The Analysis of Different Classes of Weighted Fuzzy Petri Nets and Their Features

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Abstract**—**This paper presents the analysis of six different classes of Petri nets: fuzzy Petri nets (FPN), generalized fuzzy Petri nets (GFPN), parameterized fuzzy Petri nets (PFPN), T2GFPN, flexible generalized fuzzy Petri nets (FGFPN), binary Petri nets (BPN). These classes were simulated in the special software PNeS® for the analysis of its pros and cons on the example of models which are dedicated to the decision-making process of passenger transport logistics. The paper includes the analysis of two approaches: when input values are filled with the experts' knowledge; when fuzzy expectations represented by output values are added to the point. These approaches fulfill the possibilities of triples of functions which are replaced with different combinations of t-/s-norms.

Keywords**—**Fuzzy petri net, intelligent computational techniques, knowledge representation, triangular norms

I. Introduction

THIS paper focused on the research of different classes of Petri nets (PN) with the analysis of their features as well as advantages and disadvantages. The already conducted researches on possibilities of PN can be found in authors' earlier papers [1]-[8]. The first paper described the initial scheme of the transport logistics problem based on which mathematical description was presented [1], [3]. Additionally, the application of hierarchical approach was described [3] which allowed the full development of the decision-making process for passenger transport logistics [3], [6], [8] with classical (ZtN, GtN, ZsN) and maximal (ZtN, ZtN, LsN) triples.

Moreover, there were tested triples in the range between minimal (LtN, LtN, ZsN), classical and maximal [2], [4], [5], [7]. Yet, all experiments were considered with the classical weighted fuzzy Petri net (wFPN). Thus, this paper aims to extend the knowledge of possibilities of wFPN by adding to the analysis other types of nets like: GFPN, T2GFPN, PFPN, FGFPN and BPN. Each of the above-mentioned classes is considered with weights in the research. The experiment is divided into two parts:

1. All the nets were set only with the input values provided by the experts. The decision-making process is limited to the operations exceptionally with input properties.
2. In order to overcome the problem in the previous point, fuzzy expectations were proposed and analyzed [7]. In this case, output places were also initially set with some fuzzy value in the range [0, 1]. Their aim is to correlate the decision-making process of the net with the response to the expectation of the user.

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Productions rules with logical AND were applied as the basis for the creation of PN models. A special software PNeS® was used for the simulation of the behavior and the analysis of each class of PN [9].

II. Weighted Fuzzy Petri Nets

There were created lots of modifications which are based on the classical PN [10]-[18]. The most distinguishing one for the research is wFPN [16]-[18]. It is proposed for the analysis, since all values which are set in the net can be defined in the range [0, 1]. Therefore, it is possible to estimate each value and calculations with fuzzy values which lead to more accurate results. wFPN is described as a tuple [8]:

$$N = (P, T, I, O, S, \alpha, \beta, \gamma, M_0, w) \tag{1}$$

$P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of places, $n > 0$; $T = \{t_1, t_2, \ldots, t_m\}$ is a finite set of transitions, $m > 0$; I: $T \rightarrow 2^P$ is the input function; O: $T \rightarrow 2^P$ is the output function; $S = \{s_1, s_2, \ldots, s_n\}$ is a finite set of statements, and the sets P, T, S are pairwise disjoint and $card(P) = card(S)$; α: $P \rightarrow S$ is the statement binding function; β: $T \rightarrow [0,1]$ is the truth degree function; γ: $T \rightarrow [0,1]$ is the threshold function; M_0: $P \rightarrow [0,1]$ is the initial marking, a 2^P denotes a family of all subsets of the set P; w: $P \times T \rightarrow [0,1]$ is $n \times m$ dimensional matrix $W = [w_{ij}]$.

Typically, each wFPN is represented with graphical figures, where:

- input and output places are circles;
- transitions are rectangles;
- connections are directed arcs.

Each arc connects input place to the transition and transition to the output place. It is the only acceptable direction of connections. As it was stated above, each input and output are filled with fuzzy values in the range [0,1]. Moreover, each transition t consists of the following elements:

- the truth degree function $\beta(t)$;
- the threshold function $\gamma(t)$;
- a triple of operators/functions (IN, OUT_1, OUT_2).

The truth degree function (t) describes the influence of the production rule on the input values with regards to the output object [19], [20]. The threshold function *(t)* includes a fuzzy value which is the bottom limit that should be passed in order to fire a transition. A transition t is considered fireable when the following condition is satisfied:

$$IN(w_{i1} \cdot M(p_{i1}), w_{i2} \cdot M(p_{i2}), \ldots, w_{ik} \cdot M(p_{ik})) \geq \gamma(t) > 0_j \quad (2)$$

where IN is an input operator/function, w_{ij} – weight which describes the strength of connection between input place p_{ij} and transition t, $M(p_i)$ – marking of the place [8].

Additionally, each transistor consists of three operators (IN, OUT_1, OUT_2), where IN operator corresponds to the input function $I(t)$ and OUT_1, OUT_2 operators, respectively, correspond to the output function $O(t)$. Each operator is being replaced with some function of t-/s-norm [21].

III. Classes of Petri Nets

This section is dedicated to the description of six classes of PN and their exclusivity.

Binary Petri Net (BPN) is the simplest one, since all combinations of triangular norms are based on two simplest logical operators: AND, OR forming only two combinations of triples for the test (AND, AND, OR) and (OR, AND, OR) [22], where:

(1) Boolean function AND: $f_{AND}(a,b) = a \wedge b$;
(2) Boolean function OR: $f_{OR}(a,b) = a \vee b$.

