Mobile Robot Control by Von Neumann Computer

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***Abstract*—**The digital control system of mobile robots (MR) control is considered. It is shown that sequential interpretation of control algorithm operators, unfolding in physical time, suggests the occurrence of time delays between inputting data from sensors and outputting data to actuators. Another destabilizing control factor is presence of backlash in the joints of an actuator with an executive unit. Complex model of control system, which takes into account the dynamics of the MR, the dynamics of the digital controller and backlash in actuators, is worked out. The digital controller model is divided into two parts: the first part describes the control law embedded in the controller in the form of a control program that realizes a polling procedure when organizing transactions to sensors and actuators. The second part of the model describes the time delays that occur in the Von Neumann-type controller when processing data. To estimate time intervals, the algorithm is represented in the form of an ergodic semi-Markov process. For an ergodic semi-Markov process of common form, a method is proposed for estimation a wandering time from one arbitrary state to another arbitrary state. Example shows how the backlash and time delays affect the quality characteristics of the MR control system functioning.

***Keywords*—**Mobile robot, backlash, control algorithm, Von Neumann controller, semi-Markov process, time delay.

I. Introduction

MRs are now widely used in various fields of human activity, such as industry, transport, military, ecology, etc. [1]-[4]. Despite the difference of movement habitat (aerial, terrestrial, above-water, underwater), the control systems of all type robots solve the same problems: stabilization of the vehicle spatial orientation and establishing of the required longitudinal velocity. As a rule, MR control systems are digital and realized on the Von Neumann controller basis [5], [6]. The sequential interpretation of control algorithm operators, unfolding in physical time, suggests the emerging of time delays when polling sensors and actuators [7]. In turn, in real structures the drives are loaded onto the rudders, throttles, manipulators, grips, etc., which implies the presence of backlash in the joints of a drive with an executive unit. Both time delays, generated by the digital controller when polling procedure, and the backlash in actuators, born the problem of ensuring the required quality of MR control at the system design stage [8]-[10]. Solving the problem implies the development of an adequate model that takes into account the dynamics of the MR as object under control, the dynamics of the digital controller and backlash in actuators.

For taking both factors into account in system design, it is necessary to have adequate model being based, from one side, on non-linear control theory for estimation of influence backlash and time delay on system performance, and from the other side, on the semi-Markov processes theory [11]-[13] for evaluation of time lags [14]. Methods of assessing MR performance at the design stage, which take into account soft time complexity, are not widespread, that confirms necessity and relevancy of investigation in the area.

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II. Structure of Digital Control System

Structure of the MR control system is shown on Fig. 1 [15]. As it follows from the figure, it includes actually MR and Von Neumann type digital controller (DC). Into DC aim vector $\boldsymbol{F}(s)=[F_1(s), F_2(s)]^\theta$ and feedback vector $\boldsymbol{X}_0(s)=[X_{0,1}(s), X_{0,2}(s)]^\theta$, where s is Laplace differentiation operator, θ is the transpose operation sign, and transform into computer data $\boldsymbol{F}_c(s)=[F_{c.1}(s), F_{c,2}(s)]^\theta$, $\boldsymbol{X}_{c,0}(s)=[X_{c,0,1}(s), X_{c,0,2}(s)]^\theta$ (on Fig. 1 is not shown). On the base of $\boldsymbol{F}_c(s)$ and $\boldsymbol{X}_{c,0}(s)$ controller calculates values of control signal data $\boldsymbol{U}_c(s)=[U_{c,1}(s), U_{c,2}(s)]^\theta$ (on Fig. 1 is not shown), being transmitted through the interface, are transformed to analogue control signal vector $\boldsymbol{U}(s)=[U_1(s), U_2(s)]^\theta$, which feeds the actuators inputs. Actuators through mechanical assembles with a backlash transmit the movement $\tilde{\boldsymbol{V}}(s)=[\tilde{V}_1(s), \tilde{V}_2(s)]^\theta$ on rudders, whose repositioning $\boldsymbol{V}(s)=[V_1(s), V_2(s)]^\theta$ affects MR. As a result of affection, MR state $\boldsymbol{X}(s)=[X_1(s), X_2(s)]^\theta$ changes, that is measured by sensors, which form feedback signal $\boldsymbol{X}_0(s)=[X_{0,1}(s), X_{0,2}(s)]^\theta$.

