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Vague Multiple Criteria Decision Making Analysis Method for Fighter Aircraft Selection

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***Abstract*—**Fighter aircraft selection is one of the most critical strategies for defense multiple criteria decision-making analysis to increase the decisive power of air defense and its superior power in the defense strategy. Vague set theory is an adequate approach for modeling vagueness, uncertainty, and imprecision in decision-making problems. This study integrates vague set theory and the technique for order of preference by similarity to ideal solution (TOPSIS) to support fighter aircraft selection. The proposed method is applied in the selection of fighter aircraft for the Air Force. In the proposed approach, the ratings of alternatives and the importance weights of criteria for fighter aircraft selection are represented by the vague set theory. Finally, an illustrative example for fighter aircraft selection is given to demonstrate the applicability and effectiveness of the proposed approach. The fighter aircraft candidates were selected under six criteria including costability, payloadability, maneuverability, speedability, stealthility, and survivability. Analysis results show that the best fighter aircraft is selected with the highest closeness coefficient value. The proposed method can also be applied to solve other multiple criteria decision analysis problems.

***Keywords*—**fighter aircraft selection, vague set theory, fuzzy set theory, neutrosophic set theory, multiple criteria decision making analysis, MCDMA, TOPSIS.

I. Introduction

MULTIPLE criteria decision making analysis (MCDMA) is an established branch of decision making theory[1-57]. MCDMA is a branch of a general class of operations research models that deal with decision problems in the presence of a set of often conflicting decision criteria. The MCDMA approach requires choosing among decision alternatives defined according to their characteristics. MCDMA problems are assumed to have a predetermined, limited number of decision alternatives. Thus, solving an MCDMA problem involves sorting, ranking, and selection processes.

Therefore, multiple criteria decision-making problem is a type of problem in which all alternatives in the selection set can be evaluated according to a set of evaluation criteria. An MCDMA problem can be briefly expressed in matrix format as $a=\{a_1,...,a_I\}, 1\le i\le I$ are possible alternatives that decision makers should choose, $g=\{g_1,...,g_j\}, 1\le j\le J$ are the criteria by which alternative performance is measured, x_{ij} is the rating of alternative a_i relative to the g_j criterion, and $\omega=\{\omega_1,...,\omega_j\}, 1\le j\le J$, ω_j is the weight of the g_j criterion.

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MCDMA approaches can be generally viewed as alternative methods for combining information from a problem's decision matrix with additional information from the decision-maker to determine a final ranking or selection among alternatives. Apart from the information contained in the decision matrix, all but the simplest MCDMA techniques require additional information from the decision-maker to arrive at a final ranking or selection.

MCDMA problems and the evaluation processes often involve subjective evaluations and result in qualitatively imprecise data. Mathematical, engineering or management decisions are often made through available data and information, which is often vague, imprecise, and uncertain in nature.

The decision-making process in engineering schemes, developed during the concept design phase is one of these typical situations, which often needs some method to deal with uncertain data and information that is difficult to define. During the design phase, designers often offer many alternatives. However, the subjective characteristics of the alternatives are often uncertain and need to be evaluated with insufficient knowledge and judgment of the decision maker.

In ordinary set theory, the values of elements in a set are only two possibilities: present or absent in the set. The ordinary set theory cannot handle ambiguity and uncertainty. Fuzzy sets [58-60], intuitionistic sets [61-62], vague sets [63-70], and neutrosophic sets [71-73] are considered generalizations of ordinary set theory to treat vagueness and uncertainty. A sentence is vague if and only if the sentence is neither absolutely true nor absolutely false.

Fuzzy logic, is a form of multiple-valued logic which deals with imprecise information as a way of processing data by allowing partial set membership rather than definite set membership. Fuzzy logic is a computational approach based on degrees of truth rather than the usual true or false (1 or 0) Boolean logic. In fuzzy logic, the value (degrees) for linguistic variables can be between 0 and 1. When linguistic variables are used, these degrees can be handled with special functions called membership functions. Fuzzy logic represents the degrees of truth.

In fuzzy set theory, a single membership value is assigned to each $x \in U$ element in the universe of discourse. The single membership value contains both the evidence for and against x [53]. It cannot deal with two evidences individually, or even at the same duration.

To solve this problem, the concept of vague set was introduced [53], and it allows interval-based membership function over point-based membership function. It is a further generalization of the fuzzy set theory. Vague set theory

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becomes a promising tool for dealing with imprecise, vague, and uncertain information with enhanced performance but having complex problem-solving.

Intuitionistic fuzzy sets [61] and interval valued intuitionistic fuzzy sets were also introduced. They can only handle incomplete information, not indeterminate information, and inconsistent information.

To handle indeterminate data, the concept of neutrophic logic was introduced [71], which is also a multiple-valued logic based on neutrosophy. As a generalization of fuzzy logic, indeterminacy is included in neutrosophic logic. Fuzzy theory has failed when relationships are indeterminate. The inclusion of indeterminate information with fuzzy logic is nothing more than neutrosophic logic. To apply artificial intelligence, it is often necessary to compare different multiple-valued logic in complex problem-solving. Multiple-valued logic is a nonordinary logic in which the truth values, that a proposition may have, are not restricted to two, representing only truth and falsity.

The nature of such vagueness and uncertainty is neutrosophic rather than random, especially when it comes to subjective human judgments in decision making. Multiple valued logic theory provides a possibility to deal with such data and information that includes subjective characteristics of human nature in the decision-making process.

There exist various methods for solving MCDMA problems, for which the TOPSIS technique is one of the effective multiple criteria decision analysis methods [28]. The basic principle of the TOPSIS method is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution.

In ordinary MCDMA methods, the ratings and weights of the criteria are known precisely. In the TOPSIS process, exact values are given to the weights of criteria and the performance ratings. The concept of TOPSIS was extended to develop a methodology for solving MCDM problems with interval data. A fuzzy version of the TOPSIS method based on fuzzy arithmetic operations was developed. This method was extended to solve group decision problems in a fuzzy environment. TOPSIS was extended to provide a fuzzy closeness coefficient via α-cut propagation. Most fuzzy versions of the TOPSIS method are efficient in overcoming the vagueness and uncertainty present in MCDMA problems, but their results are not able to include the hesitation present in the information provided by the decision-marker.

In real life, one may think that an object belongs to a set to a certain degree, but it is also possible to be unsure of this. In other words, the person has hesitations about the membership degree. In fuzzy set theory, there is no way to include this hesitation about the degree of suitability to which each alternative satisfies the decision maker's requirement.

To include this unknown degree in the membership function, the concept of vague sets was proposed. Vague sets are a generalized form of fuzzy sets. Vague sets are used to handle multiple criteria fuzzy decision-making problems. Some new techniques were used in vague set theory. Using vague sets, the degree of satisfiability and non-satisfiability of each alternative was presented, allowing the decision-maker to assign a different degree of importance to each criterion. A similar approach was proposed but with a more efficient score function. These proposed techniques provide a different way to assist the decision maker effectively in decision making.

