A Multigranular Linguistic Additive Ratio Assessment Model in Group Decision Making

Wiem Daoud Ben Amor, Luis Martínez López, Jr., Hela Moalla Frikha

Abstract**—**Most of the multi-criteria group decision making (MCGDM) problems dealing with qualitative criteria require consideration of the large background of expert information. It is common that experts have different degrees of knowledge for giving their alternative assessments according to criteria. So, it seems logical that they use different evaluation scales to express their judgment, i.e., multi granular linguistic scales. In this context, we propose the extension of the classical additive ratio assessment (ARAS) method to the case of a hierarchical linguistics term for managing multi granular linguistic scales in uncertain context where uncertainty is modeled by means in linguistic information. The proposed approach is called the extended hierarchical linguistics-ARAS method (ELH-ARAS). Within the ELH-ARAS approach, the decision maker (DMs) can diagnose the results (the ranking of the alternatives) in a decomposed style i.e., not only at one level of the hierarchy but also at the intermediate ones. Also, the developed approach allows a feedback transformation i.e., the collective final results of all experts are able to be transformed at any level of the extended linguistic hierarchy that each expert has previously used. Therefore, the ELH-ARAS technique makes it easier for decision-makers to understand the results. Finally, an MCGDM case study is given to illustrate the proposed approach.

Keywords**—**Additive ratio assessment, extended hierarchical linguistic, multi-criteria group decision making problems, multi granular linguistic contexts.

I. Introduction

MANY decision-making problems in the real world cannot be evaluated in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge.

In group decision-making (GDM) problems, experts give their preferences depending on their knowledge of alternatives by means of preference relations. There are some situations where the data provided by the experts may be unquantifiable due to its nature, and hence, it can be stated only in linguistic terms which results in the processes of computing with words (CW). Different linguistic computational models have been proposed to deal with CW processes in a precise way. To manage this lack of certainty, many authors also used fuzzy linguistic approach and they obtained good results in different disciplines [1]. To deal with these concerns, the ARAS method is used. The ARAS method's basic principle is focused on the ranking of alternatives from the best to the worst one according to a set of criteria and then the ranking of their degree of utility. These alternatives are evaluated by the DMs according to a set of attributes for alternatives' assessments. In decision situations with multiple experts, each one has his own knowledge and experiences to provide their preference about alternatives. Alternatives are evaluated according to a set of attributes which perplex the DMs in terms of allotting alternatives' assessments. Accordingly, a flexible and realistic multi-granular hierarchical linguistic approach based on ARAS method is proposed in this paper.

The remainder of this paper is structured as follows: Section II presents a brief state of the art survey on linguistic multi-criteria decision making (MCDM) methods. Section III reviews some related work revising in short, the different steps of the ARAS method, the linguistic hierarchies background and an aggregation process for heterogeneous information. Section IV presents the ELH-ARAS proposed approach. In Section V, a case study will be applied to an MCGDM problem with multiple linguistic scales to discuss the results. In Section VI, we will conclude and present our perspective.

II. Literature Review

The obtained results when experts give their assessment with linguistic values, are reliable and flexible but include CW. The traditional linguistic approach presents a main problem which is the loss of information and hence a lack of precision in the final results. To deal with CW processes in a precise way, different linguistic computational models have been developed such as Semantic model [2], Symbolic [3] or the 2-Tuple one [4].

DMs have disparate expertise, experience, and backgrounds which make them use different linguistic term sets to assess attributes and alternatives. To overcome these hurdles, [5] used a hybrid MCDM approach by integrating the 2-Tuple linguistic representation and soft set to solve supplier selection problems with incomplete information. The results obtained are compared with three methods called the arithmetic average, fuzzy VIKOR, and interval 2-Tuple linguistic VIKOR methods.

An extended DEMATEL method was developed by [6] for identifying risk factors of information technology (IT) outsourcing. It is a MCDM method based on the 2-Tuple fuzzy linguistic representation model and the DEMATEL method.

