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Effect of the Tidal Charge Parameter on Temperature Anisotropies of the Cosmic Microwave Background Radiation

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Abstract—We present the calculations of the temperature anisotropy of the cosmic microwave background radiation (CMBR) caused by an inhomogeneous region (the clump) within the Friedmann-Lemaitre-Robertson-Walker (FLRW) model of the Universe build in the framework of the Randall-Sundrum one brane model. We present two spherically symmetrical and statical models of the clump, the braneworld Reissner-Nordstrom black hole (bRNBH) and the perfect fluid sphere of uniform density matched to the FLRW spacetime via an external bRNBH. The boundary of the vacuum region expands, which induces an additional frequency shift to a photon of the CMBR passing through this inhomogeneity in comparison to the case of a photon propagating through a pure FLRW spacetime. This frequency shift is associated with an effective change of temperature of the CMBR in the corresponding direction. We give estimates on the changes of the effective temperature of the CMBR's photon with the change of parameters describing the brane and the induced tidal forces from the bulk.

Keywords—Braneworld, CMBR, Randall-Sundrum model, Rees-Sciama effect, Reissner-Nordstrom black hole.

I. Introduction

THE primary motivation for the construction of the Universe being represented by a $3 + 1$ dimensional braneworld embedded in a 5D bulk has been proposed in order to resolve the hierarchy problem. Particles of matter and fields are locked on this brane and only gravitons may travel out into the bulk. Thanks to this leaking of gravity into extra-dimensions, one observes that the energy scale of gravity on the brane is much smaller compared to the energy scale of other physical interactions [1]. In order to have the scale of the gravitational field to match our observations, we need to localise gravity close to the brane. One way to achieve it is to suppose that the extra-dimensions are compactified. Another possibility was emphasised by Randall and Sundrum [1] who considered curved or warped geometries of the bulk. The extra-dimensions may spread to infinity while the localisation of gravity next to the brane is ensured by a negative bulk's constant $\Lambda = -\frac{6}{l^2}$, where l is the radius of curvature of the AdS_5 spacetime. The effective gravitational constant Λ is related to the fundamental bulk's cosmological constant by the relation

$$\Lambda = \frac{1}{2}\left[\Lambda_5 - \kappa^2\lambda\right] \tag{1}$$

where λ is the brane tension and is defined by $\lambda = \frac{6k}{\kappa}$ and $\kappa_d^2 \equiv \frac{8\pi}{M_{d+1}^{d-1}}$ where M_{d+1}^{d-1} is the Planck mass in $(d+1)-$ dimensions.

In the RS II braneworld scenario [2], the $5D$ Einstein's equations in the bulk are written as ${}^{(5)}G_{AB} = -\Lambda_5^{(5)}g_{AB}$. Projecting the $5D$ curvature, imposing the Z_2 symmetry and the Israel-Darmois junction conditions, Shiromizu et al. [9] have derived the effective Einstein's equations on the brane

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu} + 6\frac{\kappa^2}{\lambda}S_{\mu\nu} - \Lambda g_{\mu\nu} - \varepsilon_{\mu\nu} \tag{2}$$

where $S_{\mu\nu}$ is the high-energy correction term and is defined as $S_{\mu\nu} \sim (T_{\mu\nu})^2$. The high-energy correction term is negligible for $\rho << \lambda$ but becomes dominant for $\rho >> \lambda$:

$$\frac{\left|\kappa^2\frac{S_{\mu\nu}}{\lambda}\right|}{\kappa^2 T_{\mu\nu}} \sim \frac{|T_{\mu\nu}|}{\lambda} \sim \frac{\rho}{\lambda}. \tag{3}$$