Therefore, a more efficient TOPSIS was developed to solve MCDMA problems with vague sets. Then, the applicability and effectiveness of the proposed method were demonstrated with a numerical example of the fighter aircraft selection process.

The remaining of this paper is organized as follows. In the next section TOPSIS method is presented with a stepwise description. The definition and notations of vague sets are briefly introduced. MCDMA based on vague sets is then proposed in section II. A numerical example of the fighter aircraft selection process and a concise conclusion are given in Sections III and VI, respectively.

II. Methodology

In this section, the basic definition of the membership functions of fuzzy set, vague set / intuitionistic fuzzy set, and neutrosophic set are recalled. Let there be a universe of discourse U, in which an element of U is denoted by u_i.

A. Fuzzy Set Theory

Definition 1. If U is a collection of objects denoted generically by u_i then a fuzzy set A in U is a set of ordered pairs:

$$A = \{(u_i, \mu_A(u_i)) \mid \mu_A(u_i) \in [0,1]\ \forall u_i \in U\} \tag{1}$$

where $\mu_A(u_i)$ is the membership function that maps X to the membership space M and $\mu_A(u_i)$) is the grade of membership (also degree of compatibility or degree of truth) of u_i in A.

B. Intuitionistic Fuzzy Set Theory

Definition 2. Let a set U be fixed. An intuitionistic fuzzy set A in U is object of the form given by

$$A = \{(u_i, \mu_A(u_i), v_A(u_i)) : u_i \in U\} \tag{2}$$

where $\mu_A(u_i) \in [0,1]$, $v_A(u_i) \in [0,1]$, denoted by $A(u_i)$ is an intuitionistic fuzzy set, $\mu_A(u_i)$ and $v_A(u_i)$ indicate the membership degree and nonmembership degree of element $\forall u_i \in U$ to set A, respectively.

Additionally, $\pi_A(u_i) = 1 - \mu_A(u_i) - v_A(u_i)$ is the hesitation degree of u_i to A. Obviously, one has $0 \le \mu_A(u_i) + v_A(u_i) \le 1$, $\forall u_i \in U$. If a membership function t_A denotes the upper bounds and a non-membership function f_A denotes the lower bounds on $\mu_A(u_i)$, then, the degree of membership of u_i in the intuitionistic fuzzy set A is bounded to a subinterval $[t_A(u_i), 1 - f_A(u_i)]$.

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The interval $[t_A(u_i),1-f_A(u_i)]$ is considered intuitionistic fuzzy set value which can be expressed as

$$A=\{(u_i,[t_A(u_i),1-f_A(u_i)]);u_i\in U\} \quad (3)$$

Intuitionistic Fuzzy Set Operations

Definition 3. Let $x=(t_x,1-f_x)$ and $y=(t_y,1-f_y)$ be two intuitionistic fuzzy sets, then operations can be defined as follows:

Definition 4. Let $x=(T_x,I_x,F_x)$ and $y=(T_y,I_y,F_y)$ be two SVNNs, then operations can be defined as follows:

$$\begin{aligned}&1.\ x^C=(f_x,1-t_x)\\&2.\ x\oplus y=(t_x+t_y-t_xt_y,f_xf_y)\\&3.\ x\otimes y=(t_xt_y,f_x+f_y-f_xf_y)\\&4.\ \lambda x=(1-(1-t_x)^{\lambda},(f_x)^{\lambda}),\ \lambda>0\\&5.\ x^{\lambda}=((t_x)^{\lambda},1-(1-(f_x)^{\lambda}),\ \lambda>0\end{aligned} \quad (4)$$

Definition 5. Let U be the universe of discourse, with a generic element of U denoted by u_i. A vague set A is characterized by a truth-membership function t_A and a false-membership function f_A, where $t_A(u_i)$ is a lower bound on the grade of membership of u, derived from the evidence for u_i; $f_A(u_i)$ is a lower bound on the negation of u_i, derived from the evidence against u_i; and $t_A(u_i)+f_A(u_i)\le 1$.

The grade of membership of u_i in the vague set A is bound to a subinterval $[t_A(u_i),1-f_A(u_i)]$ of $[0,1]$. The vague value $[t_A(u_i),1-f_A(u_i)]$ indicates that the exact grade of membership $\mu_A(u_i)$ of u_i maybe unknown, but it is bound by $t_A(u_i)\le\mu_A(u_i)\le 1-f_A(u_i)$, where $t_A(u_i)+f_A(u_i)\le 1$. Fig. 1 shows a vague set in the universe of discourse U.

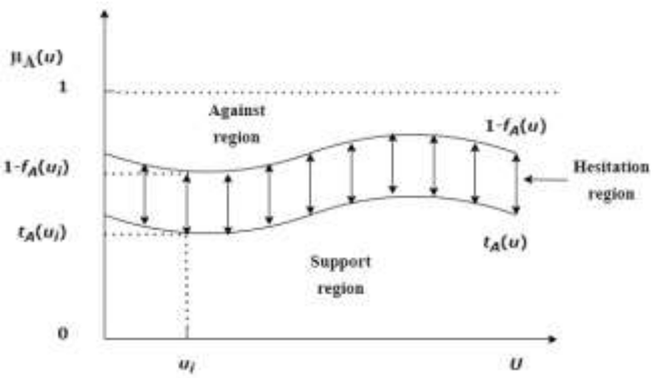

Fig. 1 A vague set in the universe of discourse U

When the universe of discourse U is continuous, a vague set A can be written as

$$A=\int_U [t_A(u_i),1-f_A(u_i)]/u_i,u_i\in U \quad (5)$$

When the universe of discourse U is discrete, a vague set A can be written as

$$A=\sum_{i=1}^{I}[t_A(u_i),1-f_A(u_i)]/u_i,u_i\in U \quad (6)$$

For example, if $[t_A(u_i),1-f_A(u_i)]=[0.7,0.9]$, then $t_A(u_i)$ =0,7, $1-f_A(u_i)$ =0.9, $f_A(u_i)$ =0,1. The result can be explained as u_i belonging to vague set A and accept evidence is 0.7; decline evidence is 0,1 and hesitation is $\pi_A(u_i)=1-f_A(u_i)-t_A(u_i)=0,2$.

By including this hesitation degree in the representation of vague sets, vague set A can be represented as $(t_A(u_i),1-f_A(u_i),\pi_A(u_i))$.

The precision of the knowledge about u_i is characterized by the difference $(1-f_A(u_i)-t_A(u_i))$. If this difference is small, the knowledge about u_i is more precise: if it is large, the knowledge about u_i is more uncertain. If $(1-f_A(u_i)=t_A(u_i))$, the knowledge about u_i is exact, the vague set theory reverts to fuzzy set theory.