Fuzzy Petri Net (FPN) is the next class [1]-[8], [12]-[17]. It consists of triangular norms: Zadeh s-/t-norm, Goguen t-norm forming also four combinations for the experiment: (GtN, GtN, ZsN), (ZtN, GtN, ZsN), (ZsN, GtN, ZsN) and (ZtN, ZtN, ZsN), where:

(1) ZtN (Zadeh t-norm): ZtN(a, b) = min(a, b);
(2) GtN (Goguen t-norm): GtN(a, b) = $a \cdot b$;
(3) ZsN (Zadeh s-norm): ZsN(a, b) = max(a, b).

Generalized Fuzzy Petri Net (GFPN) is a class of PN with a tendency to generalize output values, but at the same time it has a wider range of triangular norms for the research [1], [11], [18]. Input operator IN can be replaced with one of the following norm: Einstein t-/s-norm, Goguen t-/s-norm, Hamacher t-/s-norm, Lukasiewicz t-/s-norm, Zadeh t-/s-norm.

(1) EtN (Einstein t-norm): $T_E(a,b) = \frac{ab}{2-(a+b-ab)}$;
(2) EsN (Einstein s-norm): $S_E(a,b) = \frac{a+b}{1+ab}$;
(3) GtN (Goguen t-norm): $T_G(a,b) = ab$;
(4) GsN (Goguen s-norm): $S_G(a,b) = a + b - ab$;
(5) HtN (Hamacher t-norm): $T_H(a,b) = \begin{cases} 0 \; for \; a = b = 0 \\ \frac{ab}{a+b-ab} \; otherwise \end{cases}$;
(6) HsN (Hamacher s-norm): $S_H(a,b) = \begin{cases} 1 \; for \; a = b = 1 \\ \frac{a+b-2ab}{1-ab} \; otherwise \end{cases}$;
(7) LtN (Lukasiewicz t-norm): $T_L(a,b) = \max(0, a + +b-1)$;
(8) LsN (Lukasiewicz s-norm): $S_L(a,b) = \min(1, a + b)$;
(9) ZtN (Zadeh t-norm): $T_Z(a,b) = \min(a,b)$;
(10) ZsN (Zadeh s-norm): $S_Z(a,b) = \max(a,b)$.

First output operator OUT_1 can be replaced by one of previously mentioned t-norms and the second output operator OUT_2 can be replaced by one of previously mentioned s-norms. This class extends the number of possible combinations to 250 (125 for the case of application of logical AND in the production rules and 125 of logical OR).

Parameterized Fuzzy Petri Net (PFPN) is a class of PN where input and output places, truth degree and threshold functions include parameterized values in the range [0,1] [23], [24]. First function can be chosen from one of six parameterized t-/s-norms (Dombi parameterized t-/s-norm, Dubois-Prade parameterized t-/s-norm, Frank parameterized t-/s-norm, Hamacher parameterized t-/s-norm, Schwiezer – Sklar parameterized t-/s-norm, Yager parameterized t-/s-norm).

Second function can be set with one of six parameterized t-norms:

(1) DptN (Dombi parameterized t-norm):

$$T_D^v(a,b) = \begin{cases} T_D(a,b) \; if \; v = 0 \\ \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^v + \left(\frac{1}{b}-1\right)^v\right]^{\frac{1}{v}}} \; if \; v \in (0,\infty) \end{cases} \; where \; S_D(a,b) = \begin{cases} b \; if \; a = 1 \\ a \; if \; b = 1 \\ 0 \; otherwise \end{cases};$$

(2) DPptN (Dubois-Prade parameterized t-norm):

$$T_{DP}^v(a,b) = \begin{cases} 0 \; if \; (v,a,b) = (0,0,0) \\ \frac{ab}{\max(a,b,v)} \; if \; v \in (0,1] \end{cases};$$

(3) FptN (Frank parameterized t-norm):

$$T_F^v(a,b) = \begin{cases} \min(a,b) \; if \; v = 0 \\ ab \; if \; v = 1 \\ log_v[1 + \frac{(v^a-1)(v^b-1)}{v-1} \; if v \in (0,1) \cup (1,\infty) \end{cases};$$

(4) HptN (Hamacher parameterized t-norm):

$$T_H^v(a,b) = \begin{cases} 0 \; if \; v = a = b = 1 \\ \frac{ab}{v+(1-v)(a+b-ab)} \; if v \in [0,\infty) and \; (v,a,b) \neq (1,1,1); \end{cases};$$

(5) SSptN (Schwiezer - Sklar parameterized t-norm):

$$T_{SS}^v(a,b) = \begin{cases} ab \; if \; v = 0 \\ \max(0, a^v + b^v - 1)^{\frac{1}{v}} \; if v \in (-\infty,0) \cup (0,\infty) \end{cases};$$

(6) YptN (Yager parameterized t-norm):

$$T_Y^v(a,b) = \begin{cases} T_D(a,b) \; if \; v = 0 \\ \frac{1}{1-\min\left[1,((1-a)^v+(1-b)^v)^{\frac{1}{v}}\right]} \; if \; v \in (0,\infty) \end{cases} \; where \; S_D(a,b) = \begin{cases} b \; if \; a = 1 \\ a \; if \; b = 1 \\ 0 \; otherwise \end{cases}.$$

The third function – with one of six parameterized s-norms:

