World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:16, No:10, 2022

Overview of Adaptive Spline Interpolation

Rongli Gai, Zhiyuan Chang, Xiaohong Wang, Jingyu Liu

Abstract**—**In view of various situations in the interpolation process, most researchers use self-adaptation to adjust the interpolation process, which is also one of the current and future research hotspots in the field of CNC (Computerized Numerical Control) machining. In the interpolation process, according to the overview of the spline curve interpolation algorithm, the adaptive analysis is carried out from the factors affecting the interpolation process. The adaptive operation is reflected in various aspects, such as speed, parameters, errors, nodes, feed rates, random period, sensitive point, step size, curvature, adaptive segmentation, adaptive optimization, etc. This paper will analyze and summarize the research of adaptive imputation in the direction of the above factors affecting imputation.

Keywords**—**Adaptive algorithm, CNC machining, interpolation constraints, spline curve interpolation.

I. Introduction

NURBS interpolation technology for high-speed and high-precision has become increasingly important with the continuous development of manufacturing industry. Interpolation [1], [2] is the basis for generating tool paths in CNC machine tool processing. The interpolation accuracy directly affects the processing quality of the CNC system, and the interpolation speed directly determines the processing efficiency of the CNC machine tool. Therefore, the interpolation algorithm is the core of the entire CNC system. Because the sudden situation in the interpolation process will affect the processing process, and the manual intervention will make the processing efficiency low, the adaptive interpolation algorithm has become a current research hotspot.

The adaptive interpolation algorithm [3]-[5] is the main practical method to deal with unexpected situations in the interpolation process. In the interpolation process, there are many factors that affect the interpolation. Due to the different emphasis, the adaptive operation is mainly carried out in the main aspects such as speed, parameters, error, and curvature. With the in-depth exploration of researchers, other influencing factors risk correction is also possible through adaptive algorithms. Through the analysis of the existing adaptive interpolation algorithms, this paper summarizes and compares the interpolation algorithms of various adaptive types, shows the advantages and disadvantages of various methods, and then points out the research and development trend of adaptive interpolation algorithms.

II. Overview of Spline Interpolation Algorithm

A. Spline Curve

There are many definitions of splines. It is generally believed that splines are connected by a series of k-order polynomial curves at specific data points. Each data point connects two polynomial curves. It is continuous, and it is the first and second derivatives are also continuous [6]-[9]. Splines are defined piecewise, and the shape of the curve is generally determined by a set of control points or data points [10], [11].

Taking the cubic spline curve as an example, on the parameter interval $[u_i, u_{i+1}]$, the general cubic spline curve equation $S(u)$ can be expressed as [12]:

$$S(u) = A_i + B_i(u-u_i) + C_i(u-u_i)^2 + D_i(u-u_i)^3 \tag{1}$$

In the formula, u is the parameter of the spline curve, A_i, B_i, C_i, D_i is the coefficient matrix of the spline curve, and its dimension is equal to the dimension of the processed curve. When calculating the value of A_i, B_i, C_i, D_i, in addition to using the coordinate values corresponding to u_i and u_{i+1}, it is generally calculated by the continuous characteristics of the first-order and second-order derivatives at the endpoints of the spline curve [13].

In the field of CNC machining, commonly used spline curves include Bezier curve, B-spline curve and NURBS (Non-Uniform Rational B-Splines) curve. Because the shape control function of NURBS curve is particularly powerful and flexible, and it can accurately represent analytical curves and free-form surfaces, NURBS curves are set by the International Organization for Standardization (ISO) as the data exchange standard for CAD (Computer Aided Design) and CAM (Computer Aided Manufacturing). The NURBS curve is developed on the basis of the Bezier curve and the ordinary B-spline curve, and is well compatible with the two parametric curves, and has now become a unified curve expression.

Assuming that there are $n+1$ data points $P_0, P_1, P_2 \cdots P_n$, a NURBS curve of order k is defined as:

$$C(u) = \frac{\sum_{i=0}^{n} w_i P_i N_{i,k}(u)}{\sum_{i=0}^{n} w_i N_{i,k}(u)} \tag{2}$$

Rongli Gai is with the Dalian University of Information Engineering, Dalian, China (corresponding author, phone: 136-1095-9380; e-mail: gairli@sict.ac.cn).

Zhiyuan Chang is a graduate student studying at Dalian University (e-mail: 18263468648@163.com).

In (2), u is the parameter of the NURBS equation, $P_i(i=0,1\ldots n)$ is the data point or control point of the NURBS curve, and w_i is the weight corresponding to the control point P_i, where $w_0>0, w_n>0$, and other $w_i \geq 0$; $N_{i,k}(u)$ is the basis function of the k-order NURBS curve, determined by the node vector U $U=(u_i,u_{i+1}\ldots u_{i+k})$ the basis function satisfies the Cox-de Boor recurrence:

$$\begin{cases} N_{i,1}(u)=\begin{cases}1 & u_i \leq x < u_{i+1} \\ 0 & Otherwise\end{cases} \\ N_{i,k}(u)=\dfrac{u-u_i}{u_{i+k}-u_i}N_{i,k-1}(u)+\dfrac{u_{i+k+1}-u}{u_{i+k+1}-u_{i+1}}N_{i+1,k-1}(u) \end{cases} \quad (3)$$

Here you need to define $\frac{0}{0}=0$.