To avoid the distortion and the lack of information, as well

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as uncertainty of the assessment information provided by experts, [7] proposed an extended VIKOR method for group multi-criterion supplier selection with interval 2-Tuple linguistic information. In this paper, the authors used the Ordered Weighted Averaging (OWA) operator to aggregate DM's opinions. Additionally, [8] presented a new consensus reaching model (CRP) based on fuzzy information granulation (IG) to solve GDM with the multiplicative linguistic preference relations (MLPRs). An application of the proposed model in a real emergency decision-making case for a liquid ammonia leak and finally a comparison of the traditional CRP with the existing consensus GDM method is done. Furthermore, [9] used a hybrid MCDM approach by integrating the 2-Tuple linguistic analytic network process noted TL-ANP for determining weights of criteria and sub-criteria and the interval 2-Tuple Elimination and Choice Translating Reality II method noted IT-ELECTRE II for alternatives evaluation. A real case of supplier selection is applied to the proposed approach. Also, [10] combined two methods "multi-granular hierarchical linguistic approach" and "Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) method" to propose a new approach called a linguistic multi-granular PROMETHEE model. The lead advantage of this method is to manage the uncertainty of both performances of criteria and expert knowledge without loss of information. Moreover, [11] used a hybrid multiple attribute decision-making (MADM) approach and applied it in a site selection problem for a shopping mall project in Tehran, Iran. This approach is in fact the integration of three methods: the multi-granular interval-valued 2-Tuple linguistic variables, target valued criteria with the Best-Worst Method (BWM) and Combinative Distance-based Assessment (CODAS) method. There are nine criteria and six alternatives in this study that have been gathered based on expert opinions. In addition, the assessment of each alternative according to each criterion is done by a committee of five DMs using different linguistic sets.

Reference [12] proposed a MCGDM problem. Authors used a linguistic distribution behavioral MCGDM model for gathered a group linguistic assessment, extended TODIM method for computing the dominance of each alternative and quantum probability theory for exploring the interference effects among experts.

III. Brief Description of the Used Approach

A. ARAS Method

The ARAS method is a ranking method proposed by [13]. The main objective of this method is to select the best alternative among others according to a set of criteria. It has been applied on several disciplines to substantiate the selection of effective alternatives such as the agricultural sector [14], industrial sector [15]-[17] (environment protection, energy management, and manufacturing technology), services sector [18]-[20] (transportation, supply chain management and public health services), and information industry sector [21]-[23] (internet, finance, culture and strategy management).

The different stages for ARAS method are:

Stage1. Form the decision-making matrix X_{ij} of preferences for m alternatives and n criteria.

$$X_{ij} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{23} \\ \vdots & \vdots & \dots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix}; \; i = 1, 2, \dots, m; \; j=1,2, \dots, n$$

where x_{ij} is the performance value of the alternative i according to the criterion j, m is the number of alternatives and n is the number of criteria.

Stage2. Normalize the original decision-making matrix. The idea of any normalization technique is to unify the incommensurable measures of attributes. Howbeit, the normalization formula suggested by authors is as follows: The criteria, preferable values of which are maxima, are normalized through:

The beneficial criteria are normalized through this equation:

$$\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}} \tag{1}$$

The cost criteria are normalized through two-stage process:

$$x'_{ij} = \frac{1}{x^*_{ij}}; \; \bar{x}_{ij} = \frac{x'_{ij}}{\sum_{i=0}^{m} x'_{ij}} \tag{2}$$

where $\bar{x}_{ij}$ denotes the normalized values of the normalized decision-making matrix $\bar{X}$ and x^*_{ij} is the original value of cost criterion. Hence, the generalized structure of the normalized decision-making matrix $\bar{X}_{ij}$ is granted as:

$$\bar{X}_{ij} = \begin{pmatrix} \bar{x}_{11} & \bar{x}_{12} & \dots & \bar{x}_{1n} \\ \bar{x}_{21} & \bar{x}_{22} & \dots & \bar{x}_{23} \\ \vdots & \vdots & \dots & \vdots \\ \bar{x}_{m1} & \bar{x}_{m2} & \dots & \bar{x}_{mn} \end{pmatrix}; \text{ for all } i=1,2, \dots, ; \; j=1,2, \dots,n$$

