

# Loading and Unloading Scheduling Problem in a Multiple-Multiple Logistics Network: Modeling and Solving

Yasin Tadayonrad, Alassane Ballé Ndiaye

**Abstract**—Most of the supply chain networks have many nodes starting from the suppliers' side up to the customers' side that each node sends/receives the raw materials/products from/to the other nodes. One of the major concerns in this kind of supply chain network is finding the best schedule for loading/unloading the shipments through the whole network by which all the constraints in the source and destination nodes are met and all the shipments are delivered on time. One of the main constraints in this problem is the loading/unloading capacity in each source/destination node at each time slot (e.g., per week/day/hour). Because of the different characteristics of different products/groups of products, the capacity of each node might differ based on each group of products. In most supply chain networks (especially in the Fast-moving consumer goods (FMCG) industry), there are different planners/planning teams working separately in different nodes to determine the loading/unloading timeslots in source/destination nodes to send/receive the shipments. In this paper, a mathematical problem has been proposed to find the best timeslots for loading/unloading the shipments minimizing the overall delays subject to respecting the capacity of loading/unloading of each node, the required delivery date of each shipment (considering the lead-times), and working-days of each node. This model was implemented on Python and solved using Python-MIP on a sample data set. Finally, the idea of a heuristic algorithm has been proposed as a way of improving the solution method that helps to implement the model on larger data sets in real business cases, including more nodes and shipments.

**Keywords**—Supply chain management, transportation, multiple-multiple network, timeslots management, mathematical modeling, mixed integer programming.

## I. OVERVIEW AND MOTIVATION

ONE of the main concerns in any supply chain network is determining the best date and time to load the shipments on the source nodes (Manufacturing Plants, Central Distribution Centers, etc.; Fig. 1 (a)) to be able to deliver and unload them in the most appropriate date and time in the destination nodes (Regional Distribution Centers). Receiving the shipments in the right time and quantity is as important that Walmart, which is a leading company in FMCG industry, is implementing the “On-time, In-full (OTIF)” project since 2017. One of the major goals of this project is receiving the products at the right time because any late or early delivery imposes out-of-stock or over-stock costs to the logistics system [1]. Generally, this process is done by different planners or different

planning teams through the supply chain network individually and most likely it is done manually. As a result, not only it takes a considerable time from the planners to coordinate with different nodes to arrange the timeslots, but also it may cause some conflicts in the final schedule in the whole network that needs to be revised and rescheduled to achieve a feasible plan for the whole logistics network. Therefore, having an automated model considering all nodes simultaneously and optimizing the schedule plan with respect to the network constraints is an important issue that is worth working on it. We found this problem as a potential area for optimization in the logistics network of one of the major international companies in Europe where the logistics network is a multiple-multiple network with many source nodes (supply nodes) and many destination nodes (demand nodes) through which thousands of shipments need to be scheduled in each period of time. The goal is finding the schedule plan by which all the shipments are delivered by their required delivery dates respect to the constraints related to the capacity of each node and the working calendar of each node. In this problem, the shipments (products) are delivered to the Regional Distribution Centers in two different ways: 1. directly from the Manufacturing Distribution Centers to the Regional Distribution Centers, 2. From the Manufacturing Distribution Centers to the Central Distribution Centers and from there to the Regional Distribution Centers (Figs. 1 (b), (c)). Therefore, Central Distribution Centers are considered as both source and destination nodes in our assumptions.

A simple example for clarifying the problem: If we want to plan for loading and unloading 11 shipments all including a particular group of products (group  $i$ ) when we have 2 source nodes and 2 destination nodes and source 1 and source 2 are capable to load (send) the shipments in the day 1, day 2 and day 3 of the planning period and day 1 and day 2 of the planning period respectively, and destination 1 and destination 2 are capable to unload (receive) the shipments in the day 1, day 2 and day 3 of the planning period and day 1, day 2, day 3 and day 4 of the planning period respectively. Assuming that the maximum loading capacity of each source node is 3 shipments per day, the maximum unloading capacity of each destination node is 2 shipments per day and the lead time between source 1 and destination 1 and 2 is 0 and 1 day, respectively, and the lead time between source 2 and destination 1 and 2 is 1 and 0 day,

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respectively; Fig. 2 shows a feasible schedule plan for this problem. By formulating the capacity constraints as linear constraints and adding another constraint related to the required delivery date, we will have the set of feasible solutions. The

optimum schedule plan could be achieved by applying a linear objective function aims to minimizing the number of delayed shipments.