A. CMBR

The Cosmic Microwave Background Radiation (CMBR) is the oldest light in the universe. It is seen today as a black body radiation at a near uniform temperature of 2.73 K covering the whole sky. This radiation field is not perfectly uniform but includes small anisotropies of temperature of the order of $\Delta T/T \sim 10^{-5}$. There are several kinds of temperature anisotropies of the CMBR, they have their origin in several sources, in the intrinsic fluctuations in the electron-nucleon-photon plasma, in the Doppler effect caused by the velocity fluctuations in the plasma, in the gravitational redshift/blue shift due to the Sachs-Wolfe effect and in the gravitational redshift/blueshift due to the time varying gravitational potential taking place between the Last Scattering Surface and our present epoch. The last type of temperature fluctuation is sometimes called the Rees-Sciama effect [3] and it is the key effect we study in this work. Some physical processes that occurred in the early Universe have left their fingerprints in these anisotropies of the CMBR, therefore the temperature anisotropies of the CMBR play an important role in our understanding on the origin of the growth of fluctuations of matter density, which grew later to become galaxies and other large scale structures we may observe in the Universe

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today, they are the birth certificate of the Universe. With the Big Bang being confirmed as the correct model of creation of our Universe, it is essential to use the observations of the CMBR to better understand this model and to study the physics of the Big Bang and the processes that led to the formation of large scale structures in the Universe as the anisotropies of the CMBR are believed to be caused by inhomogeneities in the distribution of matter during the period of Recombination [8].

In this work, we have determined the temperature fluctuations of the Cosmic Microwave Background radiation in the framework of the braneworld model using the Rees-Sciama effect. We model the inhomogeneity of matter by a braneworld Reissner-Nordstrom black hole spacetime and by a halo of constant density having a braneworld Reissner-Nordstrom vacuum matched to the FLRW spacetime.

B. Braneworld Cosmology

Fields and matter are localised on a 3-brane immersed in a 5D bulk. The bulk's spacetime with a spatial 4-isotropy can be naturally foliated into 1+3 dimensional FLRW surfaces. The induced spacetime metric on the brane describing the expanding Universe is simply [7]

$$\mathrm{d}s^2 = -\mathrm{d}T^2 + a^2(T)\left[\mathrm{d}\chi^2 + \Sigma_k^2(\chi)(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)\right] \quad (4)$$

This spacetime geometry is applied to the effective Einstein's equations on the brane with the appropriate perfect fluid stress-energy tensor. The Friedman equation then reads

$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\mu}{a^4} + \frac{\Lambda}{3} - \frac{k}{a^2}. \quad (5)$$

Here we constraint ourselves to the case when $\Lambda = 0$ (we tune the bulk's cosmological constant so that the braneworld cosmological constant is zero). The parameter μ represents the mass of the bulk's black hole which affects the expansion of the brane, contributing to the matter on the brane in the form of the so called "dark radiation". The parameter k represents the curvature index of spacetime ($k = 0, \pm 1$).

Schematically, the model of the clump is illustrated in Fig. 1. We replace a FLRW sphere made of dust having a mass M by a static and spherically symmetrical spacetime of the same mass. The matching hypersurface separating the FLRW spacetime from the clump spacetime is comoving with the cosmic observers from the FLRW.

C. Black Hole on a Brane

The spherically symmetric and static black hole solution on the brane is the braneworld Reissner-Nordstrom solution [11] having the spacetime interval

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right) \quad (6)$$

where

$$f(r) = 1 - \frac{2GM}{r} + \frac{q}{r^2} \quad (7)$$

and the parameter q is the "tidal charge" representing the local tidal effects from the bulk (the leaked gravity is reflected back by the negative bulk cosmological constant).