B. Spline Curve Interpolation

A spline curve [14] refers to a curve formed by connecting polynomial curve segments. The boundary of each segment satisfies a specific continuous condition, and its shape is generally determined by a set of control points. Because the spline curve can accurately and uniformly represent the analytical curve and the free curve, it is stipulated by the International Organization for Standardization as the data exchange standard of CAD/CAM. According to STEP-NC (the numerical control system data standard ISO14649) [14]-[16], the spline curve interpolation makes the processing information of the three-dimensional geometric model conforming to the STEP standard directly as the input of the numerical control system, that is, the parameter curve is directly interpolated.

In spline interpolation, chord is usually used to approach arc, and its principle is shown in Fig. 1.

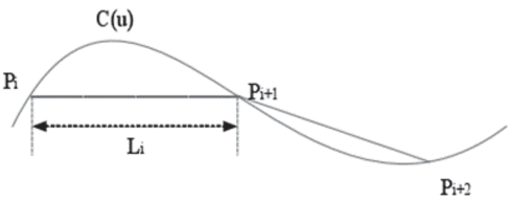

Fig. 1 Schematic diagram of spline interpolation

In spline interpolation, chords are usually used to approximate arcs [15]. $C(u)$ is the parameter curve, u is the interpolation parameter, and L_i is the interpolation step. Suppose the current interpolation point is $P_i=C(u_i)$, the next interpolation point is $P_{i+1}=C(u_{i+1})$, the ideal instantaneous speed of the tool at point P_i is $V_{ds}(u_i)$, and the interpolation period is T, according to the geometric relationship:

$$L_i=V_{ds}(u_i)\cdot T=\left\|C(u_{i+1})-C(u_1)\right\|=\left\|P_iP_{i+1}\right\| \quad (4)$$

The arc length l_i from parameter u_i to u_{i+1} is:

$$l_i=\int_{u_i}^{u_i+1}\left\|C'(u)\right\|du \quad (5)$$

The arc length is approximated by the chord length, ideally $L_i=l_i$, that is

$$V_{ds}(u_i)\cdot T=\int_{u_i}^{u_i+1}\left\|C'(u)\right\|du \quad (6)$$

Due to the nonlinear relationship between arc lengths and parameters of NURBS curves and B-spline curves, it is difficult to calculate the exact value of G during real-time spline interpolation, and only approximate values can be obtained. This approximation will cause the difference between the theoretical feed-rate and the actual feed-rate during spline interpolation, that is, feed-rate fluctuations.

After calculating the next parameter u_{i+1}, the actual feed rate V_i is:

$$V_i=\frac{\left|C(u_{i+1})-C(u_i)\right|}{T} \quad (7)$$

According to the above feed information [16], the next interpolation parameter u_{i+1} can be obtained, and then the actual interpolation point P_{i+1} can be obtained. Due to the nonlinear relationship between the arc length and the parameters of the spline curve, the exact value of u_{i+1} often cannot be obtained during interpolation, but only approximate values can be obtained. This results in a discrepancy between the ideal feed-rate and the actual feed-rate, that is, feed-rate fluctuations. The fluctuation of the feed rate will not only lead to the reduction of the machining accuracy of the workpiece, but may even cause chattering and affect the machining quality. The size of the speed fluctuation can be measured by the speed volatility δ_i [17]:

$$\delta_i=\frac{\Delta V_i}{V_{ds}(u_i)}\times 100\%=\left(1-\frac{\left\|C(u_{i+1})-C(u_i)\right\|}{V_{ds}(u_i)T}\right)\times 100\% \quad (8)$$

Through the understanding of spline curve interpolation, it can be found that there are many factors affecting the interpolation process, and there are many directions worthy of in-depth study on the point of adaptation [18].

III. Adaptive Interpolation Method

Common adaptive interpolation methods mainly include speed look-ahead adaptation, parameter adaptation, error adaptation, and other direction adaptations. These adaptive methods are analyzed below.

A. Speed Forward-Looking Adaptive

The speed look-ahead control [19], [20] is to realize the preprocessing of the program by monitoring the machining trajectory, so as to carry out the acceleration analysis and deceleration area discrimination of each linkage axis in advance, and realize the smooth transition at the turning point of the block or the deceleration point, to ensure excellent acceleration and deceleration control to avoid machine impact and ensure processing quality.

a. Speed Calculation of Continuous Micro Program Segment

Because of its simple expression, small amount of calculation and strong applicability, small segment path is the most widely expressed form of machining path of complex curve and surface parts such as impeller, blade and mold. With the rapid development of high-speed and high-precision machining technology, the machining process of complex curved surface parts takes milling instead of grinding as the development goal, and puts forward higher requirements for micro segment path machining.

Zhengjie et al. [21] inserted the bisector of the transition angle at the corner of the line segment for transition, adaptively divided the number of prospective segments, and planned forward and reverse speeds to make the speed transition between micro-line segments smooth and further improve the surface quality of the workpiece. Haihong et al. [22], [23] adopted an adaptive look-ahead control algorithm that optimizes the connection speed between trajectory segments for the current situation that most of the actual machining parts trajectories are composed of straight lines and circular arc trajectories. By judging and adjusting the accessibility, the speed between adjacent track segments can be smoothly and quickly connected, reducing the frequent start and stop of the feed axis of a large number of tiny track segments, and improving the acceleration and deceleration efficiency and machining accuracy.