Dynamics of physical processes in MR is performed with the matrix equation

$$\boldsymbol{X}(s)=\boldsymbol{W}(s)\cdot\boldsymbol{V}(s), \tag{1}$$

where $\boldsymbol{W}(s)$ is the matrix of transfer functions, which describe MR dynamics;

$$\boldsymbol{W}(s)=\begin{bmatrix} W_{11}(s) & W_{12}(s) \\ W_{21}(s) & W_{22}(s) \end{bmatrix}. \tag{2}$$

Sensors with transfer functions, defined by diagonal feedback matrix $\boldsymbol{W}_0(s)$, measure MR state and transform it to feedback signal $\boldsymbol{X}_0(s)$:

$$\boldsymbol{X}_0(s)=\begin{bmatrix} W_{0,1}(s) & 0 \\ 0 & W_{0,2}(s) \end{bmatrix}\boldsymbol{X}(s), \tag{3}$$

where $W_{0,1}(s)$, $W_{0,2}(s)$ are transfer functions of the first and second sensors, correspondingly.

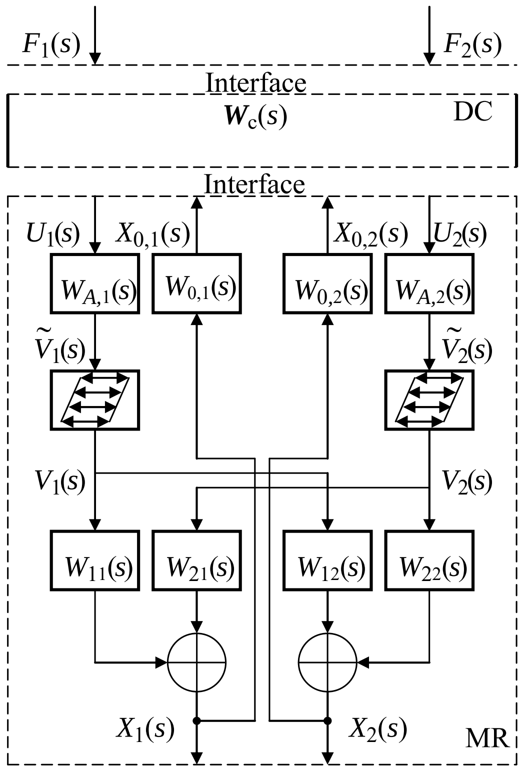

Fig. 1 Flowchart of MR digital control system

For processing signal vectors, $\boldsymbol{F}(s)$ and $\boldsymbol{X}_0(s)$ are into transformed data vectors $\boldsymbol{F}_c(s)$ and $\boldsymbol{X}_{c,0}(s)$, which are inputted into controller element-by-element. Transformation and Inputting of vectors unfold in physical time, so between forming vectors at sensors output and beginning of its processing there is time delays, and vectors $\boldsymbol{F}_c(s)$ and $\boldsymbol{X}_{c,0}(s)$ may be defined as:

$$\boldsymbol{F}_c(s) = \boldsymbol{N}_f(s) \cdot \boldsymbol{F}(s); \tag{4}$$

$$\boldsymbol{X}_{0,c}(s) = \boldsymbol{N}_0(s) \cdot \boldsymbol{X}_0(s); \tag{5}$$

where $\boldsymbol{N}_f(s)$ and $\boldsymbol{N}_0(s)$ are delay matrices;

$$\boldsymbol{N}_f(s) = \begin{bmatrix} \exp(-\tau_{f,1}s) & 0 \\ 0 & \exp(-\tau_{f,2}s) \end{bmatrix}; \tag{6}$$

$$\boldsymbol{N}_0(s) = \begin{bmatrix} \exp(-\tau_{0,1}s) & 0 \\ 0 & \exp(-\tau_{0,2}s) \end{bmatrix}; \tag{7}$$