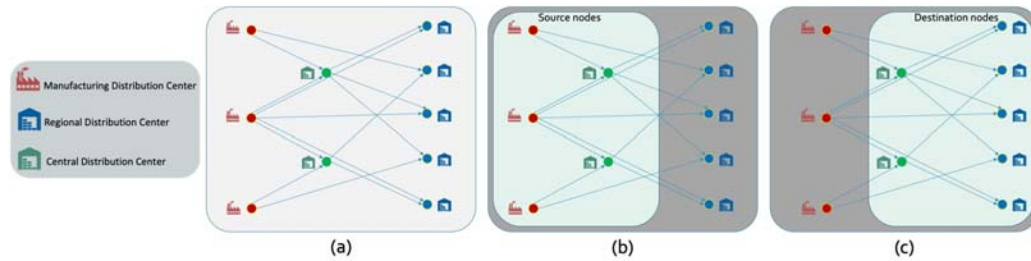


Fig. 1 A general form of a multiple-multiple logistics network with manufacturing/ central/ regional distribution centers

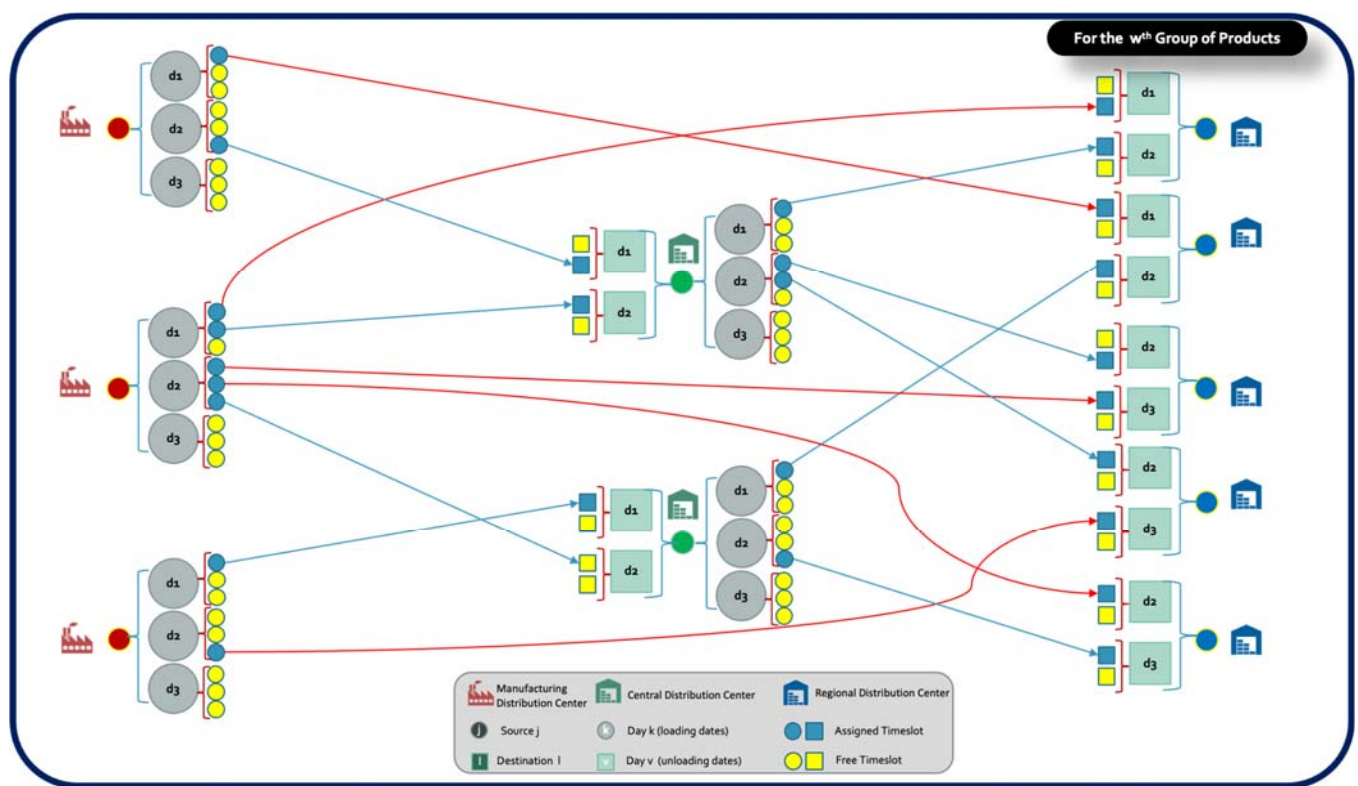


Fig. 2 A general form of assigning shipments (including a particular group of products, e.g., group i) to available loading timeslots in the source nodes and available unloading timeslots in the destination nodes

Due to the importance of transportation and supply chain scheduling, a considerable number of papers have been written in these areas during the recent decades. Matsuo is one of the first authors who introduced a scheduling problem with the weighted total tardiness, shipping times and overtime utilization [2]. He has proposed a heuristic based on a capacitated transshipment and could provide a good trade-off between overtime cost and penalties for late deliveries. Later, Hall et al. extended the scheduling problem with fixed delivery dates various objective functions (e.g., the maximum lateness, the total tardiness, and so on) forming a variety of forms applicable in different industries [3]. Liang implemented fuzzy linear programming to minimize the total production and

transportation costs as well as the total delivery time considering capacity and budget limitation in the destination nodes [4]. Formulating supply chain scheduling and transportation problems as multi-objective problems has been matter of concern in the recent years. Cakici et al. have integrated production and distribution issues as a problem of producing and delivering the products by capacitated vehicles to the customers as a multi objective problem in which the total tardiness and total distribution costs must be minimized [5]. Gupta et al. have combined linear and fractional objective functions to build a multi objective model with capacitated routes using both crisp and fuzzy parameters [6]. In order to minimize several objectives, including the sum of the total