(1) DpsN (Dombi parameterized s-norm):

$$S_D^v(a,b) = \begin{cases} S_D(a,b) \; if \; v = 0 \\ \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-v} + \left(\frac{1}{b}-1\right)^{-v}\right]^{-\frac{1}{v}}} \; if \; v \in (0,\infty) \end{cases}; where \; S_D(a,b) = \begin{cases} b \; if \; a = 0 \\ a \; if \; b = 0 \\ 1 \; otherwise \end{cases};$$

(2) DPpsN (Dubois-Prade parameterized s-norm):

$$S_{DP}^{v}(a,b) = \begin{cases} 1 \; if \; (v,a,b) = (0,1,1) \\ \frac{a+b-ab-\min(a,b,1-v)}{\max(1-a,1-b,v)} \; if \; (v=0 \; and \; (a,b) \neq (1,1)) or \; v \in (0,1]; \end{cases}$$

(3) FpsN (Frank parameterized s-norm):

$$S_{F}^{v}(a,b) = \begin{cases} \max(a,b) \; if \; v = 0 \\ a+b-ab \; if \; v = 1 \\ 1 - log_v[1 + \frac{(v^{1-a}-1)(v^{1-b}-1)}{v-1} \; if v \in (0,1) \cup (1,\infty) \end{cases};$$

(4) HpsN (Hamacher parameterized s-norm):

$$S_{H}^{v}(a,b) = \begin{cases} 1 \; if \; v = 0 \; and \; a = b = 1 \\ \frac{a+b-(2-v)ab}{1-(1-v)ab} \; if v \in (0,\infty) and \; (v,a,b) \neq (1,1,1) \end{cases};$$

(5) SSpsN (Schwiezer - Sklar parameterized s-norm):

$$S_{SS}^{v}(a,b) = \begin{cases} a+b-ab \; if \; v = 0 \\ 1 - [\max(0,(1-a)^{v} + (1-b)^{v} - 1]^{\frac{1}{v}} \; if v \\ \in (-\infty,0) \cup (0,\infty) \end{cases};$$

(6) YpsN (Yager parameterized s-norm):

$$S_{Y}^{v}(a,b) = \begin{cases} S_D(a,b) \; if \; v = 0 \\ \frac{1}{\min[1,(a^{v}+b^{v})^{\frac{1}{v}}]} if \; v \in (0,\infty); \end{cases} where \; S_D(a,b) = \begin{cases} b \; if \; a = 0 \\ a \; if \; b = 0 \\ 1 \; otherwise \end{cases}.$$

The following class is T2GFPN [25]. The input operator *IN* and the first input operator Out_1 can be replaced with one of five interval t-norms (iEtN, iGtN, iHtN, iLtN, iZtN).

(1) iEtN (interval Einstein t-norm): iT_E ([*a,a'*],[*b,b'*]) = $\left[\frac{a \cdot b}{2-(a+b-a \cdot b)}, \frac{a' \cdot b'}{2-(a'+b'-a' \cdot b')}\right]$;
(2) iGtN (interval Goguen t-norm): iT_G ([*a,a'*],[*b,b'*]) = $[a \cdot b, a' \cdot b']$;
(3) iHtN (interval Hamacher t-norm): iT_H ([a, a'], [b, b'] = $\begin{cases} [0,0] \; if \; a = a' = b = b' = 0 \\ \left[\frac{a \cdot b}{a+b-a \cdot b}, \frac{a' \cdot b'}{a'+b'-a' \cdot b'}\right] otherwise \end{cases}$;
(4) iLtN (interval Lukasiewicz t-norm): iT_L ([*a,a'*],[*b,b'*]) = [max(0, *a* + *b* − 1), max (0, $a' + b' - 1$)];
(5) iZtN (interval Zadeh t-norm): iT_Z ([*a,a'*],[*b,b'*]) = [min(*a,b*), min(a',b')].

The second output operator OUT_2 can be replaced with one of five interval s-norms (iEsN, iGsN, iHsN, iLsN, iZsN):

(1) iEsN (interval Einstein s-norm): iS_E ([*a,a'*],[*b,b'*]) = $\left[\frac{a+b}{1+a \cdot b}, \frac{a'+b'}{1+a' \cdot b'}\right]$;
(2) iGsN (interval Goguen s-norm): iS_G ([*a,a'*],[*b,b'*]) = $[a + b - a \cdot b, a' + b' - a' \cdot b']$;
(3) iHsN (interval Hamacher s-norm): iS_H ([*a,a'*],[*b,b'*]) = $\begin{cases} [1,1] \; if \; a = a' = b = b' = 1 \\ \left[\frac{a+b-2a \cdot b}{1-a \cdot b}, \frac{a'+b'-2a' \cdot b'}{1-a' \cdot b'}\right] otherwise \end{cases}$;
(4) iLsN (interval Lukasiewicz s-norm): iS_L ([*a,a'*],[*b,b'*]) = [min(1, *a* + *b*), min(1, $a' + b'$)];
(5) iZsN (interval Zadeh s-norm): iS_Z ([*a,a'*],[*b,b'*]) = [max(*a,b*), max(a',b')];

The distinguishing feature of this net is four different conditions for transition firing (also known as order relations on *L* ([0,1])):

- [*a,a'*] $\preccurlyeq_{U\,po}$ [*b,b'*] ⇔ *a* ≤ *b* and *a'* ≤ *b'* (the usual partial order);
- [*a,a'*] $\preccurlyeq_{Lex1}$ [*b,b'*] ⇔ *a* < *b* or *a* = *b* and *a'* ≤ *b'* (the first lexicographical order);
- [*a,a'*] $\preccurlyeq_{Lex2}$ [*b,b'*] ⇔ *a'* < *b'* or *a'* = *b'* and *a* ≤ *b* (the second lexicographical order);
- [*a,a'*] $\preccurlyeq_{YX}$ [*b,b'*] ⇔ *a* + *a'* < *b* + *b'* or *a* + *a'* < *b*+ + *b'* and *a'* - *a* ≤ *b'* - *b* (the order introduced by Xu and Yager in [26]).