Stage3. Form the weighted-normalized matrix $\hat{R}_{ij}$. The weighted-normalized values of all criteria are calculated as:

$$\hat{r}_{ij} = \bar{x}_{ij} * w_j \tag{3}$$

where w_j is the weights of the criterion j and $\sum_{j=1}^{n} w_j = 1$

$$\hat{R}_{ij} = \begin{pmatrix} \hat{r}_{11} & \hat{r}_{12} & \dots & \hat{r}_{1n} \\ \hat{r}_{21} & \hat{r}_{22} & \dots & \hat{r}_{23} \\ \vdots & \vdots & \dots & \vdots \\ \hat{r}_{m1} & \hat{r}_{m2} & \dots & \hat{r}_{mn} \end{pmatrix}; \text{ for all } i=1,2, \dots, m; \; j=1,2, \dots,n$$

Stage4. Compute the values of optimality function S_i such that,

$$S_i = \sum_{j=1}^{n} \hat{x}_{ij}, \; i = 0, \dots, m \tag{4}$$

Stage5. Calculate the utility degree value K_i which determines the relative efficiency of a feasible alternative. It is obtained using:

$$K_i = \frac{S_i}{S_0}, i = 0, \dots, m \quad (5)$$

Stage6. Rank the alternatives in an increasing order of the values of the utility degrees K_i. Therefore, the best one is obtained.

B. Linguistic Hierarchies Background

The extended linguistic hierarchies (ELH) [24] is a computational symbolic model based on the linguistic hierarchies (LH) [25] and the 2-Tuple linguistic representation model in order to accomplish a process of CW [4]. The main objective of this model is managing multigranular linguistic information. ELH model is composed of different hierarchical levels, where each level represents a different multigranular linguistic term set to the rest of levels of the hierarchy. This level is denoted by $l(t, n(t))$ with t is the number of hierarchy level and $n(t)$ is the granularity of the term set of the level t.

Definition1. Given an ELH, we denote as $S^{n(t)} = \{S_0^{n(t)}, \dots, S_{n(t)-1}^{n(t)}\}$ the ordered linguistic term set in LH of a linguistic variable in LH. The set of former modal point of the level t is defined as $FP_t = \{fp_t^0, \dots, fp_t^i, \dots, fp_t^{2.\delta_t}\}$, where each former modal point , $fp_t^i \in [0, 1]$ is located at:

$$fp_t^i = \frac{i}{2.\delta_t} \in [0, 1] \quad (6)$$

and,

$$\delta_t = n(t) - 1 \in \mathbb{N} \quad (7)$$

Definition2. Two extended rules are defined in order to build an ELH while keeping the former modal point from one level t to another t+1.

The extended first rule is to include a finite number of the levels that define the multigranular linguistic framework required by DMs to express their judgments. It is not necessary to keep the former modal points of the membership functions of each linguistic term from one level to the following one [24].

In the extended second rule, new level $l(t^*, n(t^*))$, $t^* = m + 1$ should be append to save all the former modal points of all the previous levels within this new level [24].

From the above concepts, the ELH is defined as the union of all levels t required by the DMs and the new level that keeps all the former modal points to provide accuracy in the processes of CW. Let:

$$\text{ELH} = \bigcup_{t=1}^{t=m+1} (l(t, n(t))) \quad (8)$$

To make the ELH computational model easier we use the least common multiple (LCM) to minimize the granularity of t^* [24].

Definition3. Let $\{S^{n(1)}, \dots, S^{n(m)}\}$ be the set of linguistic scales with any odd value of granularity. A new level, *l (t*, n (t*)* with *t*= m + 1,* that keeps the former modal points of the previous m levels can have the following granularity:

$$n(t^*) = (LCM(\delta_1, \dots, \delta_m)) + 1 = 1, \dots, m \quad (9)$$

IV. The Proposed ELH-ARAS Algorithm

The ELH-ARAS approach is proposed to deal with multi granular linguistic scales information in a symbolic and precise way without loss of accuracy. In what follows, we present the different steps of the proposed algorithm.

For $h = t$

Step1. Define the set of the benefit, cost criteria $(C_{j,\{j=1,\dots,n\}})$ and the alternative set $(ALT_{i,\{i=1,\dots,m\}})$. The selection of the set of criteria and alternative was approved by decision-makers $DM_K (K = 1, 2, \dots, k)$ based on their field of expertise.

