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Statically Fused Unbiased Converted Measurements Kalman Filter

Zhengkun Guo, Yanbin Li, Wenqing Wang, Bo Zou

Abstract—Active radar and sonar systems often report Doppler measurements in addition to the position measurements such as range and bearing. The tracker can perform better by making full use of the Doppler measurements. However, due to the high nonlinearity of the Doppler measurements with respect to the target state in the Cartesian coordinate systems, those measurements are not always fully exploited. This paper mainly focuses on dealing with the Doppler measurements as well as the position measurements in Polar coordinates. The Statically Fused Converted Position and Doppler Measurements Kalman Filter (SF-CMKF) with additive debiased measurement conversion has been presented. However, the exact compensation for the bias of the measurement conversion are multiplicative and depend on the statistics of the cosine of the angle measurement errors. As a result, the consistency and performance of the SF-CMKF may be suboptimal in the large angle error situations. In this paper, the multiplicative unbiased position and Doppler measurement conversion for two-dimensional (Polar-to-Cartesian) tracking are derived, and the SF-CMKF is improved by using those conversion. Monte Carlo simulations are presented to demonstrate the statistic consistency of the multiplicative unbiased conversion and the superior performance of the modified SF-CMKF (SF-UCMKF).

Keywords—Measurement conversion, Doppler, Kalman filter, estimation, tracking.

I. Introduction

In practical active radar and sonar tracking systems, measurements often include not only target position such as range and bearing, but also Doppler velocity. The Doppler measurements directly contain the velocity information of the target, the tracking performance can be greatly improved by sufficient utilization of the Doppler measurements. However, unlike the position measurements, the Doppler measurements have a high nonlinearity relationship with the target Cartesian state, there are many difficulties in extracting Cartesian state from Doppler measurements.

The Extended Kalman filter (EKF) [16], [13], [7] and Unscented Kalman filter (UKF) [8], [10], [4] are popular nonlinear filtering approaches for tracking, however, due to the performance deterioration for high nonlinearity of EKF and large calculation load of UKF, they are not applicable for some practical tracking systems. An alternative method, converted position measurements Kalman filter (CPMKF), which converts the measurements into pseudo linear form in Cartesian Coordinates and then utilizes Kalman filter to get the estimation of target state, is proposed in [11]. But the Doppler measurement, which contains information about target velocity to potentially improve the tracking performance, is not used in CPMKF.

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In order to use Doppler measurement better in tracking filter, the converted Doppler measurement Kalman filter (CDMKF), which uses a linear Kalman filter, instead of nonlinear filters, to estimate pseudo states from the converted Doppler measurements, is introduced [17]. Properly fusing the outputs of the CDMKF and CPMKF results in the statically fused converted measurement Kalman filter (SF-CMKF), which produces better performance due to the shifting of the nonlinear approximations outside the dynamic filtering recursions.

However, the additive debiased measurement conversion [11] is used in SF-CMKF as a fact that the exact compensation for the bias in the Polar-to-Cartesian and Spherical-to-Cartesian conversion are multiplicative and determined by the statistics of the cosine of the angle measurement errors [15]. When the azimuth measurement errors become larger, the consistency and performance of SF-CMKF may deteriorate.

The multiplicative unbiased method is proposed by [15], [5] solves a compatibility problem in the derivation of the mean and covariance of the converted measurement errors in [15], and the converted position measurement errors have been explored in [5], [1], [6], [9], [14]. However, the converted Doppler measurement is not considered in them. In this paper, the multiplicative unbiased converted Doppler measurement and the covariance between multiplicative unbiased converted position and Doppler measurement for 2D (Polar-to-Cartesian) tracking are derived. Then the SF-CMKF is modified by using the multiplicative unbiased measurement conversion derived above. Monte Carlo simulation results illustrate the better statistic consistency of the multiplicative unbiased conversion and better performance of the proposed SF-UCMKF.

The rest of this paper is organized as follows. The problem formulation for 2D tracking is presented in Section II. The multiplicative unbiased conversion is derived in Section III. In Section IV, the SF-UCMKF is developed. Monte Carlo simulations are presented in Section V, followed by conclusions in Section VI.