Flexible Generalized Fuzzy Petri Net (FGFPN) is an extension of GFPN [27]. There can be found both: classical and inverted fuzzy implications. The list of functions for the first *IN* operator is the same as for Out_1 in GFPN (it includes only a list of five t-norms). Out_2 is the same as Out_2 in GFPN (it includes only a list of five s-norms). Only Out_1 in FGFPN includes a different list of seven inverted fuzzy applications:

- FDiFI (Fodor inverted fuzzy implication): $Invl_{FD}(a,b) = b$, $1 - a < b < a$, $a \in (0,1]$;
- GDiFI (Godel inverted fuzzy implication): $Invl_{GD}(a,b) = b$, $0 \leq b < a$, $a \in (0,1]$;
- GGiFI (Goguen inverted fuzzy implication): $Invl_{GG}(a,b) = a\,b$, $0 \leq b < 1$, $a \in (0,1]$;
- KDiFI (Kleene - Dienes inverted fuzzy implication): $Invl_{KD}(a,b) = b$, $1 - a < b \leq 1$, $a \in (0,1]$;
- LKiFI (Lukasiewicz inverted fuzzy implication): $Invl_{LK}(a,b) = b + a - 1$, $1 - a \leq b < 1$, $a \in (0,1]$;
- RCiFI (Reichenbach inverted fuzzy implication): $Invl_{RC}(a,b) = \frac{a+b-1}{a}$, $1 - a \leq b \leq 1$, $a \in (0,1]$;
- YGiFI (Yager inverted fuzzy implication): $Invl_{YG}(a,b) = b^{\frac{1}{a}}$, $0 \leq b \leq 1$, $a \in (0,1]$.

IV. The Analysis of Combination of Triangular Norms for Each Class of PN

As it was described earlier, the experiment consists of two approaches: fuzzy expectations are set only on the last level of output places; fuzzy expectations are set on each level of output places.

Each input place is initially filled in with fuzzy valued set by the experts in the corresponding subject area as well as the truth degree function (*t*) and the threshold function $\gamma(t)$. Yet, in this experiment, the truth degree function was calculated as [1]:

$$\beta = {}^{k}/_{(k+1)} \tag{3}$$

The values of the truth degree function $\beta(t)$ as well as for the threshold function $\gamma(t)$ can be provided by experts or generated from data tables [28].

The following triples of each class of PN were considered in the experiment:

1) FPN:
a) minimal triple (GtN, GtN, ZsN);
b) middle triple (GtN, ZtN, ZsN);
c) middle triple (ZtN, GtN, ZsN);
d) maximal triple (ZtN, ZtN, ZsN).
2) GFPN:
a) minimal triple (LtN, LtN, ZsN);
b) middle triple (HtN, EtN, HsN);
c) middle triple (GtN, GtN, GsN);
d) middle triple (EtN, HtN, EsN);
e) maximal triple (ZtN, ZtN, LsN).
3) T2GFPN:
a) four conditions for firing of transitions;
b) periods for input places in the range;
4) PFPN:
a) minimal triple ($DptN^{\nu}$, $DptN^{\nu}$, ZsN) ($\nu \to 0$);
b) minimal triple ($HptN^{\nu}$, $HptN^{\nu}$, ZsN) ($\nu \to \infty$);
c) minimal triple ($SSptN^{\nu}$, $SSptN^{\nu}$, ZsN) ($\nu \to \infty$);
d) inner triple (LtN, LtN, ZsN);
e) inner triple (ZtN, ZtN, LsN);
f) maximal triple (ZtN, ZtN, $DpsN^{\nu}$) ($\nu \to 0$);
g) maximal triple (ZtN, ZtN, $HpsN^{\nu}$) ($\nu \to \infty$);
h) maximal triple (ZtN, ZtN, $SSpsN^{\nu}$) ($\nu \to \infty$).

The minimal and maximal triples in PFPN are applied for the experiment in order to exceed the calculation limitations available in other classes.

5) FGFPN:
a) minimal triple (LtN, LtN, ZsN);
b) middle triple (HtN, GGiFI, HsN);
c) middle triple (GtN, GGiFI, GsN);
d) middle triple (EtN, GGiFI, EsN);
e) maximal triple (ZtN, OPtN, LsN), where OPtN is the operator which chooses the most optimal triangular norm among initially set one of three options [21]:
- greatest (g);
- random (r);
- least (l).
6) BPN:
a) input/output places are defined exceptionally as values from {0,1};
b) beta and gamma are also defined with values from {0,1};
c) weights are also not applicable since the connection either exists ($w = 1$) or does not ($w = 0$).

V. The Influence of Triangular Norms on the First Two Functions of the Triple of Each Class of PN

As far as previous researches and experiments conducted by the authors were focused on the production rules with logical AND, the observations in the current experiment confirmed that the choice of t-norm between GtN and ZtN plays a vital role in wFPN. ZtN is not dependent on the number of input, while GtN is. It leads to the dependence of the number of input places for transition firing in accordance to (1). The same choice also influences the truth degree function β(t) as well. Result of application of ZtN always provides higher results over GtN. It is explained that multiplication which is performed under GtN results with lower value than the minimal value resulted from ZtN. As far as the last element of the triple remains stable – ZsN, the following sequence held true: (GtN, GtN, ZsN) ≤ (GtN, ZtN, ZsN) ≤ (ZtN, GtN, ZsN) ≤ (ZtN, ZtN, ZsN). It is also worth highlighting that triples (GtN, GtN, ZsN) and (GtN, ZtN, ZsN) have a tendency not to reach threshold function (t) and therefore not to reach final numbers. Triple (ZtN, GtN, ZsN) is considered as a golden spot which is also known as a classical triple.