Step2. Define a finite number of levels $l(t, n(t))$ of the hierarchy tree, where each level t is a linguistic term set, $S^{n(t)} = \left\{S_0^{n(t)}, \dots, S_{n(t)-1}^{n(t)}\right\}$, with different granularity $n(t)$ to the rest of the levels of the hierarchy. Then, add a new level to build an ELH, $l(t^*, n(t^*))$ with $t^* = m + 1$, and with granularity $n(t^*)$ which is calculated using (9).

Step3. Provide the linguistic preference assessment over the set of alternatives ALT_i under the criterion C_j. The evaluations of experts are based on multigranular linguistic term sets of any level of the hierarchy that he/she has chosen $l(t, n(t))$.

For h= t^*

Step4. Unify the linguistic information assessed in multiple scale in any term set of the ELH using the transformation function [24].

Step5. Aggregate the unified information of all DM's using a multi-granular linguistic 2-Tuple weighted aggregation operator (L2TOWA) [3], to obtain the combined 2-Tuple decision matrix of all experts which is based on ELH.

Step6. Compute the based normalized 2-Tuple linguistic hierarchies decision matrix based on the ELH model using (1) and (2).

Step7. Calculate the weighting of the based normalized 2-Tuple linguistic decision matrix for all the criteria using (3).

Step8. Compute the values of the optimality function for the i^{th} alternative using (4) and calculate the utility alternative degree using (5).

In this order, we obtain the partial-preorder of alternatives from ranking in decreasing order the value utility degree, K_i.

For $h = t^* - i$

Step9. Transform the collective value of the DM's (see step 5) obtained in level t^* into any level of the original linguistic term set $l(t, n(t))$ using (3). The main advantage of this step is to express the results in different linguistic term sets to facilitate the comprehension to the different DMs.

Repeat step 6.

Repeat step 7.

Step10. Calculate the new multigranular linguistic values of optimality function (S'_i) for the i^{th} alternative using (4) and the fraction of the new multigranular linguistic utility degree of alternatives (K'_i) using (5).

Based on the proposed approach, we construct the complete pre-order based at any level of the hierarchy tree $l(t, n(t))$ (i.e., ranking the alternatives according to any level of ELH).

V. An Illustrative Example

We solve a multi-expert decision-making problem by the application of the ELH-ARAS presented in this paper which is defined in a multi-granular linguistic hierarchies context. This example is related to the choice of the best option to invest a sum of money for an investment company.

There are four investment possibilities $A_i = \{A_1, \dots, A_4\}$. A_1 is an arms company, A_2 is a food company, A_3 is a computer company, A_4 is in the car industry. Four experts from four consultancy departments $DM_K = \{DM_1, \dots, DM_4\}$ are chosen by the computer company to provide their preferences throughout a set of four criteria $C_j = \{C_1, \dots, C_4\}$ being C_1 pollution, C_2 potential customer and stability of the market, C_3 company's financial profitability, C_4 ability of uncertainty anticipation (minimization of risk factors). C_1 is the cost type attribute while C_2, C_3 and C_4 are the benefit type attributes. Each department is handled by an expert. The risk analysis department is directed by DM_1, the growth analysis department is managed by DM_2, the social-political analysis department is directed by DM_3 and the environmental impact analysis department is managed by DM_4. These DMs provide their preference over the set of alternatives using different term sets of the linguistic hierarchy (see Fig. 1). More specifically, DM_1 provides his preference in l(3,9), DM_2 provides his preference in l(1,5), DM_3 provides his preference in l(2,7) and DM_4 provides his preference in l(3,9).