II. Problem Formulation

The 2D tracking system is modeled in standard discrete-time state-space form as

$$\mathbf{x}_{k+1} = \mathbf{\Phi}\mathbf{x}_k + \mathbf{\Gamma}\mathbf{v}_k \tag{1}$$

$$\begin{aligned} \mathbf{z}_k &= [r_k^m, \theta_k^m, \dot{r}_k^m]' = \mathbf{f}(\mathbf{x}_k) + \mathbf{w}_k \\ &= [r_k, \theta_k, \dot{r}_k]' + \mathbf{w}_k \end{aligned} \tag{2}$$

where

$$\begin{cases} r_k = \sqrt{x_k^2 + y_k^2} \\ \theta_k = \tan^{-1}(y_k/x_k) \\ \dot{r}_k = (x_k\dot{x}_k + y_k\dot{y}_k)/\sqrt{x_k^2+y_k^2} \\ \mathbf{w}_k = [\tilde{r}_k, \tilde{\theta}_k, \tilde{\dot{r}}_k]' \end{cases} \tag{3}$$

$\mathbf{x}_k = [x_k, y_k, \dot{x}_k, \dot{y}_k]'$ is the state vector consisting of position and velocity along x and y directions in Cartesian coordinates. $\mathbf{z}_k = [r_k^m, \theta_k^m, \dot{r}_k^m]'$ is the measurement vector consisting of range, azimuth and Doppler in polar coordinates. $\mathbf{\Phi}$ and $\mathbf{\Gamma}$ are the respective constant transition matrices [2]. $\mathbf{v}_k$ and $\mathbf{w}_k$ are mutually independent zero-mean Gaussian random process noise and measurement noise with known covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$, respectively. $\tilde{r}_k$, $\tilde{\theta}_k$ and $\tilde{\dot{r}}_k$ are the corresponding measurement noises, which are all assumed to be zero-mean Gaussian noise with known variances σ_r^2, σ_θ^2 and $\sigma_{\dot{r}}^2$. It's assumed that the measurement noises are mutually independent with the exception that $\tilde{r}_k$ and $\tilde{\dot{r}}_k$ are statistically correlated [3] with correlation coefficient ρ, i.e.,

$$\mathrm{cov}(\tilde{r}_k, \tilde{\dot{r}}_k) = \rho\sigma_r\sigma_{\dot{r}} \tag{4}$$

The covariance matrix of the original measurements can be written as

$$\mathbf{R}_k = \begin{bmatrix} \sigma_r^2 & 0 & \rho\sigma_r\sigma_{\dot{r}} \\ 0 & \sigma_\theta^2 & 0 \\ \rho\sigma_r\sigma_{\dot{r}} & 0 & \sigma_{\dot{r}}^2 \end{bmatrix} \tag{5}$$

III. Multiplicative Unbiased Conversion

The multiplicative unbiased conversion of position measurements can be given [15] as

$$\begin{bmatrix} x_k^c \\ y_k^c \end{bmatrix} = \begin{bmatrix} e^{\sigma_\theta^2/2} r_k^m \cos\theta_k^m \\ e^{\sigma_\theta^2/2} r_k^m \sin\theta_k^m \end{bmatrix} = \begin{bmatrix} x_k + \tilde{x}_k \\ y_k + \tilde{y}_k \end{bmatrix} \tag{6}$$

where $\tilde{x}_k$ and $\tilde{y}_k$ are the converted position measurement errors along x and y directions in Cartesian coordinates.

We denote the mean and covariance of the converted position measurement errors as

$$\boldsymbol{\mu}_k^p = [\mu_k^x, \mu_k^y]' \tag{7}$$

and

$$\mathbf{R}_k^p = \begin{bmatrix} R_k^{xx} & R_k^{xy} \\ R_k^{yx} & R_k^{yy} \end{bmatrix} \tag{8}$$

respectively. Then the unbiased converted position measurement for tracking can be obtained as

$$\mathbf{z}_k^{pc} = \begin{bmatrix} x_k^c \\ y_k^c \end{bmatrix} - \boldsymbol{\mu}_k^p \tag{9}$$

The expressions of the elements in (7) and (8) are given [5] as

$$\begin{cases} \mu_k^x = (e^{\sigma_\theta^2/2} - e^{-\sigma_\theta^2/2}) r_k^m \cos\theta_k^m \\ \mu_k^y = (e^{\sigma_\theta^2/2} - e^{-\sigma_\theta^2/2}) r_k^m \sin\theta_k^m \end{cases} \tag{10}$$

$$\begin{cases} R_k^{xx} = -\, e^{-\sigma_\theta^2}(r_k^m)^2\cos^2\theta_k^m \\ \qquad + ((r_k^m)^2 + \sigma_r^2)(1 + e^{-2\sigma_\theta^2}\cos(2\theta_k^m))/2 \\ R_k^{yy} = -\, e^{-\sigma_\theta^2}(r_k^m)^2\sin^2\theta_k^m \\ \qquad + ((r_m^k)^2 + \sigma_r^2)(1 - e^{-2\sigma_\theta^2}\cos(2\theta_k^m))/2 \\ R_k^{xy} = R_k^{yx} = -e^{-\sigma_\theta^2}(r_k^m)^2\sin\theta_k^m\cos\theta_k^m \\ \qquad + ((r_k^m)^2 + \sigma_r^2)e^{-2\sigma_\theta^2}\sin(2\theta_k^m)/2 \end{cases} \tag{11}$$

With the similar method in [5], the multiplicative unbiased conversion of Doppler measurement can be obtained as

$$\eta_k^c = r_k^m \dot{r}_k^m = \eta_k + \tilde{\eta}_k \tag{12}$$

where η_k is the converted Doppler [17], given by

$$\eta_k = x_k\dot{x}_k + y_k\dot{y}_k \tag{13}$$

and $\tilde{\eta}_k$ is the error in the converted Doppler measurement η_k^c.

