,1]$ becomes closer to 1, an alternate A_i is closer to the FNIS (A^*) and further from the FPIS (A^-). Therefore, using the proximity coefficient, it is possible to determine the ranking order of all alternatives and choose the best option from a list of viable options.

Step 10. Ranking the options in accordance with the principle that the choice with the highest value CC_i is the best option.

A fuzzy TOPSIS ranking is made among the alternatives based on the closeness coefficient value CC_i. The best alternative is the closest to the fuzzy ideal solution (A^*) and the farthest from the fuzzy anti - ideal solution (A^-). Here, it would be more appropriate to use a number of linguistic variables to evaluate alternatives. For this, the interval [0,1] as a result of the evaluation of an alternative was divided into five, and acceptance conditions were determined for each class with linguistic expressions. Table 4 shows the acceptance conditions according to the closeness coefficients of the alternatives.

Table 4. Acceptance conditions according to the closeness coefficients of the alternatives

Proximity coefficient (CC_i)	**Evaluation result**
[0.0 , 0,2]	Not recommended
[0.2 , 0.4]	Recommended with high risk
[0.4 , 0.6]	Recommended with low risk
[0,6 , 0,8]	Acceptable
[0.8 , 1.0]	Accepted and preferred

In sum, the following contains an algorithm for multiple criteria decision making using a fuzzy set approach for the TOPSIS technique.

Step 1. Form a committee of decision-makers, then identify the evaluation criteria.

Step 2.Select the appropriate linguistic variables that best reflect the importance weight of the criteria and the linguistic ratings of alternatives to the criteria.

Step 3.Aggregate the weight of criteria to get the aggregated fuzzy weight (ω_j) of criterion g_j and combine the ratings of the decision-makers to get aggregated fuzzy rating (x_{ij}) alternative x_i under criterion g_j.

Step 4.Construct the fuzzy decision matrix and the normalized fuzzy decision matrix.

Step 5. Create the weighted normalized fuzzy decision matrix (z_{ij}).

Step 6. Determine the fuzzy positive-ideal solution (FPIS, A^*) and fuzzy negative-ideal solution (FNIS, A^-).

Step 7.Determine the distance of each alternative from A^* and A^-.

Step 8.Determine the closeness coefficient CC_i for the ranking order of all alternatives.

Step 9. The ranking order of all alternatives can be determined according to the closeness coefficient CC_i.

III. Application

Given that unmanned technology will likely dominate air combat in the future, "Bayraktar KIZILELMA Fighter UAV," which is currently being developed entirely within Türkiye, will undoubtedly assume a greater role as time goes on. Baykar is taking stock of its extensive UAV/UCAV experience as a foundation for forging ahead fast with the deployment of the Bayraktar KIZILELMA Fighter UAV system. Given its aggressive maneuvering skills and stealthiness against radar, the Bayraktar KIZILELMA Fighter UAV will be a force to be reckoned with. Additionally, the Bayraktar KIZILELMA Fighter UAV will be able to perform missions with internally-carried munitions and launch and land on aircraft carriers with short runways. The Kizilelma is a jet-powered, single-engine, low-observable, supersonic, carrier-capable unmanned fighter aircraft, currently in development as part of Project MIUS (Turkish: Muharip İnsansız Uçak Sistemi - English: Combatant Unmanned Aircraft System) [51].

Key features of "Kizilelma" includes fully-autonomous takeoff and landing, low radar cross-section, high maneuverability, Line Of Sight (LOS) and Beyond LOS (BLOS) controlled, takeoff and landing capability from short-runway aircraft carriers (like TCG Anadolu), high situational awareness with AESA radar, and internal bays. There will be at least three variants of Kizilelma with different engine configurations. Kizilelma-A will be capable of near-supersonic speeds, being powered by the AI-25TLT engine. Kizilelma-B will fly at supersonic speeds, powered by a single Ukrainian AI-322F engine. Lastly, Kizilelma-C will incorporate 2 AI-322F engines [51].

On the other hand, potential sixth-generation fighter aircraft technology is considered to have all of the following designed features: extreme stealth, flexible payloads, an adaptable airframe, long-range sensing, highly networked, analytics and computing, autonomous weapons, laser directed-energy weapons, advanced materials, intelligent maintenance, dynamically reconfigurable architecture, cyber protection, manned-unmanned teaming (MUM-T), trusted artificial intelligence (AI) reasoning, airspace integration, hypersonic-propulsion technologies, space technologies, and a future 'wearable' cockpit. With these technological features, the sixth generation is intended to be cost-effective and to make use of cutting-edge manufacturing techniques.