weighted number of tardy jobs, the total due date assignment costs and the total batch delivery costs, Rasti-Barzoki et al. formulated an integrated due date assignment and production and batch delivery scheduling problem using integer programming for a make-to-order system and solved it by heuristic algorithm [7]. They also presented a heuristic algorithm based on branch and bound algorithm to solve an integrated problem of production and scheduling aiming at minimizing the sum of the total weighted number of tardy jobs and the delivery costs [8].

## II. METHODOLOGY, RESULTS, AND MAIN CONTRIBUTIONS

The shipment scheduling problem or timeslot management in a supply chain network could be formulated as a mathematical problem. Therefore, we modeled this problem as a linear problem in which zero-one variables and real variables have been used to formulate the problem.

### A. Model Development

The mentioned problem was first modeled using mathematical programming. To be able to start modeling, the indices were defined in Table I. Then the parameters were defined as in Table II.

$X$  is the set of zero-one variables indicating the assignment of a shipment to be loaded in a timeslot in the source node and to be unloaded in a relevant time slot in the destination node.

We have also defined  $Y$  and  $Y'$  as two groups of non-negative slack variables used to determine the number of shipments exceeding the capacity of each source and destination node.

TABLE I  
DEFINITION OF INDICES

w	Group of products; $w \in W$ .
j	Source site; $j \in S$ .
k	Day of loading products in the source site; $k = 1, 2, \dots, T$ .
l	Destination site; $l \in D$ .
v	Day of unloading products in the destination site; $v = 1, 2, \dots, T$ .
i	Shipment number; $i \in LS$ .

TABLE II  
DEFINITION OF PARAMETERS

$T$	Time horizon (planning period).
$S$	List of source nodes.
$D$	List of destination nodes.
$LS$	List of all shipments.
$C_{jkw}$	Capacity of source $j$ for sending group of products $w$ in day $k$ .
$C'_{lvw}$	Capacity of destination $l$ for unloading group of products $w$ in day $l$ .
$L_{jl}$	Standard lead time between source $j$ and destination $l$ .
$H_{kjl}$	Holidays that should be added the standard lead time duration if a shipment from source $j$ to destination $l$ is sent in day $k$ .
$DL_i$	Deadline (Required Delivery Date) for shipment $i$ .

