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Self-Organizing Control Systems for Unstable and Deterministic Chaotic Processes

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***Abstract*—**The paper proposes a method for constructing a self-organizing control system for unstable and deterministic chaotic processes in the class of catastrophe "hyperbolic umbilic" for objects with m-inputs and n-outputs. The self-organizing control system is investigated by the universal gradient-velocity method of Lyapunov vector-functions. The conditions for self-organization of the control system in the class of catastrophes "hyperbolic umbilic" are shown in the form of a system of algebraic inequalities that characterize the aperiodic robust stability in the stationary states of the system.

***Keywords*—**Gradient-velocity method of Lyapunov vector-functions, hyperbolic umbilic, self-organizing control system, stability.

I. Introduction

It is now generally accepted that real control objects are nonlinear or linearized and deterministic chaos with the generation of a "strange attractor" and instability are an internal property of any dynamic system [1]-[3]. In nonlinear systems, when a deterministic chaos is generated, the trajectories of the system are globally limited and locally unstable inside the "strange attractor" [2], [3]. Chaotic and unstable systems represent a class of uncertainty models. The ability of a system to maintain stability in uncertain environment is understood as robust stability [4], [5]. When going beyond the confines of the robust stability domain of uncertain parameters, the system gives rise to a mode of deterministic chaos and instability [6]-[8]. In the mode of deterministic chaos and instability, control objects will lose their "controllability". Therefore, the construction of a control system in the class of catastrophes, a hyperbolic umbilic [9], in the form of self-organizing control systems under uncertainty is the main factor guaranteeing for control system the protection from the mode of deterministic chaos and instability [8], [10].

Self-organizing control systems for unstable and deterministic chaotic processes in the class of catastrophes the hyperbolic umbilic are studied by the universal gradient-velocity method of the Lyapunov vector function [7], [11].

The gradient-velocity method of the Lyapunov's vector function is based on the Morse lemma from catastrophe theory and on the basic equation of gradient dynamical systems [9].

$$\frac{dx_i}{dt} = -\frac{\partial V(x)}{\partial x_i}, i = 1, \dots, n$$

This equation directly connects the required Lyapunov function $V(x) = (V_1(x), \dots, V_n(x))$ with the equation of state of a dynamical system. It should be noted that the aperiodic stability of the stationary states of the control system is investigated and the areas of aperiodic robust stability of these stationary states are determined.

II. Formulation of a Problem

The problem of studying a multivariable self-organizing control system in the catastrophe class "hyperbolic umbilic" for linearized objects with m inputs and n outputs is considered.

Let the control system be represented by the equations:

$$\frac{dx}{dt} = Ax + Bu, \quad (1)$$

$$y = Cx,$$

where $x(t) \in R^n$: control system state vector; $u(t) \in R^m$: vector function of control actions; $y(t) \in R^l$: the output vector of the system of dimensions l; $A \in R^{n\times n}$: $n\times n$ control object matrix; $B \in R^{m\times n}$: $m\times n$ control matrix; $C \in R^{l\times n}$: output matrix of the control system of dimension $l\times n$.

The first equation of the control system contains all the dynamics of the system, while the second equation of the output is a static dependence. To study the dynamic properties, only the first equation is considered.

Let the matrices A and B be given in the following form:

$$A = \begin{Vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \vdots & a_{3n} \\ \dots & \dots & \dots & & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{Vmatrix}, B = \begin{Vmatrix} b_{11} & 0 & 0 & \dots & 0 \\ 0 & b_{22} & 0 & \vdots & 0 \\ 0 & 0 & b_{33} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & b_{nn} \end{Vmatrix}$$

The law of control is given in the form of catastrophe the "hyperbolic umbilic" (a three-parameter structurally stable mapping) [9]:

$$u_i(t) = -x_i^3 - x_{i+1}^3 - k_{i,i+1}x_i x_{i+1} + k_i^1 x_i + k_i^2 x_{i+1}, \quad (2)$$
$$i = 1, \dots, n$$

The first equation of system (1) with allowance for the law of control (2) in expanded form is written as follows:

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$$\begin{cases} \frac{dx_1}{dt} = -b_{11}x_1^3 - b_{11}x_2^3 - b_{11}k_{12}x_1x_2 + \\ +(a_{11} + b_{11}k_1^1)x_1 + (a_{12} + b_{11}k_1^2)x_2 + \\ +a_{13}x_3 + \cdots + a_{1n}x_n \\ \frac{dx_2}{dt} = -b_{22}x_1^3 - b_{22}x_2^3 - b_{22}k_{12}x_1x_2 + \\ +(a_{21} + b_{22}k_2^1)x_1 + (a_{22} + b_{22}k_2^2)x_2 + \\ +a_{23}x_3 + \cdots + a_{2n}x_n \\ \vdots \\ \frac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots - \\ -b_{nn}x_{n-1}^3 - b_{nn}x_n^3 - b_{nn}k_{n-1,n}x_{n-1}x_n + \\ +(a_{n,n-1} + b_{nn}k_n^1)x_{n-1} + (a_{nn} + b_{nn}k_n^2)x_n \end{cases} \quad (3)$$

System (3) has stationary states [9]:

$$x_{1s}^1 = 0, x_{2s}^1 = 0, x_{3s}^1 = 0, \ldots, x_{ns}^1 = 0 \quad (4)$$

and other stationary states [9]-[11]:

$$x_{is}^{2,3} = \pm\frac{1}{3}\sqrt{k_i^1 + \frac{a_{ii}}{b_{ii}}}, x_{(i+1)s} = 0, \quad x_{is}^4 = \frac{1}{k_{i,i+1}}(k_i^2 + \frac{a_{i,i+1}}{b_{ii}}) \quad (5)$$

The aperiodic robust stability of the stationary state (4) and (5) of system (3) is studied by the gradient-velocity method of the Lyapunov's vector function [7], [11]-[13].

III. Research Methods

From the equation of state (3) we determine the components of the gradient vectors from the Lyapunov vector function $V(x) = (V_1(x), \ldots, V_n(x))$ [11]:

$$\begin{cases} \frac{\partial V_i(x)}{\partial x_j} = b_{ii}x_i^3 + \frac{1}{2}b_{ii}k_{i,i+1}x_ix_{i+1} - b_{ii}\left(k_i^1 + \frac{a_{ij}}{b_{ii}}\right)x_j, \\ j = i;\ i = 1, \ldots, n; j = 1, \ldots, n \\ \frac{\partial V_i(x)}{\partial x_j} = b_{ii}x_{i+1}^3 + \frac{1}{2}b_{ii}k_{i,i+1}x_ix_{i+1} - b_{ii}\left(k_i^2 + \frac{a_{ij}}{b_{ii}}\right)x_j, \\ j = i + 1;\ i = 1, \ldots, n; j = 1, \ldots, n \\ \frac{\partial V_i(x)}{\partial x_j} = -a_{ij}x_j, \\ j \neq i;\ j \neq i + 1; i = \overline{1, n}; j = \overline{1, n} \end{cases} \quad (6)$$

From the equation of state (3), the expansion of the velocity vector components in terms of the coordinates of system (3) is determined [11]:

$$\begin{cases} \left(\frac{dx_i}{dt}\right)_{x_j} = -b_{ii}x_i^3 - \frac{1}{2}b_{ii}k_{i,i+1}x_ix_{i+1} + b_{ii}\left(k_i^1 + \frac{a_{ij}}{b_{ii}}\right)x_j, \\ for\ j = i;\ i = \overline{1, n}; j = \overline{1, n} \\ \left(\frac{dx_i}{dt}\right)_{x_j} = -b_{ii}x_{i+1}^3 - \frac{1}{2}b_{ii}k_{i,i+1}x_ix_{i+1} + b_{ii}\left(k_i^2 + \frac{a_{ij}}{b_{ii}}\right)x_j, \\ for\ j = i + 1;\ i = \overline{1, n}; j = \overline{1, n} \\ \left(\frac{dx_i}{dt}\right)_{x_j} = a_{ij}x_j, \\ for\ j \neq i;\ j \neq i + 1; \\ i = \overline{1, n}; j = \overline{1, n} \end{cases} \quad (7)$$