Table 5. Key specifications of the sixth-generation fighter aircraft "Kizilelma-A"

Characteristics	
Payload capacity	1,500 kg
Length	14,7 m
Wingspan	10 m
Height	3,3 m
Max takeoff weight	6,000 kg
Powerplant	AI-25TLT
Performance	
Maximum speed	0,9 Mach
Cruise speed	0,6 Mach
Combat range	930 km
Endurance	5 hours
Service ceiling	14,000 m
Operational altitude	11,000 m
Avionics	
National AESA Radar	
ASELSAN Common Aperture Targeting System	
Electronic Warfare Pod	
National SIGINT module	
Armament	
Hardpoints: 2 × internal stations (expected), 6 × external stations on wings (expected), with provisions to carry combinations of missiles and bombs.	

Considering the aforementioned features of the sixth-generation fighter aircraft, suppose that the Turkish Air Force now wants to strengthen its defense capabilities and capacities by purchasing a sixth-generation fighter aircraft. Three sixth-generation fighter aircraft candidates—A1, A2, and A3—remain for further assessment after preliminary screening. To conduct the evaluation and choose the best sixth generation fighter aircraft, a committee made up of D1, D2, and D3 has been established. Following a review of the literature, five benefit criteria are considered for evaluation of the sixth-generation fighter aircraft: interconnectivity (g_1), paylaodability (g_2), maneuverability (g_3), speedability (g_4), and stealthibility (g_5). Solution steps of the problem will be given separately according to fuzzy POP and fuzzy TOPSIS methods.

Fuzzy POP solution: The hierarchical structure of this decision problem is shown as Fig. 2. The suggested fuzzy POP method is used to solve this problem and the computational process can be summed up as follows:

Step 1. The decision-makers use the linguistic weighting variables (shown in Table 2) to assess the importance of the criteria and present it in Table 6.

Step 2. The decision-makers use the linguistic rating variables (shown in Table 3) to evaluate the rating of alternatives with respect to each criterion and present it in Table 7.

Step 3. Converting the linguistic evaluation (shown in Tables 3 and 4) into triangular fuzzy numbers to construct the fuzzy decision matrix and determine the fuzzy weight of each criterion as Table 8.

Step 4. Constructing the normalized fuzzy decision matrix as Table 9.

Step 5. Constructing the weighted normalized fuzzy decision matrix as Table 10.

Step 6. Determining the preference optimization value (μ_i).

Step 7. Determining the elements of ideal solution (z_j^*).

$$z_j^* = [(1, 1, 1),(1, 1, 1),(1, 1, 1),(1, 1, 1),(1, 1, 1)]$$

Here, actual determined (z_j^*) values were utilized in the solution.

Step 8. Calculate the distance of each candidate from (z_j^*) and (z_{ij}), respectively, as shown in Table 11.

Step 9. According to the values of (μ_i), the ranking order of the three candidates is A2, A3, and A1. Obviously, the best selection is candidate A2. Also, according to the values of (π_i), the ranking order of the three candidates is same as A2, A3, and A1. Again, the best selection is candidate A2.

Fuzzy TOPSIS solution: The hierarchical structure of this decision problem is shown as Fig. 2. The suggested fuzzy POP method is used to solve this problem and the computational process can be summed up as follows:

Step 1. The decision-makers use the linguistic weighting variables (shown in Table 2) to assess the importance of the criteria and present it in Table 6.

Step 2. The decision-makers use the linguistic rating variables (shown in Table 3) to evaluate the rating of alternatives with respect to each criterion and present it in Table 7.

Step 3. Converting the linguistic evaluation (shown in Tables 3 and 4) into triangular fuzzy numbers to construct the fuzzy decision matrix and determine the fuzzy weight of each criterion as Table 8.

Step 4. Constructing the normalized fuzzy decision matrix as Table 9.

Step 5. Constructing the weighted normalized fuzzy decision matrix as Table 10.