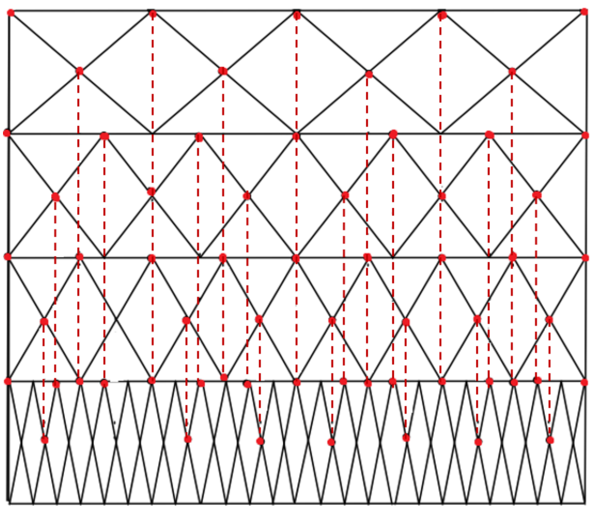

Fig. 1 ELH of 5, 7, 9 and 25 labels

A. For h = t

DM evaluates the alternatives according to criteria based at any level $l(t, n(t)$ of the linguistic hierarchy. The preference values of each expert are presented in Table I (initial decision matrix $\Re_K$).

TABLE I
HETEROGENEOUS INPUT DATA OF EACH EXPERT

DM_k	A_i	C_j			
		C_1	C_2	C_3	C_4
DM_1	A_1	S_5^9(MH)	S_2^9 (ML)	S_6^9 (MH)	S_4^9 (A)
	A_2	S_6^9(H)	S_3^9(L)	S_7^9 (VH)	S_8^9 (P)
	A_3	S_8^9(P)	S_4^9(A)	S_5^9(MH)	S_3^9(L)
	A_4	S_2^9 (ML)	S_6^9 (H)	S_2^9 (ML)	S_5^9(MH)
	W_j	0,24	0,21	0,18	0,24
DM_2	A_1	S_3^5 (H)	S_4^5 (VH)	S_2^5 (A)	S_1^5 (L)
	A_2	S_2^5 (A)	S_1^5 (L)	S_2^5(A)	S_3^5(H)
	A_3	S_1^5 (L)	S_2^5 (A)	S_4^5 (VH)	S_4^5 (VH)
	A_4	S_2^5 (A)	S_3^5 (H)	S_2^5 (A)	S_3^5 (H)
	W_j	0,2	0,24	0,24	0,24
DM_3	A_1	S_2^7 (L)	S_2^7 (L)	S_3^7 (M)	S_2^7 (L)
	A_2	S_6^7 (P)	S_5^7 (VH)	S_2^7 (L)	S_6^7 (P)
	A_3	S_5^7 (VH)	S_4^7 (H)	S_5^7 (VH)	S_5^7 (VH)
	A_4	S_6^7 (P)	S_3^7 (M)	S_2^7 (L)	S_4^7 (H)
	W_j	0,24	0,18	0,24	0,24
DM_4	A_1	S_6^9(H)	S_3^9(L)	S_7^9 (VH)	S_8^9 (P)
	A_2	S_8^9 (P)	S_5^9 (MH)	S_6^9 (H)	S_4^9 (A)
	A_3	S_2^9 (ML)	S_6^9 (H)	S_2^9 (ML)	S_5^9(MH)
	A_4	S_8^9(P)	S_4^9(A)	S_5^9(MH)	S_3^9(L)
	W_j	0,24	0,18	0,24	0,24