C. Operation between Vague Sets

Let x, y be two vague values in the universe of discourse U, $x=[t_x,1-f_x]$, $y=[t_y,1-f_y]$, where $t_x,f_x,t_y,f_y\in[0,1]$ and $t_x+f_x\le 1,t_y+f_y\le 1$; the operation and relationship between vague values is defined as follows:

Definition 6. The minimum operation of vague values x and y is defined by

$$\begin{aligned}x\wedge y&=[\min(t_x,t_y),\min(1-f_x,1-f_y)]\\&=[\min(t_x,t_y),1-\max(f_x,f_y)]\end{aligned} \quad (7)$$

Definition 7. The maximum operation of vague values x and y is defined by

$$\begin{aligned}x\vee y&=[\max(t_x,t_y),\max(1-f_x,1-f_y)]\\&=[\max(t_x,t_y),1-\min(f_x,f_y)]\end{aligned} \quad (8)$$

Definition 8. The complement of vague value x is defined by

$$\bar{x}=[f_x,1-t_x] \quad (9)$$

Let A, B be two vague sets in the universe of discourse $U=\{u_1,...,u_i\}$, $A=\sum_{i=1}^{I}[t_A(u_i),1-f_A(u_i)]/u_i$,and

$B=\sum_{i=1}^{I}[t_B(u_i),1-f_B(u_i)]/u_i$, then the operations between vague sets are defined as follows.

Definition 9. The intersection of vague sets *A* and *B* is defined by

$$A\cap B=\sum_{i=1}^{I}\left\{[t_A(u_i),1-f_A(u_i)]\wedge[t_B(u_i),1-f_B(u_i)]\right\}/u_i \quad (10)$$

Definition 10. The union of vague sets *A* and *B* is defined as

$$A\cup B=\sum_{i=1}^{I}\left\{[t_A(u_i),1-f_A(u_i)]\vee[t_B(u_i),1-f_B(u_i)]\right\}/u_i \quad (11)$$

Definition 11. The complement of vague set *A* is defined as

$$\overline{A}=\sum_{i=1}^{I}[f_A(u_i),1-t_A(u_i)]/u_i \quad (12)$$

D.Similarity Measure between Vague Sets

The similarity measure between vague values $x=[t_x,1-f_x]$, $y=[t_y,1-f_y]$ is calculated as

$$S(x,y)=1-\frac{d(x,y)}{\sqrt{2}}=1-\frac{\sqrt{(t_x-t_y)^2+(f_x-f_y)^2}}{\sqrt{2}} \quad (13)$$

where $d(x,y)=\sqrt{(t_x-t_y)^2+(1-f_x-(1-f_y))^2}$ is the distance between *x* and *y*.

Let A, B be two vague sets in the universe of discourse $U=\{u_1,...,u_i\}$, $A=\sum_{i=1}^{I}[t_A(u_i),1-f_A(u_i)]/u_i$, and $B=\sum_{i=1}^{I}[t_B(u_i),1-f_B(u_i)]/u_i$, the similarity measure between *A* and *B* is defined as

$$S(A,B)=\frac{1}{I}\sum_{i=1}^{I}S(\mu_A(u_i),\mu_B(u_i)) \quad (14)$$

E. Comparison between Vague Sets

Definition 12. For vague value $x=[t_x,1-f_x]$, $y=[t_y,1-f_y]$, the probability of $x\geq y$ is defined as

$$P(x\geq y)=\frac{\max(0,l(x)+l(y)-\max(0,1-f_y-f_x))}{l(x)+l(y)} \quad (15)$$

where $l(x)=1-f_x-t_x$, $l(y)=1-f_y-t_y$ is the length of vague value *x*, *y*. The properties can easily be obtained as follows:

Property 1. $0\leq P(x\geq y)\leq 1$

Property 2. If $P(x\geq y)=P(y\geq z)$, then $P(x\geq y)=P(y\geq x)=0.5$

Property 3. $P(x\geq y)+P(y\geq x)=1$

Property 4. For any three vague values x, y, z, if

$P(x\geq y)\geq 0.5$, $P(y\geq z)\geq 0.5$, then $P(x\geq z)\geq 0.5$.

Definition 13. Let A, B be two vague sets in the universe of discourse $U=\{u_1,...,u_i\}$, $A=\sum_{i=1}^{I}[t_A(u_i),1-f_A(u_i)]/u_i$, and $B=\sum_{i=1}^{I}[t_B(u_i),1-f_B(u_i)]/u_i$, the probability of $A\geq B$ is defined as

$$P(A\geq B)=\frac{1}{I}\sum_{i=1}^{I}S(\mu_A(u_i)\geq\mu_B(u_i)) \quad (16)$$

F. Defuzzification of Vague Sets

Definition 14. For vague value $x=[t_x,1-f_x]$, the defuzzification function to get the precise value is defined as

$$D(x)=\frac{t_x}{t_x+f_x} \quad (17)$$

G.Weighted Sum of Vague Values

Definition 15. For *I* vague values $x_i=[t_{x_i},1-f_{x_i}]$, whose weights vector $\omega=\{\omega_1,...,\omega_j\},1\leq j\leq J$, where $\sum_{j=1}^{J}\omega_j=1$, are *J* precise values, the weighted sum of x_i is defined as

$$x_i=\sum_{j=1}^{J}\omega_j x_{ij}=\left[\sum_{j=1}^{J}\omega_j t_{ij},1-\sum_{j=1}^{J}\omega_j f_{ij}\right] \quad (18)$$

H.Neutrosophic Set Theory

Definition 16. Let *X* be a space of points (objects) with a class of elements in *X* denoted by *x*. A neutrosophic set A in *X* is summarized by a truth-membership function $T_{A(x)}$, an indeterminacy-membership function $I_{A(x)}$, and a falsity-membership function $F_{A(x)}$. The functions $T_{A(x)}, I_{A(x)}, F_{A(x)}$ are real standard or non-standard subsets of $]^{-}0,1^{+}[$.

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That is $T_{A(x)}: X \to]^{-}0,1^{+}[$, $I_{A(x)}: X \to]^{-}0,1^{+}[$, and $F_{A(x)}: X \to]^{-}0,1^{+}[$.

$$^{-}0 \le \sup T_{A(x)} + \sup I_{A(x)} + \sup F_{A(x)} \le 3^{+} \tag{19}$$

Definition 17. Let X be a space of points (objects), with a class of elements in X denoted by *x*. A single valued neutrosophic set *N* in *X* is summarized by a truth-membership function $T_{N(x)}$, an indeterminacy-membership function $I_{N(x)}$, and a falsity-membership function $F_{N(x)}$. Then a SVNS *N* can be denoted as follows:

$$N = \left\{ x, \left\langle T_{N(x)}, I_{N(x)}, F_{N(x)} \right\rangle x \in X \right\} \tag{20}$$

where $T_{N(x)}$, $I_{N(x)}$, $F_{N(x)} \in [0,1]$ for $\forall x \in X$. Meanwhile, the sum of $T_{N(x)}$, $I_{N(x)}$, and $F_{N(x)}$ fulfills the condition

$$0 \le T_{N(x)} + I_{N(x)} + F_{N(x)} \le 3. \tag{21}$$

Definition 18. Let $x = (T_x, I_x, F_x)$ and $y = (T_y, I_y, F_y)$ be two SVNNs, then operations can be defined as follows:

1. $x^C = (F_x, 1 - I_x, T_x)$
2. $x \oplus y = (T_x + T_y - T_x * T_y, I_x * I_y, F_x * F_y)$
3. $x \otimes y = (T_x * T_y, I_x + I_y - I_x * I_y, F_x + F_y - F_x * F_y)$ (22)
4. $\lambda x = (1 - (1 - T_x)^{\lambda}, (I_x)^{\lambda}, (F_x)^{\lambda}),\ \lambda > 0$
5. $x^{\lambda} = ((T_x)^{\lambda}, 1 - (1 - I_x)^{\lambda}, 1 - (1 - (F_x)^{\lambda}),\ \lambda > 0$

Definition 19. For two SVNNS $x = (T_x, I_x, F_x)$ and $y = (T_y, I_y, F_y)$, if $x \le y$ then $T_x \le T_y, I_x \ge I_y, F_x \ge F_y$.

Definition 20. Let *x* and *y* be any two SVNNs, then the Hamming distance between *x* and *y* can be defined as follows:

$$d_{Ha}(x,y) = \frac{1}{3}(|T_x - T_y| + |I_x - I_y| + |F_x - F_y|) \tag{23}$$

Definition 21. Let *x* and *y* be any two SVNNs, then the Euclidean distance between *x* and *y* can be defined as follows:

$$d_E(x,y) = \sqrt{\frac{1}{3}(T_x - T_y)^2 + (I_x - I_y)^2 + (F_x - F_y)^2)} \tag{24}$$

Definition 22. Let *x* and *y* be any two SVNNs, then the normalized generalized distance between x and y can be defined as follows:

$$d_G(x,y) = \frac{1}{3}(|T_x - T_y|^{\lambda} + |I_x - I_y|^{\lambda} + |F_x - F_y|^{\lambda})^{\frac{1}{\lambda}} \tag{25}$$

with the condition of $\lambda > 0$. When $\lambda = 1$, it is the Hamming distance; when $\lambda = 2$, it is the Euclidean distance.

Definition 23. Let $x = (T_x, I_x, F_x)$ be a SVNN, then the proposed score function $S(x)$ is defined as follows:

$$S(x) = \frac{2 + T_x - I_x - F_x}{3} \tag{26}$$

Definition 24. Let $x = (T_x, I_x, F_x)$ be a SVNN, then the proposed accuracy function $H(x)$ is defined as follows:

$$H(x) = T_x - F_x \tag{27}$$

Definition 25. Let *x* and *y* be any two SVNNs, If $S(x) < S(y), x < y$, when $S(x) = S(y)$, if $H(x) = H(y)$ and then $x = y$, else if $H(x) < H(y)$ and then $x < y$.

Definition 26. Let *x* and *y* be any two SVNNs, then the normalized single-valued neutrosophic Hausdorff distance between *x* and *y* is defined as follows:

$$D_{Hd}(x,y) = \max\left\{ |T_x - T_y|, |I_x - I_y|, |F_x - F_y| \right\} \tag{28}$$

Let *x*, *y* and *z* be any three SVNNs, the above defined the weighted single valued neutrosophic Hausdorff distance among *x*, *y* and *z* satisfies the following properties (1)−(4):

1. $D_{Hd}(x,y) \ge 0$;
2. $D_{Hd}(x,y) = 0$ if only if $x = y$;
3. $D_{Hd}(x,y) = D_{Hd}(y,x)$; (29)
4. If $x \subseteq y \subseteq z$, then $D_{Hd}(x,z) \ge D_{Hd}(x,y)$ and $D_{Hd}(x,z) \ge D_{Hd}(y,z)$.

Definition 27. The normalized generalized single-valued neutrosophic Hausdorff distance between *x* and *y* is defined as follows:

$$D_{gHd}(x,y) = \max\left\{ |T_x - T_y|^{\mu}, |I_x - I_y|^{\mu}, |F_x - F_y|^{\mu} \right\}^{\frac{1}{\mu}} \tag{30}$$

$\mu > 0$, when $\mu = 1, 2, ...$, it is the Hausdorff distance. $D_{Hd} = D_{gHl}$ is obtained with any two SVNNS.

Definition 28. The weighted parameter single-valued neutrosophic distance between *x* and *y* is is defined as follows:

$$\begin{aligned} D_{\omega p}(x,y) &= vD_g(x,y) + (1-v)D_{Hd}(x,y) \\ &= \frac{v}{3}(|T_x - T_y|^{\lambda} + |I_x - I_y|^{\lambda} + |Fx - Fy|^{\lambda})^{\frac{1}{\lambda}} \\ &+ (1-v)\max\left\{ |T_x - T_y|, |I_x - I_y|, |F_x - F_y| \right\} \end{aligned} \tag{31}$$

where $\lambda > 0$, and $0 \le v \le 1$.

If we set $\lambda = 1$, then the weighted parameter is obtained from the single-valued Hamming and Hausdorff distance which is described as follows:

$$D_{\omega HH}(x,y) = vD_{H\alpha}(x,y) + (1-v)D_{Hd}(x,y)$$
$$= \frac{v}{3}(|T_x - T_y| + |I_x - I_y| + |Fx - Fy|) \quad (32)$$
$$+(1-v)\max\{|T_x - T_y|, |I_x - I_y|, |F_x - F_y|\}$$

Definition 29. Let *x*, *y* and *z* be any three SVNNs, $S(x,y)$ is a similarity measure

$$S(x,y) = 1 - D_{wHH}(x,y)$$
$$= 1 - \frac{v}{3}(|T_x - T_y| + |I_x - I_y| + |Fx - Fy|) \quad (33)$$
$$-(1-v)\max\{|T_x - T_y|, |I_x - I_y|, |F_x - F_y|\}$$

It possesses the following properties:

1. $0 \le S(x,y) \le 1$;
2. $S(x,y) = 1$ if and only if $x = y$;
3. $S(x,y) = S(y,x)$; (34)
4. If $x \subseteq y \subseteq z$, then $S(x,z) \le S(x,y)$ and $S(x,z) \le S(y,z)$.

I. Linguistic Variables

Linguistic variables are often used in various extensions of MCDM methods to facilitate and enable decision-makers, i.e. respondents, to more accurately evaluate alternatives. In addition to the use of linguistic variables, i.e., their abbreviations, respondents can express their attitudes using the recommended crisp numerical values. However, if the respondents want it or it is necessary, the respondents can express their attitudes more precisely using numbers from the interval [0,1].