Step 6. Determine FPIS and FNIS as

$$A^* = [(1, 1, 1),(1, 1, 1),(1, 1, 1),(1, 1, 1),(1, 1, 1)]$$

$$A^- = [(0, 0, 0),(0, 0, 0),(0, 0, 0),(0, 0, 0),(0, 0, 0)]$$

Here, actual determined FPIS and FNIS values were utilized in the solution.

Step 7. Calculate the distance of each candidate from FPIS and FNIS, respectively, as shown in Table 11.

Step 8. Calculate the closeness coefficient of each candidate as shown in Table 11.

Step 9. According to the closeness coefficient, the ranking order of the three candidates is A2, A3, and A1. Obviously, the best selection is candidate A2.

Table 6. The importance weight of the criteria

	D_1	D_2	D_3
g_1	H	MH	VH
g_2	VH	VH	VH
g_3	VH	H	H
g_4	VH	VH	VH
g_5	M	MH	MH

Table 7. The ratings of the three candidates by decision makers under all criteria

Criteria	Candidates	Decision makers		
		D_1	D_2	D_3
g_1	A_1	G	MG	MG
	A_2	G	G	MG
	A_3	G	VG	F
g_2	A_1	G	F	MG
	A_2	VG	VG	VG
	A_3	MG	VG	G
g_3	A_1	G	F	VG
	A_2	VG	VG	VG
	A_3	MG	G	MG
g_4	A_1	G	VG	VG
	A_2	VG	VG	VG
	A_3	VG	G	MG
g_5	A_1	F	F	F
	A_2	MG	VG	G
	A_3	G	G	MG

Comparative analysis and discussion: The effectiveness of the proposed method is validated with fuzzy TOPSIS approach. A seven scale linguistic terms are used to measure the variations within the sixth-generation fighter aircraft framework. The proposed fuzzy POP method is compared with fuzzy TOPSIS method.

The first analysis was performed using the preference optimization values (μ_i) and (π_i) of the fuzzy POP method and the same ranking order (A2 > A3 > A1) of the alternatives was obtained.

The second analysis was performed when fuzzy TOPSIS method is applied to obtain rankings for the same selection problem, the two methodologies yielded same ranking results. All methods give (A2 > A3 > A1) for the ranking order of alternatives. Since all the comparative results show the validity of the proposed fuzzy POP method, the ranking of the alternatives does not differ in the calculation process.

IV. Conclusion

Fuzzy set theory works well to handle the uncertain and imprecise data that multiple criteria decision making situations typically include. In order to resolve the multiple criteria decision-making problem in a fuzzy environment, a linguistic decision method is developed. It is frequently appropriate to employ linguistic variables rather than numerical values when evaluating alternatives in relation to criteria and importance weight throughout the decision-making process.

Here, the POP approach is extended to the fuzzy environment and present a vertex method, which is an efficient and straightforward method to calculate the distance between two triangular fuzzy numbers. In fact, the vertex method makes it simple to determine how far apart any two fuzzy numbers are if their membership functions are linear.

The importance of sixth-generation fighter aircraft is increasing within the realms of defense concept, as geopolitical risks force the Turkish Air Force to operate strategically more efficiently and effectively. There exist many solutions for countries seeking sixth-generation fighter aircraft solutions, however the question remains which of these solutions fits best to the specific needs of the Air Force. This study proposed a group decision framework based on the fuzzy POP method for evaluating and selecting a suitable sixth-generation fighter aircraft.

In order to check the applicability of the proposed framework, it is applied in a hypothetical case of the Turkish Air Force. In this study, fuzzy POP method was compared with fuzzy TOPSIS analysis. It is simple to combine the fuzzy ratings of the decision-makers in the suggested method when using different aggregation procedures in a group decision-making process. Both methods yielded same ranking order.

Although a problem involving sixth-generation fighter aircraft is utilized in this study to demonstrate the recommended strategy, a wide range of other engineering and management decision problems that involve evaluation and selection can also be solved using it.

It is important to note that, despite being created for this specific sixth-generation fighter aircraft selection challenge, the presented framework can also be used to assess other types of decision-making analysis.

Finally, future research on the issue of choosing sixth-generation fighter aircraft can combine the methodology based on fuzzy POP approach with other MCDMA methodologies, with the results being compared to those of this work.