Bolin et al. [24] segmented the curve by finding continuous, discontinuous and continuous, discontinuous breakpoints and key points with large curvature, but the current interpolation adjacent sensitive point area is adjacent to the next interpolation. In the sensitive point area, there may be situations where the velocity trajectory planning intersects. Yongqiao [25] used a continuous spline global smoothing algorithm interpolation technology, and realized the real-time smoothing and corner speed optimization of continuous small line segments in the forward-looking window based on the local smoothing of continuous small line segments and the optimization of corner speed based on the distribution of corner errors. and cross-segment adaptive speed planning. Jialan et al. [26] adopted the Cardinal spline curve transition method to establish a corner curve transition vector model to determine the corner transition curve that satisfies the conditions and then determine the maximum transfer speed to achieve high-speed connection between micro-segments.

Based on the S-type acceleration and deceleration scheme, Ren et al. [27] found the high-curvature points of the discrete machining path through cubic spline curve fitting and estimated the optimal speed at the high-curvature points, and controlled the machining path in blocks, effectively. It solves the problem of overcut easily occurring at high curvature points in traditional machining. Guangqiang et al. [28] performed pre-deceleration processing before reaching the area with large curvature or sudden change of curvature, and increased the speed planning to ensure the machining accuracy under the condition that the pre-deceleration does not exceed the acceleration limit.

With the development of numerical control technology, the linear interpolation technology based on small line segment path has been quite mature, but with the rapid development of high-speed machining, the traditional interpolation technology of small line segment can no longer meet the needs of high-speed machining. The higher the surface machining accuracy requirements, the shorter the line segments generated by the tool path, each of which is at the millimeter level or lower, which seriously restricts the improvement of the machining speed and makes the development of high-speed interpolation technology for micro-line segment paths facing a series of problems:

1) Linear high-order continuous smoothing problem: The tangent direction at the corners of adjacent line segments may suddenly change, and the curvature is discontinuous, resulting in sudden changes in acceleration at the corners and causing machine tool vibration.
2) Matching motion constraints of each axis of CNC machine tools: Only using the maximum speed and acceleration constraints cannot fully describe the performance characteristics of the machine tool.

b. Speed Optimization at the Transfer of Adjacent Program Segments

The first-order discontinuity at the corner of the linear tool path leads to the fluctuation of the feed rate and the sudden change of the acceleration at the corner. The look-ahead function was first used to solve this dilemma. By pre-reading a certain number of program segments, the micro-line segment path is analyzed and processed in advance, and then the point of the speed that needs to be processed is determined to maximize the connection speed of the line segments, thereby increasing the feed speed.

Yingqi et al. [29] used the Simpson adaptive integration method to calculate the length of the curve and adaptively segment the curve. Although this method improves the feed rate and reduces the processing time, it ignores the constraint of the rotation angle on the speed. Wei et al. [30] used backtracking and re-interpolation to predict at the speed-sensitive point and determine the deceleration point in this area, and then replace the previous adaptive storage speed with the re-interpolation speed in the curvature-sensitive area. By differentiating the ADAMS differential equation, Renping et al. [31], [32] estimated the correction iteration accuracy and retrospectively reconstructed the speed of the interpolation point, accurately predicted the position of the deceleration point, and made the feed of each speed sensitive point. The speed is maximized as far as possible under the premise of satisfying the machining accuracy. Haixia et al. [33] proposed an adaptive interpolation algorithm (S-A-NURBS) for symmetric combined sinusoidal function acceleration and deceleration control, which can effectively solve the problem of starting and stopping at the inflection point, smooth the transfer speed of adjacent sections, and reduce losses.

The speed optimization and smoothing processing at the transition of adjacent program segments can be divided into two types: the accurate passing method of the path connection point and the smooth transition method of the path connection point.

1) The accurate passing method of the path connection point: This method refers to accurately passing through each given path connection point during forward-looking speed smoothing and interpolation. The advantage of this method is that the interpolation accuracy is high and the interpolation error is not introduced. The disadvantage is that when planning the speed of the path segment, because the beginning and end speed of the optimized path segment is not zero, it is impossible to complete the interpolation of the path segment according to the expected acceleration and deceleration strategy in an integer interpolation cycle. To solve this problem, Wang et al. [34] presented an integer optimization model and solved it by a simplified method. Literature [34], [35] gives two strategies to adjust the feed speed curve for linear acceleration and deceleration, so that the adjusted path segment margin is exactly equal to the path increment of one interpolation cycle, so that the path segment can complete interpolation in an integral number of interpolation cycles. Li et al. [36] proposed a variable period interpolation method. According to the margin of less than one interpolation period at the end of the path segment, the interpolation period is adjusted accordingly. The variable interpolation period is provided by the RTX (Real Time Executive) real-time subsystem [37]. However, due to the influence of the timer precision, the feed rate at the end of the path segment is prone to fluctuations. In addition, this method is limited by the length of the path segment. When the length of the path segment is short (such as several microns to tens of microns), the connection speed of the path segment will be limited to a lower value (related to the interpolation period, the interpolation period The longer it is, the lower the connection speed is limited), so the actual feed speed is still difficult to increase sufficiently.
2) The smooth transition method of the path connection point: This method performs local smoothing processing on the pre-reading path segment, realizes continuous interpolation across segments, does not need to pass through the given path connection point precisely, can overcome the problem that the interpolation period is not an integer, and can also avoid the length of the path segments. Commonly used parametric curves are Bezier curve [38], [39], PH (Pythagorean-Hodograph) curve [40], [41], and B Spline [42] etc. The second is to optimize the connection speed of the path segment after partial smoothing according to the allowable geometric error and machine tool characteristics, and plan the speed according to the desired acceleration and deceleration strategy. Yau et al. [43], [44] optimized the joint speed according to the corner error caused by the servo lag. Tsai et al. [45], Zhang et al. [46] and Ye et al. [47] optimized the bridging speed according to the maximum allowed acceleration and path curvature. Ping et al. [48] based on the given maximum geometric error, the smoothing error and the interpolation error are evenly distributed, and the connection speed is optimized.

c. Forecast before Deceleration Point

The acceleration part of the interpolation process runs according to the normal acceleration law, even if it does not reach the maximum allowable speed, it will not affect the machining accuracy, but for the deceleration process, the deceleration must be monitored. If the speed at the speed sensitive point cannot be reduced in time as required, the machining accuracy will be affected. In the acceleration and deceleration planning, the inaccurate prediction of the deceleration point has always been a major difficulty in the acceleration and deceleration control planning.