Similarly, the mean and variance of the converted Doppler measurement error can be obtained as [17]

$$\mu_k^\eta = \rho\sigma_r\sigma_{\dot{r}} \tag{14}$$

and

$$\begin{aligned} R_k^{\eta\eta} = & (r_k^m)^2\sigma_{\dot{r}}^2 + \sigma_r^2(\dot{r}_k^m)^2 \\ & + 3(1+\rho^2)\sigma_r^2\sigma_{\dot{r}}^2 + 2r_k^m\dot{r}_k^m\rho\sigma_r\sigma_{\dot{r}} \end{aligned} \tag{15}$$

respectively. Then the unbiased converted Doppler measurements for tracking can be obtained as

$$z_k^{\eta c} = \eta_k^c - \mu_k^\eta \tag{16}$$

The measurement noises $\tilde{r}_k$, $\tilde{\theta}_k$ and $\tilde{\dot{r}}_k$ in polar coordinates are assumed to be mutually independent zero-mean Gaussian noised with known variances σ_r^2, σ_θ^2 and $\sigma_{\dot{r}}^2$ with the exception that $\tilde{r}_k$ and $\tilde{\dot{r}}_k$ are correlated with correlation coefficient ρ, in this case

$$\begin{cases} E[\tilde{r}_k^2] = \sigma_r^2, \quad E[\tilde{\dot{r}}_k^2] = \sigma_{\dot{r}}^2 \\ E[\tilde{r}_k\tilde{\dot{r}}_k] = \rho\sigma_r\sigma_{\dot{r}}, \quad E[\tilde{r}_k^2\tilde{\dot{r}}_k] = 0, \quad E[\tilde{r}_k\tilde{\dot{r}}_k^2] = 0 \\ E[\sin\tilde{\theta}_k] = 0, \quad E[\cos\tilde{\theta}_k] = e^{-\sigma_\theta^2/2} \end{cases} \tag{17}$$

Then the measurement-conditioned covariance between the multiplicative unbiased converted position and Doppler measurement can be derived as

$$\begin{aligned} R_k^{x\eta} &= E[(\tilde{x}_k - \mu_k^x)(\tilde{\eta}_k - \mu_k^\eta)|r_k^m, \theta_k^m] \\ &= E\{[e^{\frac{\sigma_\theta^2}{2}} r_k^m\cos\theta_k^m - (r_k^m - \tilde{r}_k)\cos(\theta_k^m - \tilde{\theta}_k) \\ &\quad - (e^{\frac{\sigma_\theta^2}{2}} - e^{-\frac{\sigma_\theta^2}{2}})r_k^m\cos\theta_k^m] \\ &\quad [r_k^m\dot{r}_k^m - (r_k^m - \tilde{r}_k)(\dot{r}_k^m - \tilde{\dot{r}}_k) - \rho\sigma_r\sigma_{\dot{r}}]\} \\ &= e^{-\frac{\sigma_\theta^2}{2}}(r_k^m\cos\theta_k^m\rho\sigma_r\sigma_{\dot{r}} + \dot{r}_k^m\cos\theta_k^m\sigma_r^2) \end{aligned} \tag{18}$$

$$\begin{aligned} R_k^{y\eta} &= E[(\tilde{y}_k - \mu_k^y)(\tilde{\eta}_k - \mu_k^\eta)|r_k^m, \theta_k^m] \\ &= E\{[e^{\frac{\sigma_\theta^2}{2}} r_k^m\sin\theta_k^m - (r_k^m - \tilde{r}_k)\sin(\theta_k^m - \tilde{\theta}_k) \\ &\quad - (e^{\frac{\sigma_\theta^2}{2}} - e^{-\frac{\sigma_\theta^2}{2}})r_k^m\sin\theta_k^m] \\ &\quad [r_k^m\dot{r}_k^m - (r_k^m - \tilde{r}_k)(\dot{r}_k^m - \tilde{\dot{r}}_k) - \rho\sigma_r\sigma_{\dot{r}}]\} \\ &= e^{-\frac{\sigma_\theta^2}{2}}(r_k^m\sin\theta_k^m\rho\sigma_r\sigma_{\dot{r}} + \dot{r}_k^m\sin\theta_k^m\sigma_r^2) \end{aligned} \tag{19}$$