Table 8. The fuzzy decision matrix and fuzzy weights of three alternatives

g_j	g_1			g_2			g_3			g_4			g_5		
a_1	5,67	7,67	9,33	4,00	5,33	6,67	5,33	6,33	7,00	8,33	9,67	10,00	0,00	0,00	1,00
a_2	6,33	8,33	9,67	9,00	10,00	10,00	9,00	10,00	10,00	9,00	10,00	10,00	7,00	8,67	9,67
a_3	5,33	6,33	7,00	7,00	8,67	9,67	5,67	7,67	9,33	7,00	8,67	9,67	6,33	8,33	9,67
ω_j	0,70	0,87	0,97	0,90	1,00	1,00	0,77	0,93	1,00	0,90	1,00	1,00	0,43	0,63	0,83

Table 9. The fuzzy normalized decision matrix

g_j	g_1			g_2			g_3			g_4			g_5		
a_1	0,59	0,79	0,97	0,40	0,53	0,67	0,53	0,63	0,70	0,83	0,97	1,00	0,00	0,00	0,10
a_2	0,66	0,86	1,00	0,90	1,00	1,00	0,90	1,00	1,00	0,90	1,00	1,00	0,72	0,90	1,00
a_3	0,55	0,66	0,72	0,70	0,87	0,97	0,57	0,77	0,93	0,70	0,87	0,97	0,66	0,86	1,00

Table 10. The fuzzy weighted normalized decision matrix

g_j	g_1			g_2			g_3			g_4			g_5		
a_1	0,41	0,69	0,93	0,36	0,53	0,67	0,41	0,59	0,70	0,75	0,97	1,00	0,00	0,00	0,09
a_2	0,46	0,75	0,97	0,81	1,00	1,00	0,69	0,93	1,00	0,81	1,00	1,00	0,31	0,57	0,83
a_3	0,39	0,57	0,70	0,63	0,87	0,97	0,43	0,72	0,93	0,63	0,87	0,97	0,28	0,55	0,83

Table 11. Rankings and performance evaluations of sixth generation fighter aircraft alternatives

	POP				TOPSIS			
	μ_i	Rank	π_i	Rank	d^*	d^-	CC_i	Rank
a_1	2,70	3	1,39	3	1,39	0,24	0,15	3
a_2	4,04	1	0,00	1	0,00	1,62	1,00	1
a_3	3,44	2	0,67	2	0,67	1,01	0,60	2

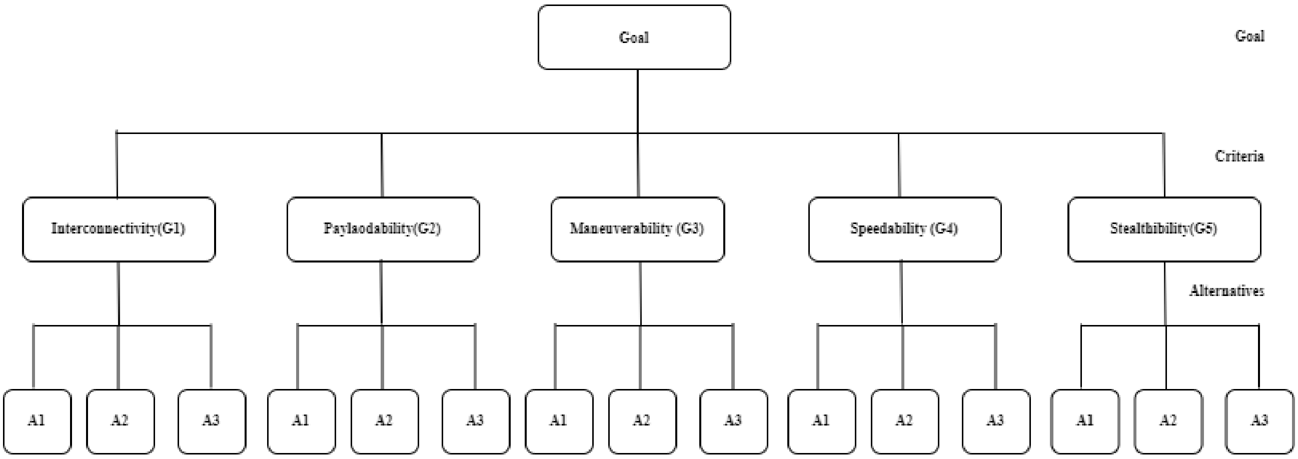

Fig. 2 Hierarchical structure of the evaluation and selection problem of sixth generation fighter aircraft.

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