Table 1. Linguistic variables for expressing confidence levels

Linguistic terms	Label	Crisp numerical value	Permissible value range
Extremely High	EH	0.9	[0.8,1.0]
Very High	VH	0,8	[0.7,0.9]
High	H	0,7	[0.6,0.8]
Moderate High	MH	0,6	[0.5,0.7]
Moderate	M	0,5	[0.4,0.6]
Moderate Low	ML	0,4	[0.3,0.5]
Low	L	0,3	[0.2,0.4]
Very Low	VL	0,2	[0.1,0.3]
Extremely Low	EL	0,1	[0.0,0.2]

J. TOPSIS Multiple Criteria Decision Making Analysis Method

The TOPSIS method is based on the idea that the best alternative should have the shortest distance from an ideal solution [28]. It is assumed that it would be easy to define an ideal solution if each attribute receives a monotonically increasing or decreasing variation. Such a solution consists of the best achievable attribute values, while the worst solution consists of all the worst achievable attribute values.

Suppose that a multiple criteria decision making analysis problem having *I* alternatives, $a = \{a_1, ..., a_I\}, 1 \le i \le I$, and *J*, criteria, $g = \{g_1, ..., g_I\}, 1 \le j \le J$. Each alternative is evaluated with respect to the *J* criteria. All the values/ratings are assigned to alternatives with respect to the decision matrix denoted by $X = [x_{ij}]_{ixj}$. Let $\omega = \{\omega_1, ..., \omega_j\}, 1 \le j \le J$ be the weight vector of criteria satisfying $\sum_{j=1}^{J} \omega_j = 1$.

The TOPSIS method consists of the following procedural steps:

Step 1. Normalization of the decision matrix

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{K} x_{kj}^2}}, 1 \le i \le I, 1 \le j \le J \quad (35)$$

Step 2. Calculation of the weighted normalized decision matrix

$$v_{ij} = \omega_j r_{ij}$$

where ω_j represents the weight of the *j*th criterion.

Step 3. Determination of the ideal and negative ideal alternatives

$$A^+ = \{v_1^+, ..., v_j^+\} = \left\{\left(\max_j v_{ij} \mid j \in \Omega_b\right), \left(\min_j v_{ij} \mid j \in \Omega_c\right)\right\} \quad (36)$$

$$A^- = \{v_1^-, ..., v_j^-\} = \left\{\left(\min_j v_{ij} \mid j \in \Omega_b\right), \left(\max_j v_{ij} \mid j \in \Omega_c\right)\right\} \quad (37)$$

where Ω_b is the set of benefit criteria, and Ω_c is the set of cost criteria.

Step 4. Calculation of the distance of the existing alternatives from ideal and negative ideal alternatives

$$S_i^+ = \sqrt{\sum_{j=1}^{J} (v_{ij} - v_j^+)^2} \quad (38)$$

$$S_i^- = \sqrt{\sum_{j=1}^{J} (v_{ij} - v_j^-)^2} \quad (39)$$

Step 5. Calculation of the relative closeness to the ideal alternatives

$$RC_i = \frac{S_i^-}{S_i^+ + S_i^-}, \ 1 \le i \le I, 0 \le RC_i \le 1 \quad (40)$$

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Step 6. Rank the alternatives

The alternatives are ranked in a descending order, the bigger is the RC_i, the better is the alternative a_i.

III. Application

In real life, one may think that an object belongs to a set to a certain degree, but it is also possible to be unsure of this. In other words, there may be hesitation about the degree of membership. In fuzzy set theory there is no way to include this hesitation in the membership degree. To incorporate this hesitancy into the TOPSIS method, this methodology was extended to vague sets. Suppose a vague set consists of $a=\{a_1,...,a_I\}, 1\le i\le I$ are I possible alternatives among which the decision makers have to choose, $g=\{g_1,...,g_J\}, 1\le j\le J$ are criteria with which alternative performance are measured, x_{ij} is the rating of alternative a_i with respect to criterion g_j. The fighter aircraft candidates were evaluated under six criteria including costability g_1, payloadability g_2, maneuverability g_3, speedability g_4, stealthability g_5, and survivability g_6. The attribute ratings of fighter aircraft are linguistic variables. Here these linguistic variables can be expressed in vague values as shown in Table 1.

A vague-valued MCDMA problem can be concisely expressed in a matrix format as

$$\begin{pmatrix} [t_{A_1}(x_1),t^*_{A_1}(x_1)] & \dots & [t_{A_1}(x_j),t^*_{A_1}(x_j)] \\ \vdots & \ddots & \vdots \\ [t_{A_i}(x_1),t^*_{A_i}(x_1)] & \cdots & [t_{A_i}(x_j),t^*_{A_i}(x_j)] \end{pmatrix}$$

$$\omega=\{\omega_1,...,\omega_j\}, 1\le j\le J$$

where $t^*_{A_i}(x_i)=1-f_{A_i}(x_i)$, ω_j is the weight of criterion g_j and is also a vague set. The vague-valued decision matrix can also be represented with hesitation degree $\pi_{A_i}(x_j)$ as

$$\begin{pmatrix} [t_{A_1}(x_1),t^*_{A_1}(x_1),\pi_{A_1}(x_1)] & \dots & [t_{A_1}(x_j),t^*_{A_1}(x_j),\pi_{A_1}(x_j)] \\ \vdots & \ddots & \vdots \\ [t_{A_i}(x_1),t^*_{A_i}(x_1),\pi_{A_i}(x_1)] & \cdots & [t_{A_i}(x_j),t^*_{A_i}(x_j),\pi_{A_i}(x_j)] \end{pmatrix}$$

Vague-valued weighted decision matrix is given by

$$\left\{\left\langle x, t_{A_i}(x_j)t_{\omega_j}(x_j), 1-t^*_{A_i}(x_j)t^*_{\omega_j}(x_j) \mid x\in X\right\rangle\right\}$$

$$\begin{pmatrix} [t_{A_1}\omega(x_1),t^*_{A_1}\omega(x_1),\pi_{A_1}\omega(x_1)] & \dots & [t_{A_1}\omega(x_j),t^*_{A_1}\omega(x_j),\pi_{A_1}\omega(x_j)] \\ \vdots & \ddots & \vdots \\ [t_{A_i}\omega(x_1),t^*_{A_i}\omega(x_1),\pi_{A_i}\omega(x_1)] & \cdots & [t_{A_i}\omega(x_j),t^*_{A_i}\omega(x_j),\pi_{A_i}\omega(x_j)] \end{pmatrix}$$

Vague positive ideal solution (VPIS) is determined by

$$a^+=\left\{\begin{matrix} \Big\langle x_j, ((\max_i t_{A_i}\omega(x_j)/j\in\Omega_b), ((\min_i t^*_{A_i}\omega(x_j)/j\in\Omega_c)), \\ ((\min_i t_{A_i}\omega(x_j)/j\in\Omega_b), ((\max_i t^*_{A_i}\omega(x_j)/j\in\Omega_c)), \Big\rangle \end{matrix}\right\}$$

$$a^+=\left\{\left\langle x_1, t_{A^+}\omega(x_1), t^*_{A}\omega(x_1)\right\rangle,...,\left\langle x_J, t_{A^+}\omega(x_j), t^*_{A^+}\omega(x_j)\right\rangle\right\}$$