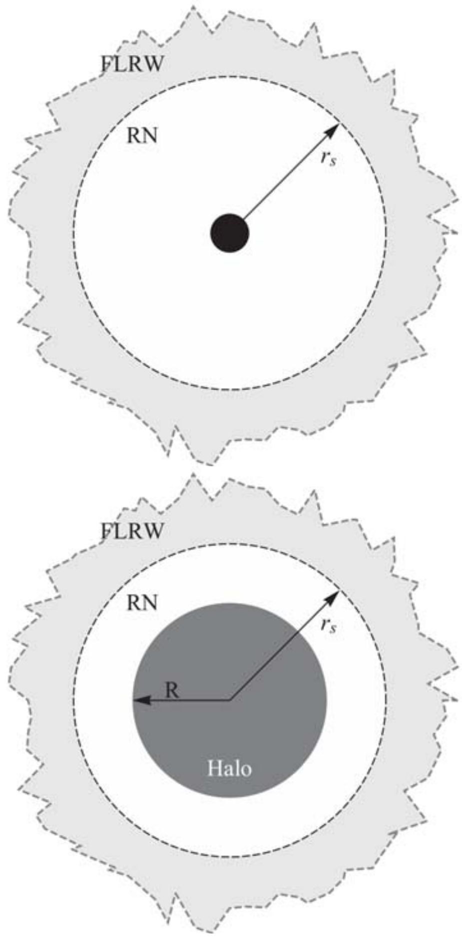

Fig. 1 Illustration of the clump models: braneworld R-N black hole (top), braneworld constant density halo (bottom)

D. The Perfect Fluid Sphere Spacetime

The spherical inhomogeneity is represented by a perfect fluid sphere of constant density, representing a galactic halo of mass M and of a radius R that is matched to the FLRW spacetime through the vacuum braneworld RN spacetime region (see Fig. 1 bottom). This system is gravitationally bounded and is separated from the rest of the expanding Universe. The spacetime interval reads [10]

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + h(r)\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right) \quad (8)$$

where

$$f(r) = \begin{cases} \left(\frac{\alpha}{\rho + p(r)}\right)^2, \text{for } r < R \\ 1 - \frac{2M}{r} + \frac{q}{r^2}, \text{for } r \geqq R \end{cases} \quad (9)$$

with

$$p(r) = \frac{\rho[A(r) - A(R)](1 + \rho/\lambda)}{[3A(R) - A(r)] + [3A(R) - 2A(r)]\rho/\lambda}, \quad (10)$$

$$A(r) = \left[1 - \frac{2M}{r}\left(\frac{r}{R}\right)^3\left(1 + \frac{\rho}{2\lambda}\right)\right]^{1/2}, \quad (11)$$

$$\alpha = \rho\, A(R), \quad (12)$$

$$M = \mathcal{M}(1 - \rho/\lambda), \quad (13)$$

$$q = -3G\,\mathcal{M}R\,\rho/\lambda \quad (14)$$

and

$$h^{-1}(r) = \begin{cases} A^2(r) & \text{for } r < R \\ 1 - \frac{2M}{r} + \frac{q}{r^2} & \text{for } r \geqq R. \end{cases} \quad (15)$$

Later we will use the above-mentioned models of the clump for the calculation of the temperature anisotropy of the CMBR.

E. The Mathing Hypersurface

Both clumps' spacetimes are matched to the external expanding braneworld FLRW spacetime. The matching hypersuface is made out of radially moving observers and is equipped with the 3-spacetime intervals of the form [5], [6]

$$\mathrm{d}s_+^2 = -\mathrm{d}T^2 + a^2(T)\Sigma_k^2(\chi_S)(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2), \quad (16)$$

$$\mathrm{d}s_-^2 = -f(r_s)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r_s)} + r_s^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \quad (17)$$

Clearly, both spacetime intervals must be the same, i.e. $\mathrm{d}s_+^2 = \mathrm{d}s_-^2$ and it implies the following conditions:

- the proper time of the preceding radial observers sitting on the matching hypersurface is identical to the cosmic time of the cosmic observers, i.e.

$$\mathrm{d}T^2 = f(r_s)\mathrm{d}t^2 + \frac{1}{f(r_s)}\mathrm{d}r^2, \quad (18)$$

- the radial coordinate r_s of the boundary and its comoving radius χ_s are related by the condition

$$r_s(T) = a(T)\Sigma_k(\chi_s). \quad (19)$$

One can be convinced that this junction condition is satisfying by calculating the relevant components of the extrinsic curvature $K_{\mu\nu}$ on both sides of the boundary and finding $K^+_{\mu\nu} = K^-_{\mu\nu}$, see e.g. [5].