The next step is to analyze wGFPN. Triple (LtN, LtN, ZsN) is considered as the minimal, since first two t-norms has two main features:

a) Lukasiewicz t-norm always returns the minimal-possible value. Definition of the formula: min (a+b−1, 0) suggests that the output is totally dependent on inputs a, b. If $a \leq 0.5$ and $b \leq 0.5$ then the output will always be equal to 0, since the left side of the formula drops below zero. The only possibility to reach the output greater than 0 is when both a, b are greater than 0.5.
b) It is the only triangular norm that can result with 0, when $0 \leq a \leq 0.5$ and $0 \leq b \leq 0.5$. In case $a = 0$ and $b = 0$, every triple results with 0. Yet, this situation should be avoided due to (1).
c) There is a direct dependency on the number of input values. Here can be considered three scenarios:
1) all inputs are below or equal 0.5 – the results of LtN is always 0 then;
2) all inputs are above 0.5 – the results of LtN are always achieved but relatively low;
3) at least one input below equal 0.5 – the result of each iteration is equal to 0 and cannot be exceeded on the next iteration (at least one input below equal 0.5 – the result decreases with each iteration (or it remains the same if the second input is always equal to 1)).

Zadeh s-norm is the lowest among other s-norms, thus in combination with a pair of LtN, they create the lowest possible result.

Middle triples (HtN, EtN, HsN), (GtN, GtN, GsN) and (EtN, HtN, EsN) are considered on the same level, but still they have a difference. The common part which can be found in HtN, GtN, EtN is the numerator which is always a multiplication of two inputs ab. The difference is in their denominators: $a + b - ab \leq 1 \leq 2 - (a + b - ab)$. The following examples can be considered to show the sequence holds true:

a) a = 0, b = 0: 0+0–0 < 1 < 2 – (0+0–0) → 0 < 1 < 2 (as far as it is impossible to get a number divided by 0, the following condition is applied for HtN: $0 \; for \; a = b = 0$);
b) a = 0.5, b = 0.5: 0.5+0.5–0.5·0.5 < 1 < 2 – (0.5+0.5-0.5·0.5)→ 0.75 < 1 < 1.25;
c) a = 1, b = 1: 1 + 1 - 1·1 ≤ 1 ≤ 2 – (1 + 1 - 1·1) → 1 ≤ 1 ≤ 1.

From the above observations, it can be highlighted that the result for $a + b - ab$ can be in the range [0, 1] (the result gets higher when a,b → 1), while the result for $2 - (a + b - ab)$ is in the range [1, 2] (the results gets higher when a,b → 0). Still, it is important to keep in mind that these output values are located in the denominator part of formulas of above-mentioned triangular norms. Considering that the numerator is equal for each of them, $a+b-ab$ which results with values in the range [0, 1], provides a higher output value over the result with $2-(a+b-ab)$ allocated in the numerator. Therefore, the following sequence is achieved: EtN ≤ GtN ≤ HtN. When it comes to the last element of the triples: HsN, GsN, EsN, then it works vice versa: HsN ≤ GsN ≤ EsN. The same approach can be considered to verify: $\frac{a+b-2ab}{1-ab} \le a+b-ab \le \frac{a+b}{1+ab}$,

a) a = 0, b = 0, follows: 0 ≤ 0 ≤ 0;
b) a = 0.5, b = 0.5, follows: 0.33(3) < 0.75 < 0.8;
c) a =1, b = 1, follows: 0 < 1 ≤ 1 (as far as it is impossible to get a number divided by 0, the following condition is applied for HsN: $1\ for\ a=b=1$) [a → 0 multiplied with b → 0 gives output which goes to the maximum (i.e. equal to 1); when division by zero occurs, the number goes to the infinitivally small value, and the result – the infinitivally large value.