TABLE II
UNIFICATION OF THE HETEROGENEOUS INFORMATION OF EACH EXPERT

DM_k	A_i	C_j			
		C_1	C_2	C_3	C_4
DM_1	A_1	$(S_{15}^{25}, 0)$	$(S_6^{25}, 0)$	$(S_{18}^{25}, 0)$	$(S_{12}^{25}, 0)$
	A_2	$(S_{18}^{25}, 0)$	$(S_9^{25}, 0)$	$(S_{21}^{25}, 0)$	$(S_{24}^{25}, 0)$
	A_3	$(S_{24}^{25}, 0)$	$(S_{12}^{25}, 0)$	$(S_{15}^{25}, 0)$	$(S_9^{25}, 0)$
	A_4	$(S_6^{25}, 0)$	$(S_{18}^{25}, 0)$	$(S_6^{25}, 0)$	$(S_{15}^{25}, 0)$
	W_j	0,24	0,21	0,18	0,24
DM_2	A_1	$(S_{18}^{25}, 0)$	$(S_{24}^{25}, 0)$	$(S_{12}^{25}, 0)$	$(S_6^{25}, 0)$
	A_2	$(S_{12}^{25}, 0)$	$(S_6^{25}, 0)$	$(S_{12}^{25}, 0)$	$(S_{18}^{25}, 0)$
	A_3	$(S_6^{25}, 0)$	$(S_{12}^{25}, 0)$	$(S_{24}^{25}, 0)$	$(S_{24}^{25}, 0)$
	A_4	$(S_{12}^{25}, 0)$	$(S_{18}^{25}, 0)$	$(S_{12}^{25}, 0)$	$(S_{18}^{25}, 0)$
	W_j	0,2	0,24	0,24	0,24
DM_3	A_1	$(S_8^{25}, 0)$	$(S_8^{25}, 0)$	$(S_{12}^{25}, 0)$	$(S_8^{25}, 0)$
	A_2	$(S_{24}^{25}, 0)$	$(S_{20}^{25}, 0)$	$(S_8^{25}, 0)$	$(S_{24}^{25}, 0)$
	A_3	$(S_{20}^{25}, 0)$	$(S_{16}^{25}, 0)$	$(S_{20}^{25}, 0)$	$(S_{20}^{25}, 0)$
	A_4	$(S_{24}^{25}, 0)$	$(S_{12}^{25}, 0)$	$(S_8^{25}, 0)$	$(S_{16}^{25}, 0)$
	W_j	0,24	0,18	0,24	0,24
DM_4	A_1	$(S_{18}^{25}, 0)$	$(S_9^{25}, 0)$	$(S_{21}^{25}, 0)$	$(S_{24}^{25}, 0)$
	A_2	$(S_{24}^{25}, 0)$	$(S_{15}^{25}, 0)$	$(S_{18}^{25}, 0)$	$(S_{12}^{25}, 0)$
	A_3	$(S_6^{25}, 0)$	$(S_{18}^{25}, 0)$	$(S_6^{25}, 0)$	$(S_{15}^{25}, 0)$
	A_4	$(S_{24}^{25}, 0)$	$(S_{12}^{25}, 0)$	$(S_{15}^{25}, 0)$	$(S_9^{25}, 0)$
	W_j	0,24	0,18	0,24	0,24

TABLE III
AGGREGATION OF THE UNIFIED 2-TUPLE INPUT DATA OF ALL EXPERTS

A_i/C_j	C_1	C_2	C_3	C_4
A_1	$(S_{14}^{25}, -0.16)$	$(S_9^{25}, 0.39)$	$(S_{14}^{25}, -0.48)$	$(S_{12}^{25}, 0)$
A_2	$(S_{18}^{25}, -0.24)$	$(S_{11}^{25}, -0.08)$	$(S_{13}^{25}, -0.42)$	$(S_{19}^{25}, -0.28)$
A_3	$(S_{11}^{25}, 0.2)$	$(S_{12}^{25}, 0.3)$	$(S_{14}^{25}, -0.1)$	$(S_{16}^{25}, 0.32)$
A_4	$(S_{15}^{25}, -0.12)$	$(S_{12}^{25}, 0.42)$	$(S_9^{25}, 0.16)$	$(S_{14}^{25}, -0.08)$
W_j	0,23	0,21	0,215	0,24

In this example, the next step consists in uniting the non-homogeneous information from a linguistic label in level t with

$l(t, n(t))$ to label in level t^* with $l(4,25)$. The 2-Tuple homogeneous linguistic decision matrix of each expert are gathered in Table II. So, in this stage, the combined 2-Tuple decision matrix of all experts is obtained by applying L2TOWA operator (Table III).

After the calculation of the normalized 2-Tuple linguistic value, the normalized 2-Tuple linguistic decision matrix is obtained. Then, we build the ELH-ARAS weighted-normalized decision-making matrix $\hat{R}_{ij}$ in which we compute the values of the optimality function (S_i), and the utility degree (K_i) to obtain a ranking of all the alternatives (Table IV).