Guoliang [49] introduced the concept of forward and reverse bidirectional interpolation to solve the problem that the deceleration starting point and the length of the curve cannot be accurately obtained during the interpolation process. Hepeng [50] proposed a bidirectional interpolation strategy based on the speed planning method of time bidirectional rounding. Lei et al. [51] performed reverse interpolation for the determined velocity abrupt point within the prospective range by using the local optimal solution of the abrupt point velocity based on the bow height error and cubic polynomial acceleration and deceleration. Nian et al. [55], [56] iterated the NURBS parameters through Steffensen algorithm to realize the adaptive planning of speed. On this basis, Dong [54] adopted the interpolation algorithm calculated by Steffensen type parameters, and adaptively adjusted the maximum jerk according to the curvature information to implement high-precision interpolation to reduce the velocity fluctuation rate. Xia et al. [55] used NURBS symmetry to predict the deceleration point, combined with real-time step size adaptive control to estimate the interpolation parameters by the simplified Adams differential equation algorithm, and finally used the prediction-correction to control the precision.

d. Other Types of Speed Look-Ahead Control

After years of hard work by many scholars at home and abroad, the research on adaptive curve interpolation has made great progress. Adaptive interpolation methods to meet different machining conditions have been proposed successively, and each adaptive interpolation algorithm has its own unique consideration for the determination of the feed rate.

Peng et al. [56], [57], based on the dual-core architecture system, studied and analyzed the optimization principle and linear assembly-level optimization of the IQ Math library, and optimized the adaptive forward-looking algorithm. significance. Jianfeng et al. [58] imposed constraints on geometric accuracy, machine tool drive characteristics and cutting characteristics, and proposed a customized FSLP (Feed Speed Customization Algorithm Based on Linear Programming) algorithm for speed planning, which could effectively control the swing speed of the tool shaft and limit the change in swing speed, greatly improving the surface quality of side milling. On the basis of RTCP linear encryption, Haitao [59] established a calculation model of limit feed rate under nonlinear error constraints, and established an adaptive feed rate prediction model under nonlinear error-machine tool dynamics and thermal properties-joint speed constraints. The interpolation method effectively reduces the vibration and impact of the machine tool.

Adaptive speed forward-looking involves knowledge in many fields, such as complex tool pose planning, machine tool kinematics and dynamics, machining error control, high-speed and high-precision interpolation algorithm and intelligent algorithm optimization. Although researchers have made some research achievements in the direction of adaptive interpolation, due to limited ability and energy, there are still many contents worthy of further research and discussion in the research field of optimal feed speed planning of NC machining.

TABLE I
COMPARISON OF SPEED LOOK-AHEAD ADAPTIVE METHODS

Method	Advantage	Disadvantage
Speed calculation of continuous Micro Program Segment	Simple expression, small amount of calculation, strong applicability	The precision of the joint is not enough, resulting in linear high-order continuous smoothing problem
Speed optimization at the transfer of adjacent program segments	Increased feed rate, reduced machining time, and high interpolation accuracy	The acceleration between segments is prone to sudden change, and the feed rate is unstable
Forecast before deceleration point	High estimation accuracy, fast convergence speed, and reduced calculation amount	The calculation process is complicated and the algorithm operation time is long
Other types of speed look-ahead control	low speed fluctuations	The stability is poor, the calculation amount of acceleration and deceleration control is too large, and most of them are limited to the given tool path

The comparison of various methods in the speed look-ahead adaptation is shown in Table I.

B. Parameter Adaptation

The concept of adaptive parametric curve interpolation has greatly enriched the connotation of feed rate control in parametric curve interpolation. Researchers can establish different feed rate planning models considering the required constraints according to different working conditions, and finally get feed speed profile curve to meet different requirements.

Zhimei [60], [61] put forward the concept of parameter intermediate point, using the distance from the parameter intermediate point to the interpolation straight line as the interpolation error, and adjusting the interpolation error in real time by changing the parameter increment. Shiming [62] used a parameter adaptive algorithm to optimize the cubic NURBS curve to determine the node vector, and then based on the chord-cut iteration method to iterate the predicted nodes to ensure the normal range of volatility. Zhiwei [63] proposed an adaptive determination method for small line segment velocity planning parameters based on statistics to solve the problem that it is difficult to determine the optimal machining parameters for CNC systems to process different workpieces. The Gear estimation correction interpolation algorithm of Hengjun et al. [64] obtains the estimated step size through parameter value estimation and the preprocessing matrix method, and then uses the adaptive ideal step size to iteratively correct the parameter values. It solves the shortcomings of poor real-time performance and large speed fluctuations. Zhibing [65] used the improved Admas differential equation method to estimate the parameters, and realized the adaptive correction of the parameters according to the constraints of chord height error, velocity and acceleration.