The radial geodesics in the vacuum region of the clump have the following form

$$\left(\frac{\mathrm{d}r_s}{\mathrm{d}\tau}\right)^2 = E_s^2 - f(r_s) = E_s^2 - 1 + \frac{2M}{r} - \frac{q}{r^2}, \quad (20)$$

$$\frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{E_s}{f(r)}. \quad (21)$$

The covariant energy of the radial geodesics E_s is associated with the expansion rate of the FLRW external spacetime. We rewrite the Friedmann equation (5) using the junction condition (19) to read

$$\left(\frac{\mathrm{d}r_s}{\mathrm{d}T}\right)^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\mu\,\Sigma_k^4(\chi_s)}{r_s^4} - k\,\Sigma_k^2(\chi_s). \quad (22)$$

Assuming that the expansion of the FLRW spacetime is dominated by the incoherent dust, i.e. $\rho\, r_s^3 = \rho_0\, r_{s0}^3$, where r_{s0} is the radius of the clump at our present epoch, and assuming $\rho \ll \lambda$, meaning that the brane tension is much larger than the energy density of matter, i.e. $\rho_{eff} = \rho(1 + \rho/(2\lambda)) \approx \rho$, we can write down the following identities, (comparing (22) with (20))

$$E_s^2 = 1 - k\,\Sigma_k^2(\chi_s), \quad (23)$$

$$q = -\mu\,\Sigma_k^4(\chi_s), \quad (24)$$

$$M = \frac{4\pi}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) r_s^3. \quad (25)$$

II. Propagation of the Photons

In order to resolve the effect of the clump on the temperature of the CMBR, we need the equations of motion of the CMBR's photon to handle its propagation through the boundary and inside the clump. The equations of motion are derived from the Hamilton-Jacobi equations and they read

- FLRW

$$k^T = \frac{1}{r_s}, \quad (26)$$

$$k^\chi = \frac{\Sigma_k(\chi_s)}{r_s^2}\sqrt{1 - b^2\frac{\Sigma_k^2(\chi_s)}{\Sigma_k^2(\chi)}}, \quad (27)$$

$$k^\phi = \frac{L}{r_s^2}\frac{\Sigma_k^2(\chi_s)}{\Sigma_k^2(\chi)} \quad (28)$$

- Clump

$$k^t = \frac{E_0}{f(r)}, \quad (29)$$

$$(k^r)^2 = \frac{1}{f(r)h(r)}\left[E_0^2 - f(r)\frac{L^2}{r^2}\right], \quad (30)$$

$$k^\phi = \frac{L}{r^2}. \quad (31)$$

Note that in the RN region of the clump there is $h(r) = 1/f(r)$ and the equation for k^r will then read

$$(k^r)^2 = E_0^2 - f(r)\frac{L^2}{r^2}. \quad (32)$$

The longer it takes the photon to propagate through the clump, the higher will be the temperature anisotropy, as we will show later. Now we must determine the constants of motion for the photon L and E_0. Due to the isotropy of the FLRW and the clump, the parameter L, the angular momentum of the photon is identical for both sides of the boundary and we treat is as a free parameter in our model. We are left with the constant E_0 now. There are at least two ways to determine E_0. Here we express the k^t component in terms of $k^{(a)}$ components with respect to the tetrade of the observers comoving with the boundary and find out that E_0 satisfies the equation

$$E_0\, r_{out} = \sqrt{1 - k\Sigma_k^2(\chi_s)} + \sqrt{1 - f(r_{out}) - k\Sigma_k^2(\chi_s)}\sqrt{1 - L^2} \quad (33)$$

This formula will be used soon to determine the change of the temperature of the CMBR.

III. The Temperature Anisotropy of the CMBR

The photons from the CMBR propagate from the Last Scattering Surface (LSS) throughout the expanding Universe toward us. As the universe is expanding, the radiation energy density decreases and the corresponding effective temperature Θ decreases too. We have the formula

$$\frac{\Theta_{LSS}}{\Theta_0} = \frac{a_0}{a_{LSS}} = (1 + z)_{LSS}. \quad (34)$$

This formula will change when we replace a part of the FLRW spacetime by a spherically symmetrical clump. We first split the formula (34) to read

$$\frac{\Theta_{LSS}}{\Theta_0} = \frac{a_0}{a_{out}}\frac{a_{out}}{a_{in}}\frac{a_{in}}{a_{LSS}} \quad (35)$$

where a_{out} is the magnitude of the scale parameter at the moment a CMBR photon leaves the clump and a_{in} corresponds to the moment when this photon enters the clump. Let us call this ratio Δ and replace it by a general frequency shift caused by the clump $(1+z)_c$.