$\tau_{f,1}$, $\tau_{f,2}$ are time delays of processing signals $F_1(s)$, $F_2(s)$ correspondingly; $\tau_{0,1}$, $\tau_{0,2}$ are time delays of processing signals $X_{0,1}(s)$, $X_{0,2}(s)$ correspondingly. Similarly, elements of vector $\boldsymbol{U}(s)$ emerging on input of actuators with time delays $\tau_{u,1}$, $\tau_{u,2}$, and

$$\boldsymbol{U}(s) = \boldsymbol{N}_u(s) \cdot \boldsymbol{U}_c(s), \tag{8}$$

where

$$\boldsymbol{N}_u(s) = \begin{bmatrix} \exp(-\tau_{u,1}s) & 0 \\ 0 & \exp(-\tau_{u,2}s) \end{bmatrix}. \tag{9}$$

Signal $\boldsymbol{U}(s)$ drives the linear part of actuator, forming the movement of actuator shaft as follows:

$$\tilde{\boldsymbol{V}}(s) = \begin{bmatrix} W_{A,1}(s) & 0 \\ 0 & W_{A,2}(s) \end{bmatrix} \boldsymbol{U}(s), \tag{10}$$

where $W_{A,1}(s)$, $W_{A,2}(s)$ are transfer functions of the liner part of actuators description (from input till mechanical assembles with a backlash).

The Von Neumann controller processes signals $\boldsymbol{F}_c(s)$ and $\boldsymbol{X}_{c,0}(s)$ are performed in the discrete form, so differentiation and integration operations of control action are computed as the finite difference and the finite summation. Applying to finite difference and finite summation Laplace transform, one can obtain so-called *Z*-transform of discrete function [16], [17], which has all main properties of Laplace transform of continual functions, which are used for obtaining transfer functions, namely, linearity and possibility of substitution operation of finite difference in discrete argument domain by the operation of *Z*-transform function multiplying on the differentiating operator. So, when sampling period tends to zero, finite-difference and finite summation operations may be performed as continual differentiation and integration [18]. So, for simulation of DC data processing may be used the same mathematical apparatus of transfer functions, that is used for simulation of MR.

With use the notion of transfer function linear data processing in controller may be performed as

$$\boldsymbol{U}_c(s) = \boldsymbol{W}_{f,c}(s) \cdot \boldsymbol{F}_c(s) - \boldsymbol{W}_{0,c}(s) \cdot \boldsymbol{X}_{0,c}(s), \tag{11}$$

where $\boldsymbol{W}_{f,c}(s)$ and $\boldsymbol{W}_{0,c}(s)$ are transfer functions' matrices;

$$\boldsymbol{W}_{f,c}(s) = \begin{bmatrix} W_{f,c,11}(s) & W_{f,c,12}(s) \\ W_{f,c,21}(s) & W_{f,c,22}(s) \end{bmatrix}; \tag{12}$$

$$\boldsymbol{W}_{0,c}(s) = \begin{bmatrix} W_{0,c,11}(s) & W_{0,c,12}(s) \\ W_{0,c,21}(s) & W_{0,c,22}(s) \end{bmatrix}. \tag{13}$$

In (11) and (12) $W_{f,c,ij}(s)$ are transfer functions, describing processing data $F_{c,i}(s)$ to obtain data $U_{c,j}(s)$, $i \in \{1,2\}$, $j \in \{1,2\}$; $W_{0,c,ij}(s)$ are transfer functions, describing processing data $X_{0,c,i}(s)$ to obtain data $U_{c,j}(s)$, $i \in \{1,2\}$, $j \in \{1,2\}$. So, (1), (3)-(5), (8), (10), (11) form linear part of the model of MR control system, from position of rudders till the movement of actuator shaft.

The static characteristics of backlash nonlinearity in the mechanical assembly may be simulated as shown in Fig. 2.