C. Error Adaptation

After the real-time error estimation of the contour is obtained, the estimated contour error needs to be used for contour control. There are currently two main classes of methods for contour control: cross-coupled controller (CCC) [66]-[68] and task coordinate method [39]-[71]. The basic principle of the cross-coupling controller is: from the ideal contour information and the actual position information of each axis, the real-time contour error is estimated by the contour error estimation algorithm, and then calculated by the cross-coupling controller, and then decoupled with the cross-coupling coefficient, so as to control the contour error as a whole. The principle of the task coordinate method: the estimated contour error is decomposed along the normal and tangential directions at the desired position point or specified point, and then the control laws for these two variables can be independently designed. Since the normal component is often considered to be approximately equal to the contour error, the control law bandwidth for the normal error is higher than that for the tangential error when designing the controller.

Zhiwei [72] combined the contour error adaptive interpolation algorithm with the path length adaptive speed planning algorithm. Zuhao [76] combined the advantages of equal parameter method and equal error method, established the adaptive error model of trisection method, and put forward the optimized error adaptive arc interpolation algorithm to obtain high machining accuracy. Yitian [74] proposed a real-time adaptive contour error estimation method (ACEE), which made the number of error estimation points more than the number of servo control points, and improved the accuracy of contour error estimation. The contour error estimation model proposed by Yang [75], [76] combined the dual fuzzy variable universe adaptive control algorithm to design the contour error controller of the contour optimal circle approximation method, which specifically avoided the estimation dependent on the tracking error. This method can effectively reduce the machining contour error. On the basis of the existing contour error calculation model, Henan [80] proposed a new contour error calculation model—the calculation model of four-point double circle weighted approximation. The model also does not depend on the tracking error value and contour shape, and the effect of contour control for large curvature is more obvious.

D. Other Direction Adaptations

Yiqiao [78] proposed a cubic spline interpolation model for node adaptive selection in the problem of the change of error data. Aiming at the problem of redundant interpolation and large approximation error in the equidistant parameterization method, Zhen [79] designed an adaptive interpolation algorithm based on NURBS curve contour curvature radius, and Qian [80] solved the equidistant curve by adaptive discrete method Self-inflicted problem. Hui [81] proposed a method to find the optimal interpolation period, and proposed an adaptive random period interpolation control method from the perspective of changing the interpolation period, which provided a new method for eliminating the frequency superposition problem in the case of a large interpolation period. way. Jipeng [85] proposed a NURBS curve adaptive fitting algorithm---FKTP (Feature Point Recognition and The Knot Placement Technique) for dense point sequences in space, and then based on FKTP performed a speed adaptive adjustment strategy for targeted two-way rendezvous to ensure the continuity of the left and right motion parameters of the rendezvous point. Shujie et al. [83] adaptively select the interpolation algorithm based on the complexity of the interpolation calculation for different interpolation accuracy and curve-related parameters, and dynamically select an efficient calculation method.

TABLE II
COMPARISON OF ADAPTIVE SPLINE INTERPOLATION METHODS

Method	Advantage	Disadvantage
Speed forward-looking adaptive	Low speed fluctuation and strong applicability	Mutation is easy to occur at the high-order continuity
Parameter adaptation	Strong real-time performance and strong convergence	There are high requirements for determining the first few parameters, error compensation is required, and the algorithm has limitations
Error adaptation	High machining accuracy and strong stability	Large amount of calculation, poor real-time performance, poor effect at high speed and large curvature
Other direction adaptation	The degree of time optimization is strong, and the convergence speed is accelerated.	The applicable direction is limited and the technology is not perfect

The comparison of various adaptive imputation methods is shown in Table II.

IV. ADAPTIVE DEVELOPMENT IN NEW DIRECTION

Hu et al. [84] used a non-dominated genetic algorithm, namely NSGA-II, based on the constraints on the tangent vector and control points, without knowing the nodes in advance, and the obtained interpolation curve is similar to the given tangent vector and data polyline, which is better than other method is more natural. In the research of robot trajectory planning, Wei [85] analyzed the basic trajectory planning algorithms in different trajectory planning spaces, and proposed a 4-5-4 polynomial hybrid interpolation algorithm based on this trajectory planning algorithm. Aiming at the problem that the standard PSO optimization algorithm is easy to fall into the local optimum and difficult to obtain the global optimum, this interpolation algorithm proposes an improved scheme that combines the standard PSO optimization algorithm with the natural selection mechanism and the adaptive inertia weighting factor. Based on the above two schemes, he proposed a scheme combining the improved PSO optimization algorithm and the 4-5-4 polynomial hybrid interpolation algorithm to realize the optimal trajectory planning of joint point to point.

V. CONCLUSION AND PROSPECT

In summary, adaptive spline interpolation has become one of the research hotspots in the field of CNC machining. In the computer numerical control system, the interpolation algorithm is the basis for generating the machining trajectory, and its pros and cons largely determine the machining efficiency and precision of the numerical control system. Based on various factors affecting the interpolation in the process of interpolation, this paper summarizes various adaptive interpolation methods, and expounds the characteristics of each method. It can be seen from the analysis that the adaptive spline curve interpolation currently has the following trends:

First, the adaptive interpolation method for the purpose of speed look-ahead is the mainstream of research. Since the feed rate is constrained and restricted by many factors, the complementary self-adaptation of many restrictive factors can be studied in depth in the horizontal direction, and the research direction of multi-axis and multi-dimensional space self-adaptive interpolation can be approached in the vertical direction.