TABLE I
THE ANALYSIS OF DIFFERENT COMBINATIONS OF ETN, GTN, HTN

Seq of t-norms	a=0.25 b=0.25	Seq of t-norms	a=0.5 b=0.5	Seq of t-norms	a=0.75 b=0.75
EtN, EtN, HsN	0,2520	EtN, EtN, HsN	0,507	EtN, EtN, HsN	0,7599
EtN, EtN, GsN	0,2527	GtN, EtN, HsN	0,515	GtN, EtN, HsN	0,7681
EtN, EtN, EsN	0,2534	EtN, EtN, GsN	0,516	HtN, EtN, HsN	0,7706
GtN, EtN, HsN	0,2540	EtN, GtN, HsN	0,518	EtN, GtN, HsN	0,7735
EtN, GtN, HsN	0,2545	HtN, EtN, HsN	0,521	EtN, HtN, HsN	0,7792
GtN, EtN, GsN	0,2553	EtN, EtN, EsN	0,523	EtN, EtN, GsN	0,7855
EtN, GtN, GsN	0,2560	GtN, EtN, GsN	0,529	GtN, GtN, HsN	0,7885
GtN, EtN, EsN	0,2567	(EtN, HtN, HsN)	0,533	HtN, GtN, HsN	0,7924
EtN, GtN, EsN	0,2575	(GtN, GtN, HsN)	0,533	GtN, HtN, HsN	0,7959
GtN, GtN, HsN	0,2588	EtN, GtN, GsN	0,535	HtN, HtN, HsN	0,8
HtN, EtN, HsN	0,2597	HtN, EtN, GsN	0,541	EtN, EtN, EsN	0,806
GtN, GtN, GsN	0,2617	GtN, EtN, EsN	0,542	GtN, EtN, GsN	0,809
HtN, EtN, GsN	0,2629	HtN, GtN, HsN	0,545	HtN, EtN, GsN	0,8161
GtN, GtN, EsN	0,2645	EtN, GtN, EsN	0,551	EtN, GtN, GsN	0,823
HtN, EtN, EsN	0,2660	GtN, HtN, HsN	0,555	EtN, HtN, GsN	0,836
EtN, HtN, HsN	0,2666	HtN, EtN, EsN	0,56	GtN, EtN, EsN	0,838
HtN, GtN, HsN	0,2702	EtN, HtN, GsN	0,562	HtN, EtN, EsN	0,846
EtN, HtN, GsN	0,2720	GtN, GtN, GsN	0,562	EtN, GtN, EsN	0,855
HtN, GtN, GsN	0,2767	HtN, HtN, HsN	0,571	GtN, GtN, GsN	0,855
EtN, HtN, EsN	0,2773	HtN, GtN, GsN	0,583	HtN, GtN, GsN	0,862
GtN, HtN, HsN	0,28	EtN, HtN, EsN	0,588	GtN, HtN, GsN	0,868
HtN, GtN, EsN	0,2831	GtN, GtN, EsN	0,588	EtN, HtN, EsN	0,870
GtN, HtN, GsN	0,2894	GtN, HtN, GsN	0,6	HtN, HtN, GsN	0,875
GtN, HtN, EsN	0,2987	HtN, GtN, EsN	0,615	GtN, GtN, EsN	0,890
HtN, HtN, HsN	0,3076	HtN, HtN, GsN	0,625	HtN, GtN, EsN	0,897
HtN, HtN, GsN	0,325	GtN, HtN, EsN	0,636	GtN, HtN, EsN	0,902
HtN, HtN, EsN	0,3414	HtN, HtN, EsN	0,666	HtN, HtN, EsN	0,909

The following tendency can be spotted: all s-norms give the result in the range [0, 1] (the result gets higher when $a,b \to 1$). Moreover, in accordance with point b) it can be mentioned that the rise of values is in the following sequence HsN ≤ GsN ≤ EsN. This knowledge allows to form a list of all possible middle triples and their sequence (Table I). For the experimental purposes, input values will be set in pairs with values equal to 0.25, 0.5, 0.75. Input values equal to 0 and 1 would not provide such a range of precise values which can be compared on the level of four digits after the comma.

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From Table I, the following observations can be extracted:

a) in case of different combination of triples HsN, GsN, EsN, the lowest result will be achieved by the triple (EtN, EtN, HsN), while the highest by the triple (HtN, HtN, EsN);
b) when other combinations of triples are considered, then there is dependency on the input values a, b for each combination of triangular norms.

Also, it should be noted that in Table I, all three elements were applied in the research as it is described in the second part of the experiment. It was done to achieve the full vision of the influence of each element of the triangular norm on every location in the combination with other triangular norms.

The maximal triple is (ZtN, ZtN, LsN). The advantage of ZtN over other t-norms lies in the fact that the operation is calculated with the logical operator and does not apply any mathematical operations. It leads to impossibility to reduce the input values. Moreover, it leads to the possibility to achieve the highest output among other t-norms. It can be easily compared with the second highest triple HtN:

- a = 0, b = 0: $\frac{0\cdot 0}{0+0-0\cdot 0} \le$ min (0, 0) → 0 ≤ 0 (as far as it is impossible to get a number divided by 0, the following condition is applied for HtN: $0\ for\ a=b=0$;
- a = 0.5, b = 0.5: $\frac{0.5\cdot 0.5}{0.5+0.5-0.5\cdot 0.5} <$ min (0.5, 0.5) → 0.33(3) < 0.5;
- a = 1, b = 1: $\frac{1\cdot 1}{1+1-1\cdot 1} \le$ min (1, 1) → 1 ≤ 1.

This analysis shows that both triples may result with the maximal possible value. Yet, the numerical rise is achieved faster by ZtN. Thus, combination of ZtN in first two places of the triples results with highest possible output among other t-norms. The last element of the triples should be s-norm which is higher than EsN. LsN which calculates the addition of two input values always results with the highest possible value (yet, there exists a limitation not to exceed 1). It results with larger output than the possible result of ZsN which takes only one maximal value. Also, it takes over GsN which includes minus sign and multiplies the values, so this multiplication on the real numbers gives lower output which is subtracted from the operation of addition which also takes place here. The same happens to HsN: it includes division which reduces the numerator which already becomes lower due to the subtraction operation of the value achieved with multiplication in two times. It also exceeds EsN which also has the division, so the divider reduces the numerator (in case input values are not equal to 1). Therefore, triples (ZtN, ZtN, LsN) results with the maximal value which can be achieved. Moreover, in accordance to the analysis, the following sequence can be stated for t-norms: LtN ≤ EtN ≤ ≤ GtN ≤ HtN ≤ ZtN and for s-norms, direction is totally opposite: ZsN ≤ HsN ≤ GsN≤EsN ≤ LsN. What is more, GFPN includes triples from FPN. Therefore, classical triples (ZtN, GtN, ZsN) can be considered in the analysis. These triples will be located after middle triples as the first element ZtN is higher over each first function in the middle triples. Yet, it will be allocated before maximal triple, since GtN ≤ ZtN. Thus, it can be considered more effective in the analysis.