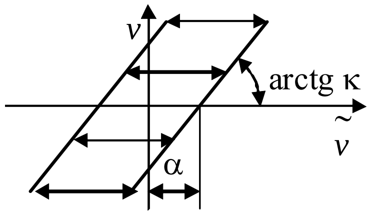

Fig. 2 Backlash static characteristics

Static characteristics is analytically simulated as:

$$v = \begin{cases} \kappa(\tilde{v} - \alpha) \text{ when} v = \kappa(\tilde{v} - \alpha) \text{ and} \dot{\tilde{v}} > 0; \\ \kappa(\tilde{v} + \alpha) \text{ when} v = \kappa(\tilde{v} + \alpha) \text{ and} \dot{\tilde{v}} < 0; \\ v = \text{const,when} \kappa(\tilde{v} + \alpha) \le v \le \kappa(\tilde{v} - \alpha), \end{cases} \quad (14)$$

where α is the backlash width; κ is the transmitting coefficient; $\tilde{V}$ is position of actuator shaft; v is position of the rudder; $\dot{\tilde{v}} = \frac{d\tilde{v}(t)}{dt}$; t is the time.

The following expression closes analytical description of MR digital control system:

$$\begin{cases} \tilde{\boldsymbol{v}}(t) = L^{-1}\left[\tilde{\boldsymbol{V}}(s)\right]; \\ \boldsymbol{V}(s) = L\left[\boldsymbol{v}(s)\right], \end{cases} \quad (15)$$

where $L[...]$ and $L^{-1}[...]$ are direct and inverse Laplace transforms, correspondingly; $\tilde{\boldsymbol{v}}(t) = [\tilde{v}_1(t), \tilde{v}_2(t)]^{\theta}$; $\boldsymbol{v}(t) = [v_1(t), v_2(t)]^{\theta}$, close analytical description of MR digital control system.

As it follows from the MR control system description, there are two aspects, which decrease performance of MR: time delays in control loops [19] and backlash in actuator. Latter may be remedied by mechanics designer and/or manufacturer of actuator. Time delays entirely depend on controller's hardware speed and software time complexity, so on the stage of software design it is necessary to estimate time delays and take precautions to ensure that lags caused by software are minimized.

III. Semi-Markov Model of DC Operation

For estimation of time intervals between quests to vectors $\boldsymbol{F}(s)$, $\boldsymbol{X}_0(s)$ $\boldsymbol{U}(s)$ the model of Von Neumann computer operation in time domain should be worked out. For simplicity it may be represented as including transaction operators only. Control algorithm has the following features:

- it is the cyclic one, but in it absent a looping effect;
- it may generate transactions in arbitrary sequence, with one exception; the same transaction cannot be generated twice at a time;
- there is the only transactions operator for input/output one type of data;

On the graph, which represents the structure of control algorithm, strong connectivity condition is imposed [20]. In common case such properties have the full oriented graph without loops (Fig. 3 (a)).

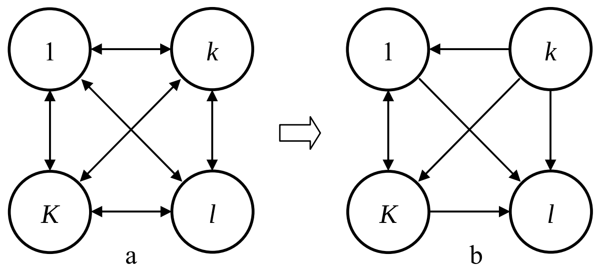

Fig. 3 Common structure of semi-Markov process (a), and the model for time interval estimation (b)

With taking into account randomness of time interval between transactions and stochastic transactions sequence for external observer, the adequate approach to algorithm simulation is the semi-Markov process being represented by the semi-Markov matrix

$$\boldsymbol{h}(t) = [h_{kl}(t)] = [g_{kl}(t)] \otimes [p_{kl}(t)], \quad (16)$$

where $p_{kl}(t)$ is probability of the direct switching from the k-th state to the l-th state; $g_{kl}(t)$ is the pure time density of residence the process (17) in the k-th state before switching into the l-th state; $1 \le k, l \le K$; K is number of transactions types (number of algorithm operator); $\otimes$ is the direct multiplication sign; t is the physical time.