The covariance derived above can be rewritten as

$$\begin{aligned}\mathbf{R}_k^{p\eta} &= \begin{bmatrix} R_k^{x\eta} \\ R_k^{y\eta} \end{bmatrix} \\ &= e^{-\sigma_\theta^2/2}(r_k^m \rho \sigma_r \sigma_{\dot{r}} + \dot{r}_k^m \sigma_r^2) \begin{bmatrix} \cos\theta_k^m \\ \sin\theta_k^m \end{bmatrix}\end{aligned} \tag{20}$$

IV. SF-UCMKF Development

The SF-CMKF is modified by replacing the additive debiased conversion [17] of measurements with the multiplicative unbiased conversion of measurements derived in Section III. Fig. 1 illustrates the structure of the SF-UCMKF.

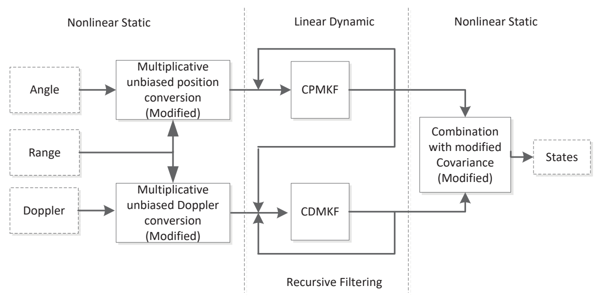

Fig. 1 Filtering structure of the SF-UCMKF

The original sensor measurements (i.e., range, Doppler and angle) are divided into two parts to be processed separately by two linear filters first. The range and Doppler measurements are transformed to the unbiased converted Doppler measurements by Doppler conversion and the CDMKF is used to estimate pseudo-states. Correspondingly, the unbiased position conversion and the CPMKF are designed to extract target Cartesian states from the range and angle measurements. Meanwhile, the Cartesian states from the CPMKF are used by the CDMKF. The estimated pseudo-states and Cartesian states are combined to fuse the final target state estimates.

The CPMKF and CDMKF are given for derivation of the SF-UCMKF as follows.

The state equation and measurement equation of CPMKF can be expressed as [17]

$$\mathbf{x}_{k+1}^p = \mathbf{\Phi}^p \mathbf{x}_k^p + \mathbf{\Gamma}^p \mathbf{v}_k \tag{21}$$

$$\mathbf{z}_k^{pc} = \mathbf{H}^p \mathbf{x}_k^p + \mathbf{w}_k^p \tag{22}$$

where (for Constant velocity model CV [12] in this paper)

$$\mathbf{\Phi}^p = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{\Gamma}^p = \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix} \tag{23}$$

$$\mathbf{v}_k = \begin{bmatrix} v_k^x \\ v_k^y \end{bmatrix}, \mathbf{H}^p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{24}$$

The formulation of the CPMKF is well known, for simplicity, the CPMKF filtering procedure is written as

$$\begin{aligned}&[\hat{\mathbf{x}}_{k+1|k+1}^p, \mathbf{P}_{k+1|k+1}^p, \mathbf{K}_{k+1}^p] = \\ &\quad \mathrm{KF}^p(\mathbf{z}_{k+1}^{pc}, \hat{\mathbf{x}}_{k|k}^p, \mathbf{P}_{k|k}^p, \mathbf{\Phi}^p, \mathbf{\Gamma}^p, \mathbf{Q}_k, \mathbf{H}^p, \mathbf{R}_{k+1}^p)\end{aligned} \tag{25}$$

where $\hat{\mathbf{x}}_{k+1|k+1}^p$, $\mathbf{P}_{k+1|k+1}^p$, $\mathbf{K}_{k+1}^p$, $\mathbf{\Phi}^p$, $\mathbf{\Gamma}^p$, $\mathbf{Q}_k$ and $\mathbf{H}^p$ stand for the estimated state, covariance of the estimates, filtering gain, state transition matrix, noise input matrix, covariance of process noises, and measurement matrix, respectively. The same definitions as in Section II apply. The superscript p here indicates that the variables or matrixes are related to the CPMKF.