PFPN is another class of PN. The know-how of this class is parameter v which includes the value in the range $[0, \infty)$. The main purpose of this class of the net is to exceed the limits of minimal and maximal triples of wFPN. In this case, previously minimal (LtN, LtN, ZsN) and maximal (ZtN, ZtN, LsN) triples become inner triples. Minimal triples are as follows: ($DptN^v$, $DptN^v$, ZsN) ($v \to 0$), ($HptN^v$, $HptN^v$, ZsN) ($v \to \infty$), ($SSptN^v$, $SSptN^v$, ZsN) ($v \to \infty$); In accordance to the definition of DptN, when $v = 0$ and $a,b \neq 1$ then the result is always undefined since it does not comply with the condition (2). Definition of HptN suggests that with the rise of parameter v, the result will be getting lower. The lowering of the resulting value by v is dependent on its location in the formula in the divisor part. As higher v is as high divisor will be. Thus, with the rise of the divisor part, the output will decrease, but it will never reach 0. SSptN has the same feature: with the rise of v the result drops down. Moreover, the range for v lies in $(-\infty, 0)\cup(0, \infty)$. The resulting value is mostly dependent on the v value in the divided part of the power of the maximal element. It is worth noting that this is the only parameterized norm which may have a negative sign for v. Additionally, SSptN is lowering result faster than HptN with the rise of v. ZsN function is equal to FpsN when $v = 0$ and it remains the lowest triangular norm on the third place of the triple. The main concern regarding group of minimal triples is that the parameter v should be set in the way that the calculations will achieve lower non-zero output values compared to the classical minimal triple (LtN, LtN, ZsN) and it will comply with condition (2), so the result is not undefined.

When it comes for the maximal triples, the observations are totally vice versa: the highest t-norm is ZtN which can be replaced with FptN when v = 0 and the last element of the triple should be changed with DpsN, HpsN or SSpsN. So, there is the following group: maximal triple (ZtN, ZtN, $DpsN^v$) ($v \to 0$), (ZtN, ZtN, $HpsN^v$) ($v \to \infty$), (ZtN, ZtN, $SSpsN^v$) ($v \to \infty$). It is worth noting that in case the output place is empty then there is no influence of the third element of the triple. DpsN gives as the output the maximum value when v = 0 and a,b ≠ 0, so it is the fastest way to achieve 1 on the output under these conditions. Yet, the difference between HpsN and SSpsN is as follows: with the rise of v output for SSpsN rises faster than for HpsN. The only concern regarding the group of maximal triples, they can easily reach maximum output value equal to 1. So, the properly set parameter v can exceed the classical meaning of minimal and maximal triples presented in GFPN. PFPN includes all the same features but extends their possibilities with parameter v as well as the range of output which can be achieved.

T2GFPN is another class of PN which stands over previously described PNs due to the following features: a) a list of interval triangular norms; b) intervals (ranges of representation) of fuzzy values; c) four different rules for transition firing. First two points are the most definitive from the other classes of PNs. It allows setting the range of fuzziness of some value. This approach is being applied on input/output values, weights, beta and gamma values. All interval triangular norms are the same as for GFPN. The ranges of parameters can easily be compared with defined fuzzy value by simple calculation of mathematical average meaning. Interval of extension [0,0] is equal to the initial fuzzy value and has no range in fact. It can be extended with some interval values $[k, l]$, where $k = l$ or $k \neq l$ and $k,l \neq 0$ (defined by the initial knowledge about inputs). The initial input values a, b will be represented as follows ($[a,a']$,$[b,b']$), where $a = a+k$, $a'=a+l$ and $b = b+k$, $b'=b+l$. Yet, the maximum interval could not exceed the maximum of the fuzzy value which is equal to [1,1]. When the interval is [0,0], then the results will be the same as for GFPN. When it is extended with some values $[k, l]$, then the average mathematical meaning will show a different value. For the classical triple of function, the average meaning is slowly rising with the rise of values k, l. For the maximal triple, the average meaning may vary with the rise of k, l, which leads to the suggestion that the decisions become fuzzier and more uncertain. One more thing which is worth to be highlighted is whether weights are also represented with intervals. With the addition of intervals to the weights, the extension of intervals in the calculation led to faster increase of difference of intervals at the outputs.

To sum it up, the resulting values in T2GFPN are dependent on the intervals in which values are represented (as larger the interval is, as large the fuzziness in the result is). Also, the intervals in the results are extending faster with the extension of intervals on the input and weights compared to the results without application of the intervals to the weights. Moreover, there exists four types of conditions for transition firing which are also dependent on the initial intervals ($[a,a']$,$[b,b']$). The benefits of T2GFPN are:

a) the possibility to define fuzzy values in the ranges, in this manner extending the range of the truth degree of the value;
b) the extended possibilities with interval triangular norms over classical ones which are also covered when values k, l in the interval are equal;
c) the extended list of conditions for transition firing, so there can be selected the one that fits best for the research as well acknowledgement of limitations of every condition in a different context.

The main disadvantage of T2GFPN is that with the rise of the intervals, it reduces the precision of fuzzy values at the outcome. Therefore, this class of net offers more possibilities than GFPN with the introduction of intervals, but these intervals may lead to the decrease of the precision of the calculations at the end by blurring the range of intervals at each round of calculations.