Due to control algorithm properties, semi-Markov process (16) is the ergodic one, and does not include both absorbing, and partially absorbing states. Due to semi-Markov process' ergodicity on densities $g_{k,l}(t)$ and probabilities $p_{k,l}(t)$ following restrictions are imposed:

$$0 < T_{kl}^{\min} \le \arg[g_{kl}(t)] \le T_{kl}^{\max} < \infty, \; 1 \le k, l \le 2K; \quad (17)$$

$$\sum_{k=1}^{2K} p_{kl} = 1; \quad (18)$$

$$\int_{T_{k,l}^{\min}}^{T_{k,l}^{\max}} g_{kl}(t) dt = 1, \quad (19)$$

where $2K$ is the common quantity of transaction operators; $T_{kl}^{\min}$ and $T_{kl}^{\max}$ are upper and lower bounds of density $g_{kl}(t)$ domain.

When estimating time intervals between transactions it is no matter how semi-Markov process (16) reaches the *l*-th state from the *k*-th one, either due to the direct switching, or by means of a wandering through the process. Decisive in considering case is that achievement must be the first, but not second, third, etc. For time interval estimation initial semi-Markov process should be transformed into the process with the structure, shown on Fig. 3 (b), in which *k*-th state is the starting one, and *l*-th state is the absorbing one. For getting such structure: *k*-th column and *l*-th row of $\boldsymbol{h}(t)$ should be reset to zeros; probabilities $p_{ij}(t)$ in all rows, excluding the *l*-th, and in all columns, excluding *k*-th, should be recalculated as:

$$p'_{ij} = \frac{p_{ij}}{1 - p_{ik}},\ 1 \le i, j \le 2K,\ i \ne k,\ j \ne l\cdot \tag{20}$$

in such a way

$$\boldsymbol{h}(t) \to \boldsymbol{h}'(t) = \left[g_{kl}(t)\cdot p'_{kl}\right]. \tag{21}$$

After recalculation probabilities according to (22), partially absorbing states are annihilated, and events of getting the *l*-th state from the *k*-th state begin to make up a full group of incompatible events. In such a way, time density of wandering from the *k*-th state to the *l*-th state may be estimated as:

$$g_{kl}^{\Sigma}(t) = \boldsymbol{I}_k^r \cdot L^{-1}\left[\sum_{y=1}^{\infty}\{L[\boldsymbol{h}'(t)]\}^y\right]\cdot \boldsymbol{I}_l^c, \tag{22}$$

where $\boldsymbol{I}_k^r$ is the row-vector, *k*-th element of which is equal to one, and other elements are equal to zeros; $\boldsymbol{I}_l^c$ is the column-vector, *l*-th element of which is equal to one, and other elements are equal to zeros.

For time density (24) the expectation and the dispersion may be calculated [21]:

$$T_{kl}^{\Sigma} = \int_0^{\infty} t \cdot g_{kl}^{\Sigma}(t)dt; \tag{25}$$

$$D_{kl}^{\Sigma} = \int_0^{\infty}\left(t - T_{kl}^{\Sigma}\right)^2 \cdot g_{kl}^{\Sigma}(t)dt\cdot \tag{26}$$

Expectations $T_{kl}^{\Sigma}(t) = \tau_{kl}$ give middle estimations of time delays. Also, time intervals may be estimated with using “three sigma rule”, as [22]:

$$\tau_{kl} = T_{kl}^{\Sigma} + 3\sqrt{D_{kl}^{\Sigma}}\,. \tag{25}$$

Estimations (24)-(26) define lags of input/output *l*-th element with respect to *k*-th. Changing indices *k* and *l*, one can calculate elements of matrices $\boldsymbol{N}_f(s)$, $\boldsymbol{N}_0(s)$ and $\boldsymbol{N}_u(s)$.