Vague negative ideal solution (VNIS) is determined by

$$a^-=\left\{\begin{matrix} \Big\langle x_j, ((\min_i t_{A_i}\omega(x_j)/j\in\Omega_b), ((\max_i t^*_{A_i}\omega(x_j)/j\in\Omega_c)), \\ ((\max_i t_{A_i}\omega(x_j)/j\in\Omega_b), ((\min_i t^*_{A_i}\omega(x_j)/j\in\Omega_c)), \Big\rangle \end{matrix}\right\}$$

$$a^-=\left\{\left\langle x_1, t_{A^-}\omega(x_1), t^*_{A^-}\omega(x_1)\right\rangle,...,\left\langle x_J, t_{A^-}\omega(x_j), t^*_{A^-}\omega(x_j)\right\rangle\right\}$$

Vague distance measures (VPIS S_i^+, VNIS S_i^-) are respectively calculated by

$$S_i^+=\{\frac{1}{2J}\sum_{j=1}^{J}[(t_A\omega(x_1)-t_{A^+}\omega(x_1))^2+$$

$$(t^*_{A_i}\omega(x_1)-t^*_{A^+}\omega(x_1))^2+$$

$$(\pi_{A_i}\omega(x_1)-\pi_{A^+}\omega(x_1))^2]\}^{1/2}$$

$$S_i^-=\{\frac{1}{2J}\sum_{j=1}^{J}[(t_{A_i}\omega(x_1)-t_{A^-}\omega(x_1))^2+$$

$$(t^*_{A_i}\omega(x_1)-t^*_{A^-}\omega(x_1))^2+$$

$$(\pi_{A_i}\omega(x_1)-\pi_{A^-}\omega(x_1))^2]\}^{1/2}$$

The relative closeness to the ideal alternatives is calculated by

$$RC_i=\frac{S_i^-}{S_i^++S_i^-},\ 1\le i\le I, 0\le RC_i\le 1$$

Conclusively, TOPSIS model integrated with the vague set theory was applied to the fighter aircraft selection problem by following the procedure through equations (35) - (40). The results of the procedural computations are shown in Tables 2-6. After analyzing the results, the fighter aircraft a_4 was selected as the best candidate for the Air Force.

IV. Conclusion

For the selection of fighter aircraft, a TOPSIS method based on vague set theory is proposed to solve multiple criteria decision-making problems characterized by uncertain human judgments. To demonstrate the effectiveness of the proposed TOPSIS method, a case study was considered to evaluate and compare the quality of five fighter aircraft. This study helps defense decision makers in the selection of fighter aircraft to know the requirements of fighter aircraft and the priority of increasing their defense capabilities.

Given that in some cases it is difficult to determine the exact values of the attributes, and their values are considered as vague data. However, fuzzy sets cannot resolve the uncertainty in the data in the form of hesitation.

As vague sets are efficient to deal with this hesitation available in the information provided by the decision maker, TOPSIS is expanded for vague sets to determine the most preferred choice among all possible options when the data is vague. Here, the ratings of the alternatives are taken as vague sets. In this approach, a normalized Euclidean distance measure is also taken into account to calculate the distance of an alternative from the vague positive ideal solution and its distance from the negative ideal solution. The smaller the distance of the evaluated alternative from the positive ideal solution and the further away from the vague negative ideal solution, the better the ranking.

Table 2. Vague valued decision matrix

g_j	g_1 min		g_2 max		g_3 max		g_4 max		g_5 max		g_6 max	
ω_j	0,12		0,13		0,14		0,16		0,23		0,22	
a_1	0,8	1	0,7	0,9	0,4	0,6	0,6	0,8	0,4	0,6	0,3	0,5
a_2	0,3	0,5	0,3	0,5	0,5	0,7	0,4	0,6	0,3	0,5	0,6	0,8
a_3	0,4	0,6	0,4	0,6	0,6	0,8	0,2	0,4	0,5	0,7	0,4	0,6
a_4	0,6	0,8	0,7	0,9	0,8	1	0,3	0,5	0,8	1	0,7	0,9
a_5	0,2	0,4	0,6	0,8	0,4	0,6	0,7	0,9	0,1	0,3	0,5	0,7

Table 3. The weights of the criteria

g_j	g_1	g_2	g_3	g_4	g_5	g_6
ω_j	0,12	0,13	0,14	0,16	0,23	0,22

Table 4. Vague valued weighted decision matrix

g_j	g_1 min			g_2 max			g_3 max			g_4 max			g_5 max			g_6 max		
ω_j	0,12			0,13			0,14			0,16			0,23			0,22		
a_1	0,096	0,12	0,024	0,091	0,117	0,026	0,056	0,084	0,028	0,096	0,128	0,032	0,092	0,138	0,046	0,066	0,11	0,044
a_2	0,036	0,06	0,024	0,039	0,065	0,026	0,07	0,098	0,028	0,064	0,096	0,032	0,069	0,115	0,046	0,132	0,176	0,044
a_3	0,048	0,072	0,024	0,052	0,078	0,026	0,084	0,112	0,028	0,032	0,064	0,032	0,115	0,161	0,046	0,088	0,132	0,044
a_4	0,072	0,096	0,024	0,091	0,117	0,026	0,112	0,14	0,028	0,048	0,08	0,032	0,184	0,23	0,046	0,154	0,198	0,044
a_5	0,024	0,048	0,024	0,078	0,104	0,026	0,056	0,084	0,028	0,112	0,144	0,032	0,023	0,069	0,046	0,11	0,154	0,044

Table 5. Vague Positive-Ideal Solution (VPIS) and the Vague Negative-Ideal Solution (VNIS)

a^+	0,024	0,048	0,024	0,091	0,117	0,026	0,112	0,14	0,028	0,112	0,144	0,032	0,184	0,23	0,046	0,154	0,198	0,044
a^-	0,096	0,12	0,024	0,039	0,065	0,026	0,056	0,084	0,028	0,032	0,064	0,032	0,023	0,069	0,046	0,066	0,11	0,044

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Table 6. Vague distance measures (VPIS S_i^+, VNIS S_i^-), relative closeness coefficient (RC_i) and normalized rankings (J_i) of the stealth fighter aircraft alternatives

a_i	S_i^+	R_i	S_i^-	R_i	RC_i	R_i	J_i
a_1	0,064	2	0,044	4	0,406	5	0,169
a_2	0,059	3	0,043	5	0,425	3	0,177
a_3	0,055	4	0,045	3	0,449	2	0,187
a_4	0,033	5	0,082	1	0,715	1	0,297
a_5	0,072	1	0,050	2	0,410	4	0,170