Second, we can carry out divergent innovation research according to the factors affecting adaptive interpolation by innovating and improving the basic kinematics basis and kinematics related models.

Third, it can be cross applied with algorithms in robot trajectory planning or research methods in other fields, such as the combination with machine learning and neural network. In recent years, the cross application of knowledge between subject directions is frequent. We can find suitable adaptive interpolation methods in experiments and explore new research directions.

REFERENCES

[1] Lin Wang, Jianfu Cao. "A look-ahead and adaptive speed control algorithm for high-speed CNC equipment (J). The International Journal of Advanced Manufacturing Technology,2012(5/8). DOI:10.1007/s00170-012-3924-7.
[2] Zhang Lixian, Sun Ruiyong, Gao Xiaoshan, et al. High speed interpolation for micro-line trajectory and adaptive real-time look-ahead scheme in CNC machining (J). Science China Technological Sciences, 2011(6). DOI:10.1007/s11431-011-4329-9.
[3] Jingkui Zheng, Haina Zhong, Speed control method of 5-axis linkage numerical control system (J). Computer Integrated Manufacturing System, 2007(05):950-954+966.
[4] Yunan Cao, Tianmiao Wang, Youdong Chen, Hongxing Wei, Application of S acceleration and deceleration before interpolation in CNC foresight (J). Journal of Beijing University of Aeronautics and Astronautics,2007(05):594-599.
[5] Heng Ye, Minghong Xie, Adaptive forward planning algorithm for