The total time derivative of the Lyapunov vector function is calculated as the scalar product of the gradient vector (6) by the expansion of the velocity vector components (7):

$$\frac{dV(x)}{dt} = -b_{11}^2x_1^3 + \frac{1}{2}k_{12}x_1x_2 - \left(k_1^1 + \frac{a_{11}}{b_{11}}\right)x_1 - b_{11}^2\left[x_2^3 + \frac{1}{2}k_{12}x_1x_2 - \left(k_1^2 + \frac{a_{12}}{b_{11}}\right)x_2\right]^2 - a_{13}^2x_3^2 - \cdots - a_{1n}^2x_n^2 - b_{22}^2\left[x_1^3 + \frac{1}{2}k_{12}x_1x_2 - \left(k_2^1 + \frac{a_{21}}{b_{22}}\right) \times x_1\right]^2 - \cdots - a_{n1}^2x_1^2 - a_{n2}^2x_2^2 - \cdots - b_{nn}^2[x_{n-1}^3 + \frac{1}{2}k_{n-1,n}x_{n-1}x_n - \left(k_n^1 + \frac{a_{n,n-1}}{b_{nn}}\right)x_{n-1}\Big]^2 - b_{nn}^2 \times \left[x_n^3 + \frac{1}{2}k_{n-1,n}x_{n-1}x_n - \left(k_n^2 + \frac{a_{nn}}{b_{nn}}\right)x_n\right]^2 \quad (8)$$

Function (8) is a sign-negative function; that is, the sufficient condition for the aperiodic stability of system (3) is satisfied.

From (6), the components of the gradient vectors are used to construct the Lyapunov vector function in scalar form:

$$V(x) = \frac{1}{4}b_{11}x_1^4 + \frac{1}{4}b_{11}k_{12}x_1^2x_2 - \frac{1}{2}b_{11}\left(k_1^1 + \frac{a_{11}}{b_{11}}\right)x_1^2 + +\frac{1}{4}b_{11}x_2^4 + \frac{1}{4}b_{11}k_{12}x_1x_2^2 - \frac{1}{2}b_{11}\left(k_1^2 + \frac{a_{12}}{b_{11}}\right)x_2^2 - -\frac{1}{2}a_{13}x_3^2 - \cdots - \frac{1}{2}a_{1n}x_n^2 + \cdots - \frac{1}{2}a_{n1}x_1^2 - \frac{1}{2}a_{n2}x_2^2 - \ldots - \frac{1}{2}a_{n,n-2}x_{n-2}^2 + \frac{1}{4}b_{nn}x_{n-1}^4 + \frac{1}{4}b_{nn}k_{n-1,n}x_{n-1}^2x_n - -\frac{1}{2}b_{nn}\left(k_n^1 + \frac{a_{n,n-1}}{b_{nn}}\right)x_{n-1}^2 + \frac{1}{4}b_{nn}x_n^4 + \frac{1}{4}b_{nn}k_{n-1,n} \times \times x_{n-1}x_n^2 - -\frac{1}{2}b_{nn}\left(k_n^2 + \frac{a_{nn}}{b_{nn}}\right)x_n^2 \quad (9)$$

By the form of function (9) existence condition of the Lyapunov function, that is, its positive definiteness is not obvious, but function (9) is a continuously differentiable function and at the origin of coordinates turns to zero; that is, it satisfies the conditions of the Morse lemma from catastrophe theory [9]. In accordance with the Morse lemma, function (9) can be locally represented in the neighborhood of the origin in the form of a quadratic form:

$$V(x) \approx -[b_{11}k_1^1 + a_{11} + b_{22}k_1^2 + a_{21} + a_{31} + \cdots + +a_{n1}]x_1^2 - [b_{11}k_2^1 + a_{12} + b_{22}k_2^2 + a_{22} + a_{32} + \cdots + -a_{n2}]x_2^2 - \cdots - [b_{n-1,n-1}k_n^1 + a_{n-1,n} + b_{nn}k_n^2 + -a_{nn} + a_{1n} + \cdots + a_{n-2,n}]x_n^2 \quad (10)$$

Existence condition of the Lyapunov function; that is, positive definiteness of the Lyapunov vector function is given by the system of inequalities:

$$\begin{cases} -[b_{11}k_1^1 + b_{22}k_1^2 + a_{11} + a_{21} + \cdots + a_{n1}] > 0 \\ -[b_{11}k_2^1 + b_{22}k_2^2 + a_{12} + a_{22} + \cdots + a_{n2}] > 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ -[b_{n-1,n-1}k_n^1 + b_{nn}k_n^2 + a_{1n} + \cdots + a_{nn}] > 0 \end{cases} \quad (11)$$