REFERENCES

[1] Ardil, C., Bilgen, S. (2017). Online Performance Tracking. SocioEconomic Challenges, 1(3), 58-72. ISSN (print) – 2520-6621.
[2] Ardil, C. (2018) Multidimensional Performance Tracking. International Journal of Computer and Systems Engineering, Vol:12, No:5,320-349
[3] Ardil, C. (2021). Architectural Acoustic Modeling for Predicting Reverberation Time in Room Acoustic Design Using Multiple Criteria Decision Making Analysis. International Journal of Architectural and Environmental Engineering, 15(9), 418 - 423.
[4] Ardil, C. (2019). Aircraft Selection Using Multiple Criteria Decision Making Analysis Method with Different Data Normalization Techniques. International Journal of Industrial and Systems Engineering, 13(12), 744 - 756.
[5] Ardil, C. , Pashaev, A. , Sadiqov, R. , Abdullayev, P. (2019). Multiple Criteria Decision Making Analysis for Selecting and Evaluating Fighter Aircraft. International Journal of Transport and Vehicle Engineering, 13(11), 683 - 694.
[6] Ardil, C. (2019). Military Fighter Aircraft Selection Using Multiplicative Multiple Criteria Decision Making Analysis Method. International Journal of Mathematical and Computational Sciences, 13(9), 184 - 193.
[7] Saaty, T. L. (1990). How to make a decision: The Analytic Hierarchy Process. European Journal of Operational Research, 48(1), 9-26. doi: 10.1016/0377-2217(90)90057-I
[8] Saaty, T. L. (2008). Decision making with the analytic hierarchy process. International Journal of Services Sciences, 1(1), 83-98. doi: 10.1504/IJSSCI.2008.017590
[9] Saaty, T.L. (1980). Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation. McGraw-Hill, New York.
[10] Ardil, C. (2021). Advanced Jet Trainer and Light Attack Aircraft Selection Using Composite Programming in Multiple Criteria Decision Making Analysis Method. International Journal of Aerospace and Mechanical Engineering, 15(12), 486 - 491.
[11] Ardil, C. (2021). Comparison of Composite Programming and Compromise Programming for Aircraft Selection Problem Using Multiple Criteria Decision Making Analysis Method. International Journal of Aerospace and Mechanical Engineering, 15(11), 479 - 485.
[12] Ardil, C. (2018) Multidimensional Compromise Optimization for Development Ranking of the Gulf Cooperation Council Countries and Turkey. International Journal of Mathematical and Computational Sciences Vol:12, No:6, 131-138.
[13] Ardil, C. (2018) Multidimensional Compromise Programming Evaluation of Digital Commerce Websites. International Journal of Computer and Information Engineering Vol:12, No:7, 556-563.
[14] Ardil, C. (2018) Multicriteria Decision Analysis for Development Ranking of Balkan Countries. International Journal of Computer and Information Engineering Vol:12, No:12, 1118-1125.
[15] Ardil, C. (2021). Freighter Aircraft Selection Using Entropic Programming for Multiple Criteria Decision Making Analysis. International Journal of Mathematical and Computational Sciences, 15(12), 125 - 132.
[16] Ardil, C. (2020). A Comparative Analysis of Multiple Criteria Decision Making Analysis Methods for Strategic, Tactical, and Operational Decisions in Military Fighter Aircraft Selection. International Journal of Aerospace and Mechanical Engineering, 14(7), 275 - 288.
[17] Ardil, C. (2020). Aircraft Selection Process Using Preference Analysis for Reference Ideal Solution (PARIS). International Journal of Aerospace and Mechanical Engineering, 14(3), 80 - 93.
[18] Ardil, C. (2020). Regional Aircraft Selection Using Preference Analysis for Reference Ideal Solution (PARIS). International Journal of Transport and Vehicle Engineering, 14(9), 378 - 388.
[19] Ardil, C. (2020). Trainer Aircraft Selection Using Preference Analysis for Reference Ideal Solution (PARIS). International Journal of Aerospace and Mechanical Engineering, 14(5), 195 - 209.
[20] Ardil, C. (2020). Software Product Quality Evaluation Model with Multiple Criteria Decision Making Analysis. International Journal of Computer and Information Engineering, 14(12), 486 - 502.
[21] Roy, B. (1991). The outranking approach and the foundation of ELECTRE methods. Theory and Decision, 31(1), 49–73.
[22] Fei, L., Xia, J., Feng, Y., Liu, L. (2019) An ELECTRE-Based Multiple Criteria Decision Making Method for Supplier Selection Using Dempster-Shafer Theory. IEEE Access, 7, 84701-84716.
[23] Brans JP., Mareschal B. (2005). Promethee Methods. In: Multiple Criteria Decision Analysis: State of the Art Surveys. International Series in Operations Research & Management Science, vol 78, pp 163-186. Springer, New York, NY. https://doi.org/10.1007/0-387-23081-5_5.
[24] Brans, J., Ph. Vincke. (1985). A Preference Ranking Organisation Method: (The PROMETHEE Method for Multiple Criteria Decision-Making). Management Science, 31(6), 647-656.
[25] Brans, J.P., Macharis, C., Kunsch, P.L., Chevalier, A., Schwaninger, M., (1998). Combining multicriteria decision aid and system dynamics for the control of socio-economic processes. An iterative real-time procedure. European Journal of Operational Research 109, 428-441.
[26] Brans, J.P., Vincke, Ph., Mareschal, B., (1986). How to select and how to rank projects: the PROMETHEE method. European Journal of Operational Research, 24, 228-238.
[27] Ardil, C. (2020) Facility Location Selection using Preference Programming. International Journal of Industrial and Systems Engineering, 14(1), 1 - 12.
[28] Hwang, C.L.; Yoon, K. (1981). Multiple Attribute Decision Making: Methods and Applications. New York: Springer-Verlag.
[29] Chu, T.C. (2002. Facility location selection using fuzzy TOPSIS under group decisions", International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, Vol. 10 No. 6, pp. 687-701.
[30] Choudhary, D. and Shankar, R. (2012. A STEEP-fuzzy AHP-TOPSIS framework for evaluation and selection of thermal power plant location: a case study from India", Energy, Vol. 42 No. 1, pp. 510-521.
[31] Zavadskas, E.K., Mardani, A., Turskis, Z., Jusoh, A., Nor, K.M. (2016) Development of TOPSIS method to solve complicated decision-making problems: An overview on developments from 2000 to 2015. International Journal of Information Technology & Decision Making, 15, 645-682.
[32] Ardil, C. (2019) Scholar Index for Research Performance Evaluation Using Multiple Criteria Decision Making Analysis. International Journal of Educational and Pedagogical Sciences, Vol:13, No:2, 93-105.
[33] Ardil, C. (2019). Fighter Aircraft Selection Using Technique for Order Preference by Similarity to Ideal Solution with Multiple Criteria Decision Making Analysis. International Journal of Transport and Vehicle Engineering, 13(10), 649 - 657.
[34] Ardil, C. (2022). Military Attack Helicopter Selection Using Distance Function Measures in Multiple Criteria Decision Making Analysis.