The state equation and measurement equation of CDMKF can be expressed as [17]

$$\boldsymbol{\eta}_{k+1} = \mathbf{\Phi}^\eta \boldsymbol{\eta}_k + \mathbf{G}\mathbf{u}_k + \mathbf{\Gamma}^x \mathbf{v}_k^x + \mathbf{\Gamma}^s \mathbf{v}_k^s \tag{26}$$

$$z_k^{\eta c} = \mathbf{H}^\eta \boldsymbol{\eta}_k + w_k^\eta \tag{27}$$

where

$$\boldsymbol{\eta}_k = \begin{bmatrix} \eta_k \\ \dot{\eta}_k \end{bmatrix}, \mathbf{G} = \mathbf{\Gamma}^s = \begin{bmatrix} T^3/2 & T^3/2 \\ T^2 & T^2 \end{bmatrix} \tag{28}$$

$$\mathbf{\Phi}^\eta = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \mathbf{u}_k = E\begin{bmatrix} (v_k^x)^2 \\ (v_k^y)^2 \end{bmatrix} = \begin{bmatrix} q \\ q \end{bmatrix} \tag{29}$$

$$\mathbf{\Gamma}^x = \begin{bmatrix} T & 3T^2/2 \\ 0 & 2T \end{bmatrix}, \mathbf{v}_k^s = \begin{bmatrix} (v_k^x)^2 - q \\ (v_k^y)^2 - q \end{bmatrix} \tag{30}$$

$$\mathbf{v}_k^x = \mathbf{x}_\Gamma \mathbf{v}_k = \begin{bmatrix} x_k & y_k \\ \dot{x}_k & \dot{y}_k \end{bmatrix} \begin{bmatrix} v_k^x \\ v_k^y \end{bmatrix} \tag{31}$$

In the above, $\mathbf{H}^\eta = [1\ 0]$ is the measurement matrix, $z_k^{\eta c}$ is the unbiased converted Doppler measurement as described in (16), w_k^η is zero-mean Gaussian measurement noise with known variance $R_k^{\eta\eta}$ given in (15).

The conditions of zero-mean and whiteness of the noises in (30) and (31) are approximately satisfied. The covariance of $\mathbf{v}_k^x$ can be obtained [17] as

$$\begin{aligned}\mathbf{Q}_k^x = q&\left\{\begin{bmatrix} \hat{x}_k^2 & \hat{x}_k\hat{\dot{x}}_k \\ \hat{\dot{x}}_k\hat{x}_k & \hat{\dot{x}}_k^2 \end{bmatrix} + \begin{bmatrix} \hat{y}_k^2 & \hat{y}_k\hat{\dot{y}}_k \\ \hat{\dot{y}}_k\hat{y}_k & \hat{\dot{y}}_k^2 \end{bmatrix}\right\} \\ -q&\left\{\begin{bmatrix} P_k^{xx} & P_k^{x\dot{x}} \\ P_k^{\dot{x}x} & P_k^{\dot{x}\dot{x}} \end{bmatrix} + \begin{bmatrix} P_k^{yy} & P_k^{y\dot{y}} \\ P_k^{\dot{y}y} & P_k^{\dot{y}\dot{y}} \end{bmatrix}\right\}\end{aligned} \tag{32}$$

The matrix elements in (32) are the corresponding elements of the Cartesian state estimation from the CPMKF at time k. In other words, $\hat{\mathbf{x}}_{k|k}^p$, $\mathbf{P}_{k|k}^p$ is required to calculate $\mathbf{Q}_k^x$. The process of CPMKF is given before.

The covariance of $\mathbf{v}_k^s$ is

$$\mathbf{Q}_k^s = \begin{bmatrix} 2q^2 & 0 \\ 0 & 2q^2 \end{bmatrix} \tag{33}$$

The implementation of CDMKF is carried out as follows

$$\mathbf{M}_{k+1}^\eta = \mathbf{\Phi}^\eta \mathbf{P}_k^\eta (\mathbf{\Phi}^\eta)' + \mathbf{\Gamma}^x \mathbf{Q}_k^x \mathbf{\Gamma}^{x'} + \mathbf{\Gamma}^s \mathbf{Q}_k^s \mathbf{\Gamma}^{s'} \tag{34}$$

$$\mathbf{K}_{k+1}^\eta = \mathbf{M}_{k+1}^\eta \mathbf{H}^{\eta'} [\mathbf{H}^\eta \mathbf{M}_{k+1}^\eta \mathbf{H}^{\eta'} + R_{k+1}^{\eta\eta}]^{-1} \tag{35}$$

$$\begin{aligned}\hat{\boldsymbol{\eta}}_{k+1} &= \mathbf{\Phi}^\eta \hat{\boldsymbol{\eta}}_k + \mathbf{G}\mathbf{u}_k \\ &\quad + \mathbf{K}_{k+1}^\eta [z_{k+1}^{\eta c} - \mathbf{H}^\eta(\mathbf{\Phi}^\eta \hat{\boldsymbol{\eta}}_k + \mathbf{G}\mathbf{u}_k)]\end{aligned} \tag{36}$$

$$\mathbf{P}_{k+1}^\eta = [\mathbf{I} - \mathbf{K}_{k+1}^\eta \mathbf{H}^\eta] \mathbf{M}_{k+1}^\eta \tag{37}$$

With the pseudo-state $\hat{\boldsymbol{\eta}}_{k+1}$ from the CDMKF and the Cartesian state $\hat{\mathbf{x}}_{k+1}^p$ from the CPMKF, the final state estimations can be derived under the MMSE criterion as follows.