The priority order of the investment company can be represented as:

$$\mathrm{A}_3 \succ \mathrm{A}_2 \succ \mathrm{A}_1 \succ \mathrm{A}_4$$

TABLE IV
WEIGHTED NORMALIZED VALUES AND SOLUTION RESULTS

	C_1	C_2	C_3	C_4	S_i	K_I	Rank
	0,09426696	0,08001896	0,07463786	0,09087379	0,33979757	1	
A_1	0,0762854	0,06051185	0,07281219	0,05825243	0,26786187	0,78829836	3
A_2	0,05944763	0,06946919	0,06754995	0,09087379	0,28734056	0,84562278	2
A_3	0,09426696	0,07802844	0,07463786	0,0792233	0,32615656	0,95985549	1
A_4	0,07095363	0,08001896	0,04918581	0,06757282	0,26773121	0,78791386	4

B. For $h = t^* - 1$

For facilitating the comprehension to the different DMs, we transform the collective value of the DMs (Table III) into level 3 of the original linguistic term set $l(3,9)$, because in our case, most of the experts have expressed their preference in it. The transformation is done by applying (5) and (6) (Table V). So, the new based normalized extended linguistic 2-Tuple-ARAS value is obtained. Therefore, the optimality function (S'_i), the utility degree (K'_i) and a final ranking are obtained (Table VI).

TABLE V
TRANSFORMATION OF THE UNIFIED 2-TUPLE INPUT DATA OF DMS FROM $l(4,25)$ INTO $l(3,9)$

A_i/C_j	C_1	C_2	C_3	C_4
A_1	$(S_5^9, -0.386)$	$(S_3^9, 0.13)$	$(S_5^9, -0.493)$	$(S_4^9, 0)$
A_2	$(S_6^9, -0.08)$	$(S_4^9, -0.36)$	$(S_4^9, 0.193)$	$(S_6^9, 0.24)$
A_3	$(S_4^9, -0.266)$	$(S_4^9, 0.1)$	$(S_5^9, 0.366)$	$(S_5^9, 0.44)$
A_4	$(S_5^9, -0.04)$	$(S_4^9, 0.14)$	$(S_3^9, 0.053)$	$(S_5^9, -0.36)$
W_j	0,23	0,21	0,215	0,24

The priority order of the investment company can be represented as:

$$\mathrm{A}_3 \succ \mathrm{A}_2 \succ \mathrm{A}_1 \succ \mathrm{A}_4$$

The company should choose computer company for its investment.

As can be noticed, the same ranking is obtained when we use linguistic terms sets of the third and the fourth levels of the ELH. Experts could acquire the collective values in their expression domains either in any linguistic terms set of the ELH or for the set of linguistic terms of the new level obtained. Subsequently, the optimality function and the utility degree are calculated to obtain a ranking of all the alternatives. Therefore, the ranking will be the same for all linguistic terms sets of the ELH.

TABLE VI
WEIGHTED NORMALIZED VALUES AND SOLUTION RESULTS EXPRESSED IN LEVEL 3

	C_1	C_2	C_3	C_4	S'_i	K'_i	Rank
	0,05485389	0,04539948	0,04739112	0,05638554	0,20403003	1	
A_1	0,04441857	0,03432376	0,04609817	0,03614458	0,16098507	0,78902635	3
A_2	0,03458946	0,03991645	0,04289034	0,05638554	0,17378179	0,85174614	2
A_3	0,05485389	0,04496084	0,04739112	0,04915663	0,19636248	0,96241946	1
A_4	0,04128419	0,04539948	0,03122924	0,04192771	0,15984062	0,78341713	4

VI. CONCLUSION

In this paper, we proposed an outranking method based on an extended linguistic hierarchical structure of assessment called the ELH-ARAS method. This developed model is the result of an integration between the aggregation operators of the ARAS method, the extended linguistic hierarchical model and the 2-Tuples weighted aggregation operator (L2TOWA). This method adopts a feedback approach to facilitate the comprehension of the final results by the DM. The main objective of this model is to manage multi-granular linguistic information in the GDM problem without loss of information. Nevertheless, we intend to deal with fuzzy data in future research to permit the model to be applicable in the case of uncertainty. Thus, we will develop the interval rough number with the ELH-ARAS method called the IR-ELH-ARAS method.

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