The aperiodic robust stability of the stationary state (5) of system (3) is investigated by the gradient-velocity method of the Lyapunov vector function [7], [11]-[13]. To do this, we write system (3) in deviations relative to the stationary state (5):

$$\begin{cases} \frac{dx_1}{dt} = -b_{11}x_1^3 - b_{11}x_2^3 - b_{11}k_{12}x_1x_2 - \\ -3b_{11}\sqrt{k_1^1 + \frac{a_{11}}{b_{11}}}x_1^2 - 3b_{11}\sqrt{k_1^2 + \frac{a_{12}}{b_{11}}} - \\ -2b_{11}\left(k_1^1 + \frac{a_{11}}{b_{11}}\right)x_1 - 2b_{11}\left(k_1^2 + \frac{a_{12}}{b_{11}}\right)x_2 + \\ +a_{13}x_3 + \cdots + a_{1n}x_n \\ \frac{dx_2}{dt} = -b_{22}x_1^3 - b_{22}x_2^3 - b_{22}k_{12}x_1x_2 - \\ -3b_{22}\sqrt{k_2^1 + \frac{a_{21}}{b_{22}}}x_1^2 - 3b_{22}\sqrt{k_2^2 + \frac{a_{22}}{b_{22}}} - \\ -2b_{22}\left(k_2^1 + \frac{a_{21}}{b_{22}}\right)x_1 - 2b_{22}\left(k_2^2 + \frac{a_{22}}{b_{22}}\right)x_2 + \\ +a_{23}x_3 + \cdots + a_{2n}x_n \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + \cdots + b_{nn}x_{n-1}^3 - b_{nn}x_n^3 - \\ -b_{nn}k_{n-1,n}x_{n-1}x_n - 3b_{nn}\sqrt{k_n^1 + \frac{a_{n,n-1}}{b_{n-1,n}}}x_{n-1}^2 - \\ -3b_{nn}\sqrt{k_n^2 + \frac{a_{nn}}{b_{nn}}}x_n^2 - 2b_{nn}\left(k_n^1 + \frac{a_{n,n-1}}{b_{nn}}\right)x_{n-1} - \\ -2b_{nn}\left(k_n^2 + \frac{a_{nn}}{b_{nn}}\right)x_n \end{cases} \tag{12}$$

From (12) the components of the gradient vectors from the Lyapunov vector function $V(x) = \left(V_1(x), \ldots, V_n(x)\right)$ are determined.

$$\begin{cases} \frac{\partial V_i(x)}{\partial x_j} = b_{ii}x_i^3 + \frac{1}{2}b_{ii}k_{i,i+1}x_ix_{i+1} + \\ +3b_{ii}\sqrt{k_i^1 + \frac{a_{ij}}{b_{ii}}}x_i^2 + 2b_{ii}\left(k_i^1 + \frac{a_{ij}}{b_{ii}}\right)x_j \\ for\ j = i;\ i = 1, \ldots, n; j = 1, \ldots, n \\ \frac{\partial V_i(x)}{\partial x_j} = b_{ii}x_{i+1}^3 + \frac{1}{2}b_{ii}k_{i,i+1}x_ix_{i+1} + \\ +3b_{ii}\sqrt{k_i^2 + \frac{a_{ij}}{b_{ii}}}x_{i+1}^2 + 2b_{ii}\left(k_i^2 + \frac{a_{ij}}{b_{ii}}\right)x_j \\ for\ j = i + 1;\ i = 1, \ldots, n; j = 1, \ldots, n \\ \frac{\partial V_i(x)}{\partial x_j} = -a_{ij}x_j, for\ j \neq i; \\ j \neq i + 1; i = 1, \ldots, n; j = 1, \ldots, n \end{cases} \tag{13}$$

The Lyapunov vector function is determined in scalar form by the components of the gradient (13) in the following form:

$$\begin{gathered} V(x)=\tfrac{1}{4}b_{11}x_1^4+\tfrac{1}{4}b_{11}k_{12}x_1^2x_2+b_{11}\sqrt{k_1^1+\tfrac{a_{11}}{b_{11}}}x_1^3+ \\ +b_{11}\left(k_1^1+\tfrac{a_{11}}{b_{11}}\right)x_1^2+\tfrac{1}{4}b_{11}x_2^4+\tfrac{1}{4}b_{11}k_{12}x_1x_2^2+ \\ +b_{11}\sqrt{k_1^2+\tfrac{a_{12}}{b_{11}}}x_2^3+b_{11}\left(k_1^2+\tfrac{a_{12}}{b_{11}}\right)x_2^2-\tfrac{1}{2}a_{13}x_3^2- \\ -\cdots-\tfrac{1}{2}a_{1n}x_n^2-\cdots-\tfrac{1}{2}a_{n1}x_1^2-\tfrac{1}{2}a_{n2}x_2^2-\cdots+ \\ +\tfrac{1}{4}b_{nn}x_{n-1}^4+\tfrac{1}{4}b_{nn}k_{n-1,n}x_{n-1}^2x_n+b_{nn}\times \\ \times\sqrt{k_n^1+\tfrac{a_{n,n-1}}{b_{nn}}}x_{n-1}^3+b_{nn}\left(k_n^1+\tfrac{a_{n,n-1}}{b_{nn}}\right)x_{n-1}^2+ \\ +\tfrac{1}{4}b_{nn}x_n^4+\tfrac{1}{4}b_{nn}k_{n-1,n}x_{n-1}x_n^2+ \\ +b_{nn}\sqrt{k_n^2+\tfrac{a_{nn}}{b_{nn}}}x_n^3+b_{nn}\left(k_n^2+\tfrac{a_{nn}}{b_{nn}}\right)x_n^2 \end{gathered} \tag{14}$$