$$\frac{\Theta_{LSS}}{\Theta_0^c} = \frac{a_0}{a_{out}}(1+z)_c \frac{a_{in}}{a_{LSS}}. \tag{36}$$

We measure the effect of the clump on the temperature of the CMBR by the ratio [4]

$$\frac{\Theta_0^c}{\Theta_0} = \frac{\Delta}{(1+z)_c} \tag{37}$$

where we have used (34 and 36). Note, due to (19) the parameter is $\Delta = r_{out}/r_{in}$.

Now, let's assume that a CMBR photon leaves the clump when its radius is r_{out}. We want to determine what was the radius, r_{in}, of the clump when a photon entered the clump. This is determined by comparing the coordinate interval Δt_c with the coordinate time interval Δt_p it takes the clump to grow from r_{in} to r_{out} and therefore how long it takes a photon to travel through the clump, i.e.

$$\Delta t_c = \Delta t_p \tag{38}$$

and one arrives to (we apply the definition of Δ):

$$\Delta t_c = \int_{r_{out}/\Delta}^{r_{out}} \frac{\sqrt{1-k\,\Sigma_k^2(\chi_s)}\mathrm{d}r_s}{f(r_s)\sqrt{1-f(r_s)-k\,\Sigma_k^2(\chi_s)}} \tag{39}$$

and

$$\begin{aligned}\Delta t_p &= \int_{r_t}^{r_{out}/\Delta}\sqrt{\frac{h(r)}{f(r)}}\frac{\mathrm{d}r}{\sqrt{1-f(r)L^2/(r^2\,E_0^2)}} \\ &+ \int_{r_t}^{r_{out}}\sqrt{\frac{h(r)}{f(r)}}\frac{\mathrm{d}r}{\sqrt{1-f(r)L^2/(r^2\,E_0^2)}}\end{aligned} \tag{40}$$

where r_t is the turning point of the null geodesics in the clump satisfying the equation

$$E_0^2\,r_t^2 = f(r_t)L^2. \tag{41}$$

The last thing we need to determine is the temperature anisotropy of the CMBR, $(1+z)_c$. It is simply the ratio of the energy of the CMBR's photon measured at the clump boundary in the moment it enters the clump to its energy when it leaves the clump, i.e.

$$(1+z)_c = \frac{u^\alpha k_\alpha|_{in}}{u^\alpha k_\alpha|_{out}}. \tag{42}$$

One easily arrive at:

$$(1+z)_c = \frac{f_o}{f_i}\frac{\sqrt{1-k\,\Sigma_{k(s)}^2}+\sqrt{1-k\,\Sigma_{k(s)}^2-f_i}\sqrt{1-f_i\,b^2/(r_i\,l_0^2)}}{\sqrt{1-k\,\Sigma_{k(s)}^2}-\sqrt{1-k\,\Sigma_{k(s)}^2-f_o}\sqrt{1-f_o\,b^2/(r_o\,l_0^2)}} \tag{43}$$

IV. Results

The procedure to calculate the temperature anisotropies of the CMBR due to a spherically symmetrical clump is the following:

1) Setting up the spacetime parameters: M, r_{s0}, q, k.
2) Set the clump radius r_{out}.
3) Set the parameter b of the photon (angular momentum).
4) Determine the constant l_0 from the formula (33).
5) Solve the equation (38) for Δ.
6) Determine $(1+z)_c$ from the formula (43).
7) Determine Θ_0^c/Θ_0 from (37).

Following the steps of this procedure, we have constructed several plots illustrating the effect of the braneworld parameter q on the temperature anisotropy.