International Journal of Aerospace and Mechanical Engineering, 16(2), 20 - 27.
[35] Opricovic, S. (1998). Multicriteria Optimization of Civil Engineering Systems. PhD Thesis, Faculty of Civil Engineering, Belgrade (in Serbian).
[36] Opricovic, S. (2007). A fuzzy compromise solution for multicriteria problems. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 15(3), 363–380.
[37] Opricovic, S., Tzeng, G.-H. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. European Journal of Operational Research, 156(2), 445–455.
[38] Modarres, M., Sadi-Nezhad, S. (2005). Fuzzy simple additive weighting method by preference ratio, Intelligent Automation and Soft Computing, 235-244.
[39] Kaur, P., Kumar, S. (2013). An Intuitionistic Fuzzy Simple Additive Weighting Method for selection of vendor. ISOR Journal Business and Management, 78-81.
[40] Sagar, M.K., Jayaswal, P., Kushwah, K. (2013). Exploring Fuzzy SAW Method for Maintenance strategy selection problem of Material Handling Equipment, (2013), ISSN 22 77 – 4106.
[41] Wang,Y.J. (2015). A fuzzy multi-criteria decision making model based on additive weighting method and preference relation, Applied Soft Computing, 30,412-420.
[42] Roszkowska, E., Kacprzak, D. (2016). The fuzzy saw and fuzzy TOPSIS procedures based on ordered fuzzy numbers. Inf. Sci., 369, 564-584.
[43] Zhang, L., Xu, X., Tao, L. (2013) Some Similarity Measures for Triangular Fuzzy Number and Their Applications in Multiple Criteria Group Decision-Making, Journal of Applied Mathematics, vol. 2013, Article ID 538261, 1-7 pages, 2013.
[44] Ardil, C. (2021). Fighter Aircraft Evaluation and Selection Process Based on Triangular Fuzzy Numbers in Multiple Criteria Decision Making Analysis Using the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). International Journal of Computer and Systems Engineering, 15(12), 402 - 408.
[45] Ardil, C. (2021). Military Combat Aircraft Selection Using Trapezoidal Fuzzy Numbers with the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). International Journal of Computer and Information Engineering, 15(12), 630 - 635.
[46] Ardil, C. (2021). Neutrosophic Multiple Criteria Decision Making Analysis Method for Selecting Stealth Fighter Aircraft. International Journal of Aerospace and Mechanical Engineering, 15(10), 459 - 463.
[47] Ardil, C. (2022). Aircraft Selection Problem Using Decision Uncertainty Distance in Fuzzy Multiple Criteria Decision Making Analysis. International Journal of Mechanical and Industrial Engineering, 16(3), 62 - 69.
[48] Ardil, C. (2022). Fighter Aircraft Selection Using Neutrosophic Multiple Criteria Decision Making Analysis. International Journal of Computer and Systems Engineering, 16(1), 5 - 9.
[49] Ardil, C. (2021). Airline Quality Rating Using PARIS and TOPSIS in Multiple Criteria Decision Making Analysis. International Journal of Industrial and Systems Engineering, 15(12), 516 - 523.
[50] Ardil, C. (2022).Fuzzy Uncertainty Theory for Stealth Fighter Aircraft Selection in Entropic Fuzzy TOPSIS Decision Analysis Process. International Journal of Aerospace and Mechanical Engineering, 16(4), 93 - 102.
[51] Chen, S.J., Hwang, C.L. (1992). Fuzzy Multiple Factor Decision Making Methods and Applications, Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, Berlin.
[52] Dyer,J.S., Fishburn,P.C., Steuer,R.E., Wallenius,J., Zionts,S. (1992). Multiple criteria decision making, Multiattribute utility theory: The next ten years, Management Sci. 38 (5), 645–654.
[53] Delgado,M., Verdegay,J.L., Vila,M.A. (1992). Linguistic decision making models, Int. J. Intelligent System 7, 479–492.
[54] Herrera, F., Herrera-Viedma,E., Verdegay, J.L.(1996). A model of consensus in group decision making under linguistic assessments, Fuzzy Sets and Systems 78,73–87.
[55] Hsu, H.M., Chen, C.T. (1997). Fuzzy credibility relation method for multiple criteria decision-making problems, Inform. Sci. 96,79–91.
[56] Chen, C.-T. (2000). Extensions of the TOPSIS for group decision-making under fuzzy environment. Fuzzy Sets Syst. 114, 1–9.
[57] Chen, S. H., Wang, S. T., Chang, S. M. (2006). Some properties of Graded Mean Integration representation of L-R type fuzzy numbers. Tamsui Oxford Journal of Mathematical Sciences, 22(2), 185-208.
[58] Zadeh L.A., (1965). Fuzzy Sets. Information and Control, 8, 338-353.
[59] Bellman, R.E., Zadeh, L.A. (1970). Decision-making in a fuzzy environment. Management Science, 17(4), 141–164.
[60] Zadeh, L.A. (1975). The concept of a linguistic variable and its application to approximate reasoning, Inform. Sci. 8, 199–249(I), 301–357(II).
[61] Atanassov K. (1986). Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, Vol. 20(1), 87-96.
[62] Atanassov K., Gargov G. (1989).Interval-valued intuitionistic fuzzy sets. Fuzzy Sets Syst. 31:343–349.
[63] Gau W.L., Buehrer D.J. (1993). Vague sets. IEEE Trans. Syst. Man Cybern. 23:610–613.
[64] Chen S.M. (1995). Measures of similarity between vague sets. Fuzzy Sets yst. 74:217–223.
[65] Bustince, H., Burillo, P.(1996). Vague sets are intuitionistic fuzzy sets.Fuzzy Sets and Systems 79, 403-405.
[66] Xu W., Ma J., Wang S., Hao G. (2010). Vague soft sets and their properties. Comput. Math. Appl.59:787–794.
[67] Wang C., Qu A. (2013). Entropy, similarity measure and distance measure of vague soft sets and their relations. Inf. Sci.244:92–106.
[68] Selvachandran, G., Garg, H., Quek, S. G. (2018). Vague Entropy Measure for Complex Vague Soft Sets. Entropy, 20(6), 403.
[69] Zhang,Q., Zhao, F.,Yang, J. (2019). The Uncertainty Analysis of Vague Sets
[70] Xu,G., Wang, Y., Leng, J. (2022). Improved PBFT Algorithm Based on Vague Sets. Sec. and Commun. Netw. 2022.
[71] Smarandache, F. (2019). Neutrosophic Set is a Generalization of Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set, Spherical Fuzzy Set, and q-Rung Orthopair Fuzzy Set, while Neutrosophication is a Generalization of Regret Theory, Grey System Theory, and Three-Ways Decision (revisited) . Journal of New Theory, (29), 1-31.
[72] Smarandache, F. (2018). Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets - Revisited. Neutrosophic Sets and Systems: 21 / 2018 pp. 153-166.
[73] Smarandache, F. (2021). Plithogenic Probability & Statistics are generalizations of MultiVariate Probability & Statistics.Neutrosophic Sets and Systems, Vol. 45.

Open Science Index, Aerospace and Mechanical Engineering Vol:16, No:5, 2022 publications.waset.org/10012555.pdf