From the formulations of the CDMKF and CPMKF presented above, the cross-covariance between the estimated pseudo state from the CDMKF and the Cartesian state from the CPMKF can be derived as

$$\begin{aligned}\mathbf{P}_{k+1}^{p\eta} &= [\mathbf{I}-\mathbf{K}_{k+1}^{p}\mathbf{H}^{p}]\mathbf{\Phi}^{p}\mathbf{P}_{k}^{p\eta}\mathbf{\Phi}^{\eta'}(\mathbf{I}-\mathbf{K}_{k+1}^{\eta}\mathbf{H}^{\eta})' \\ &+ [\mathbf{I}-\mathbf{K}_{k+1}^{p}\mathbf{H}^{p}]\mathbf{\Gamma}^{p}\mathbf{Q}_{k}(\mathbf{\Gamma}^{x}\mathbf{x}_{\Gamma})'[\mathbf{I}-\mathbf{K}_{k+1}^{\eta}\mathbf{H}^{\eta}]' \\ &+ \mathbf{K}_{k+1}^{p}\mathbf{R}_{k+1}^{p\eta}\mathbf{K}_{k+1}^{\eta}{}'\end{aligned} \quad (38)$$

Given the state estimate of the CPMKF, the prior mean of the state to be estimated is

$$\bar{\mathbf{x}} \rightarrow \hat{\mathbf{x}}_{k+1}^{p} = E[\mathbf{x}_{k+1}|\hat{x}_{k+1}^{p}] \quad (39)$$

A "Measurement"

$$\mathbf{z}_{k+1} \rightarrow \hat{\boldsymbol{\eta}}_{k+1} = \boldsymbol{\eta}_{k+1} - \tilde{\boldsymbol{\eta}}_{k+1} \quad (40)$$

is made to update the state of interest.

The prior mean of measurement is given as

$$\bar{\mathbf{z}}_{k+1} = \bar{\boldsymbol{\eta}}_{k+1} = \mathbf{C}(\hat{\mathbf{x}}_{k}^{p}) + \frac{1}{2}\sum_{i=1}^{2}\mathbf{e}_{i}\mathrm{tr}(\ddot{\mathbf{C}}^{i}\mathbf{P}_{k}^{p}) \quad (41)$$

where the function $\mathbf{C}$ is defined as

$$\mathbf{C}[\mathbf{x}_k] = \begin{bmatrix} \eta_k \\ \dot{\eta}_k \end{bmatrix} = \begin{bmatrix} x_k\dot{x}_k + y_k\dot{y}_k \\ \dot{x}_k^2 + \dot{y}_k^2 \end{bmatrix} \quad (42)$$

and tr denotes the trace of matrix.

The covariance between the states to be estimated and the "measurement" can be got as

$$\mathbf{P}_{k}^{\mathbf{xz}} = \mathbf{P}_{k}^{p}\dot{\mathbf{C}}' - \mathbf{P}_{k}^{p\eta} \quad (43)$$

The covariance of the "measurement" is

$$\begin{aligned}\mathbf{P}_{k}^{\mathbf{zz}} =& \dot{\mathbf{C}}\mathbf{P}_{k}^{p}\dot{\mathbf{C}}' + \mathbf{P}_{k}^{\eta} + \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\mathbf{e}_{i}\mathbf{e}_{j}'\mathrm{tr}(\ddot{\mathbf{C}}^{i}\mathbf{P}_{k}^{p}\ddot{\mathbf{C}}^{j}\mathbf{P}_{k}^{p}) \\ &- \dot{\mathbf{C}}\mathbf{P}_{k}^{p\eta} - (\dot{\mathbf{C}}\mathbf{P}_{k}^{p\eta})'\end{aligned} \quad (44)$$

In the above, the ith Cartesian basis vector $\mathbf{e}_i$ is

$$\mathbf{e}_1 = [\ 1 \quad 0\ ]', \mathbf{e}_2 = [\ 0 \quad 1\ ]' \quad (45)$$

Jacobian of $\mathbf{C}$ is

$$\dot{\mathbf{C}} = \begin{bmatrix} \dot{x}_k & \dot{y}_k & x_k & y_k \\ 0 & 0 & 2\dot{x}_k & 2\dot{y}_k \end{bmatrix} \quad (46)$$

Hessian for the first element and the second element are

$$\ddot{\mathbf{C}}^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ddot{\mathbf{C}}^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad (47)$$

Then, the final state is obtained as

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k}^{p} + \mathbf{P}_{k}^{\mathbf{xz}}(\mathbf{P}_{k}^{\mathbf{xz}})^{-1}(\hat{\boldsymbol{\eta}}_k - \bar{\mathbf{z}}_k) \quad (48)$$

The covariance associated with this combined estimate is

$$\mathbf{P}_k = \mathbf{P}_{k}^{p} - \mathbf{P}_{k}^{\mathbf{xz}}(\mathbf{P}_{k}^{\mathbf{zz}})^{-1}(\mathbf{P}_{k}^{\mathbf{xz}})' \quad (49)$$

For details of the derivations of the combination presented above, see [17].