FGFPN is a class of PN which can be associated with GFPN in an advanced mode. The difference between these classes lies in the introduction of interval triangular norms in FGFPN. The only exception is that these interval forms can be applied to the operator OUT_1. In the list of triangular norms, there can be found analogue for GtN (from GFPN) - GGiFI in FGFPN (with the condition: $0 \leq b < 1$, $a \in (0,1]$). That is why, triples (HtN, GGiFI, HsN), (GtN, GGiFI, GsN) and (EtN, GGiFI, EsN) are considered as middle triples. Yet, the biggest know-how of this class of PN is OPtN, operator which chooses the most optimal triangular norm. It has parameters: greatest (g), random (r), least (l). In case of parameters greatest (g) and least (l), they will be replaced with ZtN and LtN in accordance with the previously highlighted sequence (LtN ≤ EtN ≤ GtN ≤ HtN ≤ ZtN). In case of choice of random (r) as a parameter, then one of five triangular norms is chosen by random. Thus, the advantage of FGFPN over GFPN is the ability to apply optimization to tn (t-norm) and iFI (inverted fuzzy implication). Therefore, a wide range of flexibility is achieved in network modeling, where it is easy to adapt to specified requirements with additional opportunities for its expansion.

The last class of PN to be disclosed is BPN. It is the most limited class of PN in a sense of values representation as well as number of combinations of triples of functions. All values in this class of PN can be represented as [0, 1] and there are only two combinations of triples to be tested: (AND, AND, OR) and (OR, AND, OR). Weights are not applicable here, since binary value “1” means the existence of the connection (in accordance to the production rule) and “0” means luck of the connection. It can be easily spotted that the output value relies on the second and third element of the triple with the dependency on the choice of the first element. First element of the triple has two options:

a) in case it is logical AND: if it least one input value is equal to 0 then, no matter what is set ahead, the output will be equal to what is set on the output place; each of logical AND at first two places will keep output of calculations equal to zero and then logical OR choses what is set at the output, because its application to zero value does not change the result (i.e. no changes on the output place);
b) in case first operator is logical OR, the issue described in point a) is solved when the truth degree function $\beta(t) \neq 0$, otherwise, the result is dependent on the last operator – logical OR as in point a).

It leads to the conclusion that the result of calculations of this class of PN stands out in dependency on two factors: the choice of first operator between AND/OR; the value set at the output place.

VI. The Influence of Added Output Values on Each Class of PN

This part analyzes the application of fuzzy expectations (FE) [7] on the calculation process of different classes of PN. In the first part of the analysis, the output places were initially set to 0, thus, the last operator (no matter which one was chosen) did not influence the output. As for FPN, the last operator is ZsN which compares the result of calculations of the first two functions with the output and takes the biggest value out of them. Therefore, if the value set on the output place is larger than the achieved calculations, then it will simply overwrite the decision-making process. On the other side, if the calculations resulted with 0 value and the condition (1) is satisfied, then the value set on the outplace may save its object from the exclusion of the analysis and calculations on the next level of transitions. The same analysis applies for GFPN with a difference of a possibility to choose one of 5 s-norms. It should be considered that the choice of s-norm is influential on the output as far as ZsN gives the lowest result among s-norms while LsN – the biggest. PFPN has a feature to extend the outputs of the lower and upper limits. In case of minimal triples, there will be applied ZsN again with the same properties as for FPN. For the maximal triples, there is a choice between DpsN^{ν} ($\nu \to 0$), HpsN^{ν} ($\nu \to \infty$), SSpsN^{ν} ($\nu \to \infty$). Therefore, the output depends on the chosen norm as well as the parameter which is set there, since the rise of numerical values are different for different values of parameter ν: the output value with the application of SSpsN^{ν} is always rising faster than with HpsN^{ν} when $\nu \to \infty$. T2GFPN holds the same features as GFPN with the addition of the possibility to set intervals of output values in the same manner as all other values. The application of FE on FGFPN has the same effect as on GFPN which also includes features from FPN. The addition of values on the output place for BPN will define the priority between the decision-making process and expectation with human interaction. In case, the output value is set to “1”, then the output will always be equal to 1 due to logical OR, otherwise, it will take output of the second function of the triple as the output is equal to 0.

VII. Conclusion

This paper presented the analysis of six different classes of PN, which achieved the following sequence of possibilities and effectiveness of each class: BPN ⊆FPN ⊆ GFPN ⊆ PFPN ⊆ T2GFPN ⊆ FGFPN. The analysis on each class of PN resulted as follows:

a) BPN is the most limited and not flexible class which is not effective on the analysis;
b) FPN and GFPN are both effective in a classical sense of the research with the difference of number of triples to be tested;
c) FGFPN extends the possibilities of the above-mentioned classes with the addition of operator OPtN for the choice of the most optimal form;
d) T2GFPN may provide a solution for the cases when fuzzy values are not well defined. Therefore, it can be presented in some range. Yet, this range increases risk of the precision of already fuzzy values and therefore on the analysis and comparison of the results;
e) PFPN is a class which is highly effective on achieving extremely low and high values upon a properly set parameter ν;

Fuzzy expectations should be treated carefully in order not to overwrite the calculation process of the first two functions on the triples. Moreover, it can serve as a saving tool for some object when the first two functions resulted with 0.

The following experiments will include the analysis of an array of input values on the behavior of each class of net with different combinations of triangular norms in it. Additionally, the correlation of output values with different sets of fuzzy expectation can be included in the research [7].

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