continuous micro-line segment flexible acceleration and deceleration (J). Journal of Huaqiao University (Natural Science Edition), 2021,42(01):1-8.
[6] Dierckx, Paul. “Curve and surface fitting with splines.” Monographs on numerical analysis, 1996.
[7] Späth, Helmuth. “One-dimensional spline interpolation algorithms.”, 1995.
[8] Späth, Helmuth. “Two-dimensional spline interpolation algorithms.”, 1993.
[9] Salomon, David. “The Computer Graphics Manual.” Texts in Computer Science, 2011.
[10] Piegl, Les A. and Wayne Tiller. “The NURBS book (2nd ed.).”, 1997.
[11] Fazhong Shi. Computer-aided geometric design and non-uniform rational B-splines: CAGD & NURBS. Beijing University of Aeronautics and Astronautics Press, 1994.
[12] Yu Shilin, Zhu Guobao. "Mathematical Processing of List Curves in CNC Machining Programming." Journal of Wuhan Jiaotong University ,2000.
[13] Zhang Xiaohui et al. "Research on Smooth Compression Interpolation Algorithm in NC Machining." Chinese Journal of Mechanical Engineering 47.5,2011,:8.
[14] Feibiao Pang, Research on high-quality machining technology for complex contour and curve parts (D). Xi'an University of Architecture and Technology, 2020.
[15] Yanwei Wu, Research on spline interpolation algorithm and speed optimization of CNC system (D). Beijing Jiaotong University,2020.
[16] Mingrui Yin, Research on adaptive speed planning method based on dynamic characteristics of feed system (D). Shandong University of Technology, 2020.
[17] Shoujin Xu, Research on forward-looking adaptive interpolation algorithm based on NURBS curve (D). Zhejiang Sci-Tech University,2016.
[18] Lin Chen, Xufeng Huang, Meng Liu, Shanshan Yuan, Feixiang He, Jian Yi, Haihong Pan, Continuous Trajectory Look-Ahead Algorithm for Integrated Multiple Constraints Optimization (J). Chinese Journal of Mechanical Engineering,2019,55(13):151-159.
[19] Guang Yang, Research and implementation of key algorithms for forward-looking direct interpolation of NURBS curve in high-speed machining CNC system (D). Zhejiang University, 2019.
[20] Anjiang Cai, Jie Wang, Lixia Li, Dan Zhao, Five-axis linkage tool path non-uniform rational B-spline interpolation technology (J). Science Technology and Engineering, 2017,17(09):153-158.
[21] Zhengjie Chen, Dongbiao Zhao, Keqinag Li, An adaptive forward-looking smooth interpolation algorithm for micro-line segment velocity (J). Mechanical Design and Manufacturing Engineering,2014,43(03):41-44.
[22] Haihong Pan, Jian Yi, Meng Liu, Research on the forward-looking interpolation algorithm of adaptive jerk NURBS curve (J). Combined Machine Tool and Automated Machining Technology, 2020(11):54-59+64.
[23] Haihong Pan, Zengqi Yang, Lin Chen, Haitao Dong, Bingqiong Hunag, Huaqing Tan, An Adaptive Prospective Control for Optimizing the Convergence Speed between Trajectory Segments (J). Chinese Journal of Mechanical Engineering,2015,51(05):151-159.
[24] Bolin Dong, Wenjie He, Research on regional velocity trajectory planning of adjacent sensitive points in NURBS interpolation (J). Mechanical Design and Manufacturing,2020(06):177-180+184.
[25] Yongqiao Jin. Research on trajectory optimization modeling and forward-looking interpolation technology for high-speed machining of tiny line segments (D). Shanghai Jiaotong University, 2015.
[26] Jialan Yuan, Yan Huang, Rongli Gai, Adaptive look-ahead interpolation algorithm based on Cardinal spline curve (J). Combined Machine Tool and Automated Machining Technology, 2018(10):1-5.
[27] Kun Ren, Jianzhong Fu, Zichen Chen, Research on a new algorithm for speed look-ahead control in high-speed machining (J). Journal of Zhejiang University(Engineering Science),2006(11):1985-1988.
[28] Guangqiang He, Dong Yu, Xiaohui Zhang, Five-axis NURBS curve interpolation method based on speed prospective control (J). Combined Machine Tool and Automation Machining Technology, 2011(05):38-41+45.
[29] Yingqi Li, Guirong Wang, NURBS interpolation control method for polynomial acceleration and deceleration (J). Combined Machine Tool and Automated Machining Technology, 2018(05):73-77.
[30] Wei Tang, Guanghui Cai, Hengfeng Qin, Real-time look-ahead algorithm for NURBS curve parameter interpolation (J). Computer Application Research,2016,33(02):454-457.
[31] Renping Ye, Dehuai Zeng, Research on Adaptive Real-time Prospective Interpolation Algorithm Based on NURBS Curve (J). Machine Tool and Hydraulics, 2016, 44(13): 46-52.
[32] Renping Ye, Research on motion modeling and simulation of five-degree-of-freedom serial manipulator (D). Shenzhen University, 2016.
[33] Haixia Song, Research on modeling and trajectory planning of stone engraving robotic arm (D). Huaqiao University, 2019.
[34] Wang, Y. et al. “An Optimal Feed Interpolation Algorithm for High-Speed Five-Axis Machining.” Materials Science Forum 532-533, 2006: 873 - 876.
[35] Feng, Jing Chun et al. “An Optimal Feed Interpolation with Jerk-Limited Acceleration for Five-Axis Grinding.” Key Engineering Materials 359-360, 2007: 204 - 209.
[36] Li, Yuyao et al. “Variable-period feed interpolation algorithm for high-speed five-axis machining.” The International Journal of Advanced Manufacturing Technology 40, 2009: 769-775.
[37] Li Zhiwei, and Zeng Yanming. "CNC Machining Technology of Large Guide Vane Mixed-Flow Pump Blades." Drainage and Irrigation Machinery, 2006.
[38] Bi, Qingzhen et al. “A Practical Continuous-Curvature Bézier Transition Algorithm for High-Speed Machining of Linear Tool Path.” ICIRA ,2011.
[39] Bi, Qingzhen et al. “Analytical curvature-continuous dual-Bézier corner transition for five-axis linear tool path.” International Journal of Machine Tools & Manufacture 91, 2015: 96-108.
[40] Shi, Jing et al. “Development of Real-Time Look-Ahead Methodology Based on Quintic PH Curve with G2 Continuity for High-Speed Machining.” Applied Mechanics and Materials 464 ,2013: 258 - 264.
[41] Jahanpour, Javad et al. “High-speed contouring control with NURBS-based C2 PH spline curves.” The International Journal of Advanced Manufacturing Technology 49, 2010: 663-674.
[42] Zhao, Huan et al. “A real-time look-ahead interpolation methodology with curvature-continuous B-spline transition scheme for CNC machining of short line segments.” International Journal of Machine Tools & Manufacture 65, 2013: 88-98.
[43] Yau, Hong-Tzong and Junbin Wang. “Fast Bezier interpolator with real-time lookahead function for high-accuracy machining.” International Journal of Machine Tools & Manufacture 47, 2007: 1518-1529.
[44] Yau, Hong-Tzong et al. “Development and implementation for real-time lookahead interpolator by using Bezier curve to fit CNC continuous short blocks.” IEEE International Conference on Mechatronics, 2005. ICM '05. 2005: 78-83.
[45] Tsai, Meng-Shiun et al. “Development of a real-time look-ahead interpolation methodology with spline-fitting technique for high-speed machining.” The International Journal of Advanced Manufacturing Technology 47, 2010: 621-638.
[46] Yeh, Syh-Shiuh and Hsin-Chuan Su. “Implementation of online NURBS curve fitting process on CNC machines.” The International Journal of Advanced Manufacturing Technology 40, 2009: 531-540.
[47] Ye, Peiqing et al. “Interpolation of continuous micro line segment trajectories based on look-ahead algorithm in high-speed machining.” The International Journal of Advanced Manufacturing Technology 37, 2008: 881-897.
[48] He Jun, You Youpeng, and Wang Huaming. "Ferguson spline transition algorithm for high-speed machining of micro-segments." China Mechanical Engineering 19.17, 2008:0-2049.
[49] Guoliang Yin, Research on NURBS curve interpolation and speed look-ahead algorithm in motion control system (D). Harbin Institute of Technology, 2019.
[50] Hepeng Ni, Research on Robot Milling Trajectory Planning and Flutter Stability (D). Shandong University, 2019.
[51] Lei Zhang, Research and implementation of speed-adaptive cubic polynomial look-ahead interpolation algorithm for high-speed CNC machining (D). Chang'an University, 2017.
[52] Nian Liu, Research and implementation of real-time NURBS curve interpolation algorithm for embedded CNC system (D). Nanjing University of Aeronautics and Astronautics,2016.
[53] Nian Liu, Dongbiao Zhao, Jianbo Yang, NURBS curve interpolation algorithm based on Steffensen iteration method (J). Machinery and Electronics, 2016,34(04):3-7.
[54] Dong Wei, Shuyou Zhang, Xiaojian Liu, High-precision low-velocity fluctuation interpolation algorithm for non-uniform rational B-spline curves (J). Journal of Zhejiang University (Engineering Science), 2016,50(11):2215-2223.
[55] Xia Qin, Dezhao Li, Hua Deng, Research on Position Interpolation Algorithm of Industrial Robot Based on NURBS Curve (J).