V. Simulation Results

In this section the statistical consistency of the unbiased conversion given in Section III and the performance of SF-UCMKF derived in Section IV are tested.

A. Statistical Consistency Comparison

The simulation here is similar to that in [8]. The true target state is at 10 km with azimuth $45°$ and Doppler 20 m/s in the plane. A 2D radar located at the origin measures the range, azimuth and Doppler of the target. The range measurement error is a zero-mean Gaussian noise with standard deviation 100 m. The azimuth measurement error is also a zero-mean Gaussian noise with varied standard deviation between $[0° - 30°]$, and is independent of the range measurement error. The Doppler measurement error is a zero-mean Gaussian noise as well with standard deviation 3 m/s, and is correlated with the range measurement error with correlation coefficient $\rho = -0.9$. The average normalized conversion error square (ANCES) of the additive debiased conversion and the multiplicative unbiased conversion over 5000 Monte Carlo runs are shown in Fig. 2. The ANCES is defined as

$$\mathrm{ANCES} = \frac{1}{Nn}\sum_{i=1}^{N}(\mathbf{x}_i - \hat{\mathbf{x}}_i)\mathbf{P}_i^{-1}(\mathbf{x}_i - \hat{\mathbf{x}}_i) \quad (50)$$

where $(\mathbf{x}_i - \hat{\mathbf{x}}_i)$ and $\mathbf{P}_i$ are the state conversion error and error covariance in the ith run, n is the state dimension, and N is the total number of runs. If the conversion error and the conversion variance match each other, the ANCES would be close to 1.

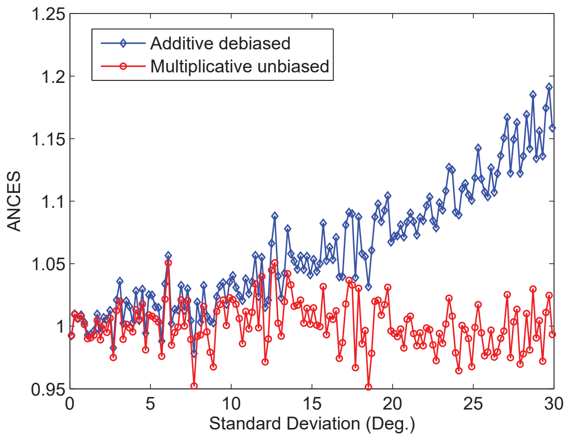

Fig. 2 ANCES comparison of unbiased conversion and debiased conversion

It can be seen from Fig. 2 that both the multiplicative unbiased conversion and additive debiased conversion perform well for practical noise levels, but the debiased conversion becomes inconsistent for very large noise levels owing to a small bias in additive converted measurements.

B. Performance Comparison in Tracking

To compare the performance of SF-UCMKF with that of SF-CMKF in tracking, a CV model in the polar coordinate with different azimuth measurement errors is considered.

The radar locates at the origin, which measures the range, azimuth and Doppler at a sampling period $T = 1$ s. The trajectory starts at position $(10\text{ km}, 10\text{ km})$ with an initial speed $(20\text{ m/s}, 20\text{ m/s})$. The acceleration disturbances along x and y directions are both independent zero-mean Gaussian noises with a common standard deviation $q = 0.01\text{ m/s}^2$. The range and Doppler measurement errors are correlated zero-mean Gaussian noised with standard deviations $\sigma_r = 100$ m, $\sigma_{\dot{r}} = 10$ m/s and correlation coefficient $\rho = -0.9$. The azimuth measurement error is also zero-mean Gaussian noise and the standard deviation varies in two cases. Simulations are performed over 100 time steps with 250 Monte Carlo runs.

In order to distinguish the performances, the relative Root Mean Squared Error (RMSE)

$$\frac{\text{RMSE of the tracking filter}}{\text{standard deviation given by the PCRLB}} \times 100\%,$$

is used as a metric to compare the performances of the tracking filters. The relative RMSE of position and velocity with varied azimuth measurement error standard deviations is illustrated as follows. The log scale is used for the vertical axis to present a clear comparison.