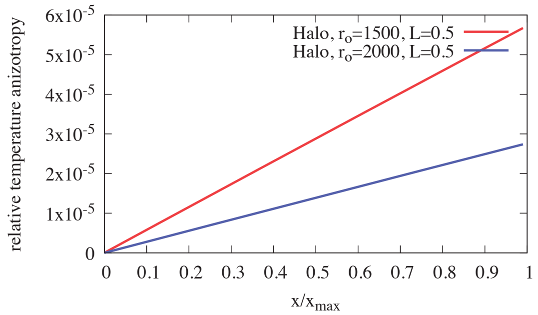

Fig. 2 Plots of relative temperature anisotropy $1-(\Theta_0^c/\Theta_0)_{BW}/(\Theta_0^c/\Theta_0)$ with respect to the normalized ratio $x \equiv \rho_S/\lambda$. The clump is represented by a perfect fluid halo of constant density attached to the FLRW spacetime via the vacuum R-N shell. Here we compare the effect of different radii when the photon is leaving the clump, r_o

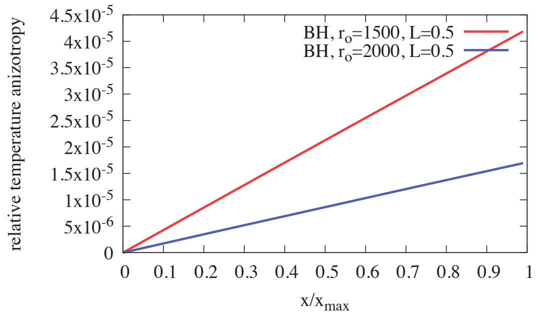

Fig. 3 Plots of relative temperature anisotropy $1-(\Theta_0^c/\Theta_0)_{BW}/(\Theta_0^c/\Theta_0)$ with respect to the normalized ratio $x \equiv \rho_S/\lambda$. The clump is represented by a braneworld R-N black hole spacetime. Here we compare the effect of different radii when the photon is leaving the clump, r_o

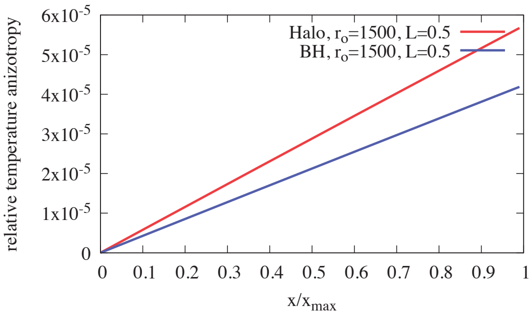

Fig. 4 Plots of relative temperature anisotropy $1-(\Theta_0^c/\Theta_0)_{BW}/(\Theta_0^c/\Theta_0)$ with respect to the normalized ratio $x \equiv \rho_S/\lambda$. The clump is represented by a perfect fluid halo of constant density attached to the FLRW spacetime via the vacuum R-N shell (red) and by the braneworld R-N black hole spacetime (blue). The radius when the photon is leaving the clump is $r_o = 1500$M

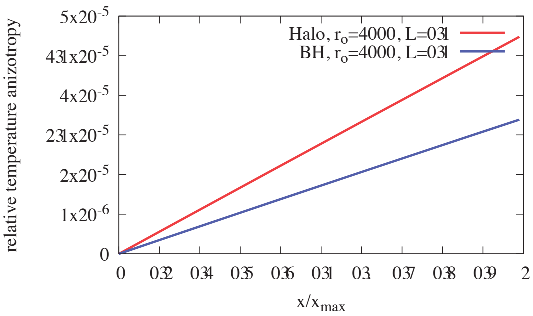

Fig. 5 Plots of relative temperature anisotropy $1-(\Theta_0^c/\Theta_0)_{BW}/(\Theta_0^c/\Theta_0)$ with respect to the normalized ratio $x \equiv \rho_S/\lambda$. The clump is represented by a perfect fluid halo of constant density attached to the FLRW spacetime via the vacuum R-N shell (red) and by the braneworld R-N black hole spacetime (blue). The radius when the photon is leaving the clump is $r_o = 2000$M

V. Conclusion

We have built a vacuola model where in the Friedmann background is joined a clump made of a bRNBH or a perfect fluid sphere. The junction conditions defines the expansion of the clump in time. As a result of this expansion, we obtain a redshift that changes the temperature fluctuations of the CMBR. We observe from our simulations that the fluctuations are higher for smaller clumps and the presence of a perfect fluid sphere increases the fluctuations of the CMBR compared to the model without a perfect sphere.

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