As can be seen from (14), the conditions for the positive or negative definiteness of the Lyapunov function are not obvious, as a result of which we apply the Morse lemma [9]. In this case, we write the Lyapunov function (14) locally in the neighborhood of stationary states (5) in the form of a quadratic form:

$$\begin{gathered} V(x)\approx[b_{11}k_1^1+a_{11}+b_{22}k_1^2+a_{21}+a_{31}+\cdots+ \\ +a_{n1}]x_1^2+[b_{11}k_2^1+a_{12}+b_{22}k_2^2+a_{22}+a_{32}+\cdots+ \\ +a_{n2}]x_2^2+\cdots+[b_{n-1,n-1}k_n^1+a_{n-1,n}+b_{nn}k_n^2+ \\ +a_{nn}+a_{1n}+\cdots+a_{n-2,n}]x_n^2 \end{gathered} \tag{15}$$

Then from (15) the condition of positive definiteness of the Lyapunov function, that is the necessary condition for aperiodic robust stability will be determined by the inequalities:

$$\begin{cases} b_{11}k_1^1+b_{22}k_1^2+a_{11}+a_{21}+\cdots+a_{n1}>0 \\ b_{11}k_2^1+b_{22}k_2^2+a_{12}+a_{22}+\cdots+a_{n2}>0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ b_{n-1,n-1}k_n^1+b_{nn}k_n^2+a_{1n}+\cdots+a_{nn}>0 \end{cases} \tag{16}$$

To summarize, it can be observed that the stability of the control system built in the catastrophe class "hyperbolic umbilic" is in an infinitely wide range of changes in the uncertain parameters of the control object. The existence and stability of the stationary state (4) is possible when the uncertain parameters of the control object in the region (11) change. In the case of loss of stability of this stationary state, other stationary states are generated (5) and their existence simultaneously is impossible. The last stationary states (5) will be stable when the system of inequalities (16) is satisfied.

Carried out studies show that a self-organizing control system with m inputs and n outputs is a periodically stable at any changes in the undetermined parameter of the system.

IV. Conclusion

Real dynamic control objects are mostly non-linear or linearized, and control systems are constructed and operated under uncertainty. The ability of a control system to maintain stability under uncertainty is understood as robust stability. When going beyond the confines of the robust stability domain of uncertain parameters, instability and a deterministic chaotic regime with the generation of a “strange attractor” are generated in the system. A practically important class of problems arises when an unstable and deterministic chaotic process must be controlled, completely suppressing instabilities and the degree of chaotic state. Therefore, a self-organizing control system in the class of catastrophes "hyperbolic umbilic" is the main protection against the mode of deterministic chaos and instability.

Control systems in the class of catastrophes "hyperbolic umbilic" have several stable stationary states. These stationary states do not exist at the same time and are not stable at the same time. If the undetermined parameters change, then the system passes from one stable stationary state to another due to "bifurcations". In this case, the initial stationary state loses its robust stability, and the other stationary state takes on the properties of robust stability. Consequently, self-organization occurs in the system and unstable and deterministic chaotic modes are excluded from the scenarios of process development in the system.

Consideration of a self-organizing control system in the hyperbolic umbilic catastrophe class as gradient dynamical systems, and Lyapunov functions as potential functions from catastrophe theory, made it possible to propose a universal approach to constructing the Lyapunov vector function. The application of the Morse lemma to the Lyapunov vector function allows us to represent the conditions for aperiodic robust stability of the system in the form of a system of algebraic inequalities regarding the undetermined and settable parameters of the controller.

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