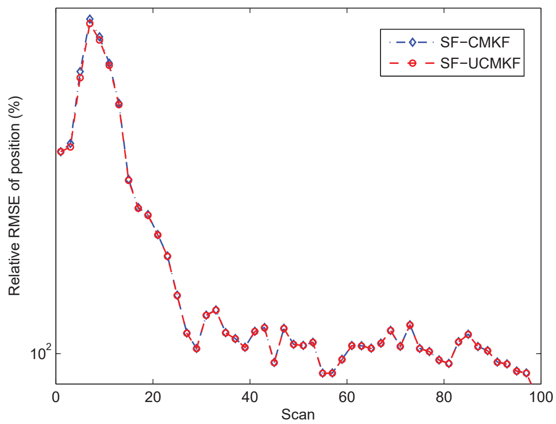

Fig. 3 Relative RMSE of position with $\sigma_\theta = 0.8°$

From the above simulation results, it can be seen that SF-UCMKF has smaller RMSE of position and velocity in both different azimuth measurement noise conditions. As the noise becomes larger, the relative RMSE between SF-UCMKF and SF-CMKF goes larger as well, and the effectiveness of SF-UCMKF is more obvious. It means that SF-UCMKF which uses the unbiased converted measurements in this paper is more accurate than SF-CMKF in [17]. In fact, the difference between the multiplicative unbiased conversion in this paper and the additive debiased conversion in [17] is small when σ_θ is not very large. The additive debiased converted measurements are slightly biased but acceptable for practical noise levels. As σ_θ becomes larger, the additive debiased

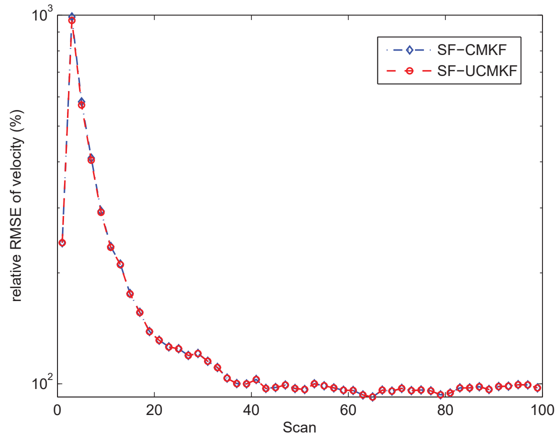

Fig. 4 Relative RMSE of velocity with $\sigma_\theta = 0.8°$

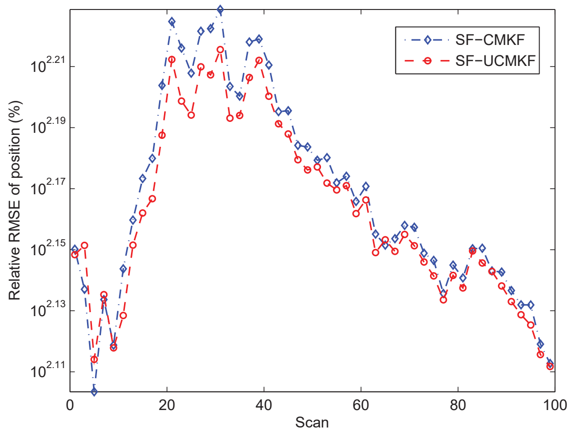

Fig. 5 Relative RMSE of position with $\sigma_\theta = 2.5°$

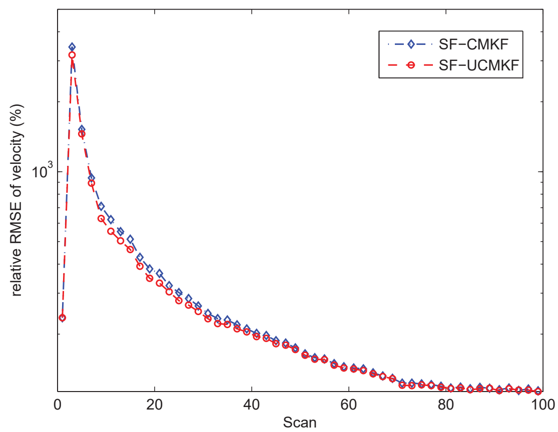

Fig. 6 Relative RMSE of velocity with $\sigma_\theta = 2.5°$

conversion goes to be more biased, which is revealed by the suboptimal performance of SF-CMKF.

VI. Conclusion

The multiplicative unbiased converted Doppler measurement and the covariance between multiplicative unbiased converted position and Doppler measurement are derived in this paper. The unbiased conversion and the convariance show good statistic consistency and robustness. A filter (SF-UCMKF) which uses the unbiased converted measurements has been proposed and has better performance than SF-CMKF for tracking, since it uses the measurement conversion in a more exact way.

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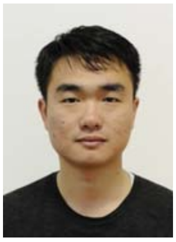

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