IV. Example of Control System Analysis

As an example, the digital system of MR longitudinal movement and direction control with the structure, shown in Fig. 1, is analyzed. Transfer functions of linear part of object under control are as follows:

$$W_{11}(s) = W_{21}(s) = W_{21}(s) = \frac{1}{0{,}1s + 1};\ W_{12}(s) = \frac{-1}{0{,}1s + 1}\cdot$$

In the system proportional feedback is realized. Inputs $F_1(s)$ and $F_2(s)$ are Laplace transform of Heaviside function $L^{-1}[F_1(s)] = 1\cdot\eta(t)$. Linear part of actuators is performed with transfer functions

$$W_{A,1}(s) = \frac{1{,}2}{s(0{,}05s + 1)};\ W_{A,2}(s) = \frac{10}{s(0{,}05s + 1)}\cdot$$

Feedback transfer functions are equal to $W_{0,1}(s) = W_{0,2}(s) = 1$. On Fig. 4 system performance without lag and backlash is shown.

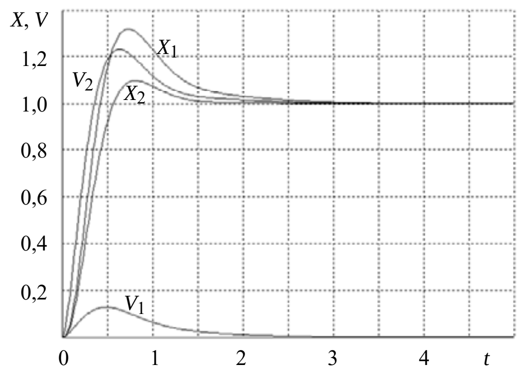

Fig. 4 System performance without lag and backlash

On Fig. 5 system performance with lag and without backlash is shown. Lags in line $X_{0,2}(s)$ are equal to zero. Lags in lines $U_1(s)$, $U_2(s)$, $X_{0,2}(s)$ are equal to 0,01, 0,015 and 0.02 s, correspondingly.

On Fig. 6 system performance with lag and backlash is shown. Backlash has the following parameters: $\kappa = 1$, $\alpha = 0{,}01$.

As one can see from the plots, lags in control loops increase both overshooting and time of control; one should take into account when designing Von Neumann computer based digital control system and working out a software for it.

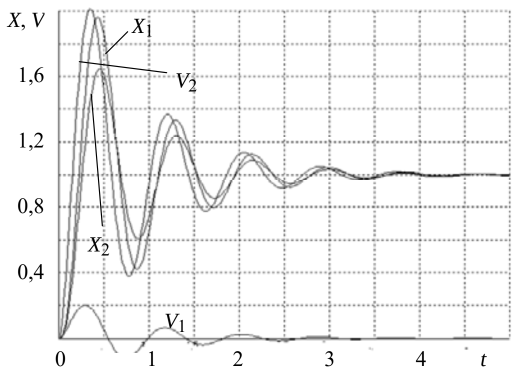

Fig. 5 System performance with lag and without backlash

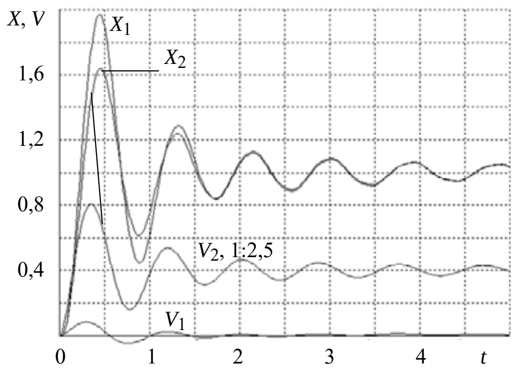

Fig. 6 System performance with lag and backlash

V. Conclusion

The mathematical model of Von Neumann computer control of MR is worked out. It is proved that real time characteristics of DC, namely feedback lags, may be calculated for control algorithm of any complexity. So, this time characteristics should be taken into account when working out both common configuration the system as a whole and software of the system. Increasing algorithm complexity for reaching quality characteristics of control process may have opposite effect due to appearance of excessive lags in control loops.

Further investigations in the domain may be directed to working out methods of practical digital control algorithms synthesis, optimal to complexity-quality ratio.

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