TABLE III  
DEFINITION OF DECISION VARIABLES

Decision Variables & Slack Variables:	
$x_{i w j k l v}$	$\begin{cases} 1; & \text{If shipment } i \text{ including group of products } w \text{ from source } j \text{ to destination } l \\ & \text{is loaded in day } k \text{ to be unloaded in day } v. \\ 0; & \text{Otherwise.} \end{cases}$
$y_{jkw}$	Slack variable for 2 <sup>nd</sup> constraint, indicating the number of shipments exceeded the capacity of source $j$ in day $k$ for product group $w$ .*
$y'_{lvw}$	Slack variable for 3 <sup>rd</sup> constraint indicating the number of shipments exceeded the capacity of destination $l$ in day $v$ for product group $w$ .

Pre-processing Phase: In order to make sure that we can achieve a feasible solution for the problem, we do a pre-processing to see if we have enough capacity in the source and destination nodes. To do that we calculate the total number of shipments sending from/to source/destination sites and compare to sites' capacities. (Overall capacity of each site based on different groups for next time horizon).

If there are any excess shipments in any node, we must send a change request to that site to increase the capacity or decrease the number of shipments based on the capacity to make sure that we can obtain a feasible solution for the problem.

if  $\sum_i \sum_k \sum_l \sum_v x_{i w j k l v} > \sum_k C_{jkw}$ ; for  $j \in S$  and  $w \in W$ ;  
→ Then send a change request to the relevant source node

if  $\sum_i \sum_j \sum_k \sum_v x_{i w j k l v} > \sum_v C'_{lvw}$ ; for  $l \in D$  and  $w \in W$ ;  
→ Then send a change request to the relevant destination node

\*  $y$  and  $y'$  are nonnegative variables

Objective function and constraints:

- Objective function aims to minimize the number of excess trucks in each site's capacity (1).
- The first constraints represent that all shipments should be sent (2),
- The second constraints show that the total number of trucks sending from each source site should not exceed the capacity of that site (3).
- The third constraints show that the total number of trucks sent to each destination site should not exceed the capacity of that site (4).
- The fourth constraints represent that each shipment should be delivered to its destination before its deadline (5).

Mathematical modeling:

Objective Function

$$\min \sum_f \sum_d \sum_m \sum_n y'_{lvw} + \sum_f \sum_s \sum_i \sum_j y_{jkw} \quad (1)$$

Constraints

$$\sum_l \sum_v \sum_j \sum_k \sum_w x_{iwjklv} = 1; \text{ for } i \quad (2)$$

$$\sum_i \sum_l \sum_v x_{iwjklv} - y_{jkw} \leq C_{jkw}; \text{ for } j, k, w \quad (3)$$

$$\sum_i \sum_j \sum_k x_{iwjklv} - y'_{lvw} \leq C'_{lvw}; \text{ for } l, v, w \quad (4)$$

$$\sum_l \sum_v \sum_j \sum_k \sum_w (x_{iwjklv})(k + L_{jl} + H_{kl}) \leq DL_i; \text{ for } i \quad (5)$$

*B. Solution Methods*

In this paper we have proposed two approaches to tackle with given problem. First, we have used Mixed Integer Programming and solved the problem to obtain the exact solution. Then a heuristic algorithm based on Genetic Algorithm has been proposed to be able to find the solutions on larger data bases for more complex supply chain networks including higher number of shipments with more nodes.

1. Exact Solution: Mixed Integer Programming

In this paper, mathematical modeling has been used to model the mentioned problem. As the objective function and constraints are linear and some of decision variables are zero-one variables, the model has been implemented on Python 3.7 and solved using CBC (Customized Branch & Cut) solver on `python-mip` toolbox.

The proposed mathematical model was solved on a small data set including 40 shipments on a part of a supply chain network with four supply nodes and demand destination nodes using MIP-Python toolbox and CBC solver.

III. CONCLUSION AND FUTURE WORKS

A schedule optimization in any component of supply chain enables us to lower costs and increase service level. In supply chain networks, loading and unloading time slot management is a major problem that needs to be solved as a link between inventory management and transportation. We presented in this article a mathematical model to address this issue by considering multiple sources and destinations with limited capacity to load and unload shipments. Furthermore, we investigated the possibility of having different product groups as well as various working calendars in different nodes through a supply chain network. For future studies, integrated inventory-transportation models could be developed to optimize the products and quantities of each product in each shipment and loading and unloading schedule simultaneously.

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