Manufacturing Automation, 2018,40(04):67-72.

[56] Peng Peng, Yi Hu, Dong Yu, Zhiyin Chen, Xiaohu Lu, Design and optimization of adaptive speed look-ahead algorithm based on IQMath (J). Combined Machine Tool and Automated Machining Technology, 2014(07):49-52.

[57] Peng Peng, Research and Development of Algorithm Optimization Mechanism for High Performance Embedded CNC System (D). Graduate School of Chinese Academy of Sciences (Shenyang Institute of Computing Technology), 2014.

[58] Jianfeng Zhou, Research on the feedrate customization method for five-axis machining of complex surfaces (D). Dalian University of Technology, 2014.

[59] Zhimei Cao, Three-point circle method parameter adaptive interpolation algorithm (J). Mechatronic Engineering Technology,2021,50(05):35-39.

[60] Zhimei Cao, Research on Adaptive Interpolation Algorithm of Non-circular Curve Parameters (J). Manufacturing Automation,2014,36(08):4-8.

[61] Shiming Shi, Research on Path and Trajectory Planning Algorithm for Robot Offline Programming (D). Guangdong University of Technology,2017.

[62] Zhiwei Su, Research on speed planning method of three-axis CNC machining based on SAT identification speed limit interval (D). Huazhong University of Science and Technology, 2018.

[63] Hengjun Liu, Dongbiao Zhao, Zhenying Qin, Research on NURBS curve interpolation algorithm based on Gear prediction correction method (J). Mechanics and Electronics,2017,35(01):3-7.

[64] Zhibing Qu, Design and development of industrial robot embedded controller based on SERCOS III (D). Southeast University, 2016.

[65] Zhiwei Lin, Realization of NURBS curve adaptive interpolation in high-speed CNC system and research on acceleration and deceleration control strategy (D). University of Science and Technology of China, 2014.

[66] Koren, Yoram. “Cross-Coupled Biaxial Computer Control for Manufacturing Systems.” Journal of Dynamic Systems Measurement and Control-transactions of The Asme 102, 1980: 265-272.

[67] Koren, Yoram and C. C. Lo. “Variable-Gain Cross-Coupling Controller for Contouring.” CIRP Annals 40, 1991: 371-374.

[68] Yeh, Syh-Shiuh and Pau-Lo Hsu. “Theory and Applications of the Robust Cross-Coupled Control Design.” Journal of Dynamic Systems Measurement and Control-transactions of The Asme 121, 1999: 524-530.

[69] Chiu, George T.-C. and Masayoshi Tomizuka. “Coordinated Position Control of Multi-Axis Mechanical Systems.” Journal of Dynamic Systems Measurement and Control-transactions of The Asme 120, 1998: 389-393.

[70] Chiu, George T.-C. and Masayoshi Tomizuka. “Contouring control of machine tool feed drive systems: a task coordinate frame approach.” IEEE Trans. Control. Syst. Technol. 9 (2001): 130-139.

[71] Sencer, Burak et al. “Modeling and Control of Contouring Errors for Five-Axis Machine Tools—Part I: Modeling.” Journal of Manufacturing Science and Engineering-transactions of The Asme 131, 2009: 031006.

[72] Zuhao Weng, Research and design of efficient and flexible numerical control algorithm with continuous jerk and error adaptive characteristics (D). Shanghai Jiaotong University,2015.

[73] Yitian Cai, Research on Adaptive Contour Error Estimation and Contour Control Based on Real-time Ethernet (D). Shanghai Jiaotong University, 2016.

[74] Yang Chen, Research on Contour Accuracy Control Method of Numerical Control System (D). Tianjin University, 2016.

[75] Yang Chen, Taiyong Wang, Jingchuan Dong, Bo Li, Zhiqiang Yu, Contour error control based on optimal circle approximation method for contours (J). Computer Engineering and Science,2016,38(06):1281-1285.

[76] Henan Xu, Research on Contour Control Method of CNC Machine Tool Feed Servo System (D). Tianjin University, 2017.

[77] Yiqiao Huang, Efficient detection and comprehensive compensation of geometric errors of five-axis CNC machine tools (D). Shanghai Jiaotong University, 2016.

[78] Zhen Sang, Research on machining path generation method of multi-axis single-channel double-head woodworking CNC machine tool (D). Tianjin University,2017.

[79] Qian Wang, Research on tool radius compensation algorithm of CNC system supporting NURBS interpolation (D). Harbin Institute of Technology, 2017.

[80] Hui Luo, Research on Motion Control Method of Open CNC System (D). Shanghai University of Engineering Technology,2016.

[81] Jipeng Yuan, Research on Adaptive NURBS Curve Fitting and Its Velocity Planning Algorithm (D). Shandong University, 2019.

[82] Shujie Sun, Hu Lin, Liaomo Zheng, A NURBS Curve Prospective Interpolation Algorithm for Backward Interpolation (J). Journal of Computer Aided Design and Graphics,2014,26(09):1543-1549.

[83] Shujie Sun, Hu Lin, Limo Zheng, Feng Liu, NURBS Curve Interpolation Algorithm with Adaptive Computational Complexity (J). Small and Microcomputer Systems, 2014,35(04):895-899.

[84] Liangchen Hu,Wensheng Zhang. NSGA-II approach for proper choice of nodes and knots in B-spline curve interpolation (J). Computer-Aided Design,2020,127:

[85] Wei Ren, Research on Trajectory Planning of Six Degrees of Freedom Industrial Robot (D). Anhui Engineering University,2020.