

Fuzzy Uncertainty Theory for Stealth Fighter Aircraft Selection in Entropic Fuzzy TOPSIS Decision Analysis Process

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Abstract—The purpose of this paper is to present fuzzy TOPSIS in an entropic fuzzy environment. Due to the ambiguous concepts often represented in decision data, exact values are insufficient to model real-life situations. In this paper, the rating of each alternative is defined in fuzzy linguistic terms, which can be expressed with triangular fuzzy numbers. The weight of each criterion is then derived from the decision matrix using the entropy weighting method. Next, a vertex method is proposed to calculate the distance between two triangular fuzzy numbers. According to the TOPSIS concept, a closeness coefficient is defined to determine the ranking order of all alternatives by simultaneously calculating the distances to both the fuzzy positive-ideal solution (FPIS) and the fuzzy negative-ideal solution (FNIS). Finally, an illustrative example of selecting stealth fighter aircraft is shown at the end of this article to highlight the procedure of the proposed method. Correlation analysis and validation analysis using TOPSIS, WSM, and WPM methods were performed to compare the ranking order of the alternatives.

Keywords—stealth fighter aircraft selection, fuzzy uncertainty theory (FUT), fuzzy entropic decision (FED), fuzzy linguistic variables, triangular fuzzy numbers, multiple criteria decision making analysis, MCDMA, TOPSIS, WSM, WPM.

I. INTRODUCTION

DECISION problems are the process of finding the best option among all possible alternatives. Decision analysis is a framework in which various types of analyzes are applied for the formulation and characterization of decision alternatives that best apply the decision maker's priorities, given the decision maker's state of knowledge. The Decision analysis process is used to support decision-making bodies to help evaluate technical, cost, and schedule issues, alternatives, and their uncertainties. Decision models have the capacity to accept and measure human subjective inputs: the judgments of experts and the preferences of decision makers. The outputs of this process support the decision maker's difficult task of deciding between competing alternatives without full knowledge; therefore, it is crucial to understand and document the assumptions and limitations of any tool or methodology and integrate them with other factors when deciding among viable options.

Complex decisions may require more formal decision analysis when contributing factors have complex or ill-defined relationships. Because of this complexity, formal decision analysis has the potential to consume significant resources and time. Typically, its application to a particular

decision is only guaranteed if some of the conditions are met: complexity, uncertainty, multiple attributes, diversity of stakeholders.

Satisfaction of all these conditions is not a requirement to initiate decision analysis. The point here is rather the increasing need for decision analysis as a function of the above conditions. Additionally, often these decisions have the potential to result in high-risk effects on cost, safety, or mission success criteria that must be identified and addressed in the process. When the decision analysis process is triggered, the decision need, identified alternatives, issues or problems, supporting data, and analysis support requests are the inputs.

Decisions are based on facts, qualitative and quantitative data, engineering judgment, and open communication to facilitate the flow of information through the hierarchy of forums where technical analysis and evaluations are presented and evaluated and decisions are made. The extent of technical analysis and evaluation required should be commensurate with the consequences of the issue requiring the decision. The work required to make a formal assessment is significant and applicability should be based on the nature of the problem to be resolved.

Decision criteria are necessary conditions for individually evaluating the options and alternatives under consideration. Typical decision criteria include cost, schedule, risk, security, mission success, and supportability. However, evaluations should also include technical criteria specific to the decision taken. The criteria should be objective and measurable. The criteria should also allow for the distinction between options or alternatives. An option that does not meet the mandatory criteria should be ignored. For complex decisions, criteria can be grouped into categories or targets. With a good understanding of the decision need, alternatives can be identified that fit the mission and system context. There may be several alternatives that could potentially meet the decision criteria. Alternatives can be found from design options, operational options, cost options, and/or scheduling options. Depending on the decision to be made, various approaches can be applied to evaluate the identified alternatives. When choosing the approach, the task and system context should be kept in mind, and the complexity of decision analysis should be appropriate to the complexity of the task, the system, and the relevant decision.

Evaluation methods and tools/techniques to be used should be chosen based on the purpose of analyzing a decision and

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the availability of information used to support the method and/or tool. The performance of each alternative against each selected performance measure is evaluated. Regardless of the method or tools used, the results should include: evaluation of the assumptions about the evaluation criteria and the evidence supporting the assumptions; and evaluating whether uncertainty in values for alternative solutions affects the evaluation. When decision criteria have different measurement bases, normalization can be used to establish a common basis for mathematical operations. The process of "normalizing" is making a scale so that all different types of criteria can be compared or aggregated. For complex decisions, decision tools often provide an automated way to normalize. It is important to question and understand operational definitions for tool weights and scales. After the decision alternative assessment is complete, recommendations should be brought back to the decision maker, including an assessment of the robustness of the ranking. Usually only single alternative should be recommended. However, if the alternatives do not differ significantly or reduction of uncertainty can convincingly alter the ranking, the recommendation should include all alternatives ranked closely for a final choice by the decision maker. In any case, the decision maker is always free to choose any alternative or request additional alternatives for consideration. The highest score is usually the option recommended to management. If a different option is suggested, an explanation should be given as to why the lower score is preferred.

In almost all such decision problems [1-66], a multiplicity of criteria for judging alternatives is common. This means that in any of such decision problems, the decision maker wants to solve a multiple criteria decision making analysis (MCDMA) problem. Generally speaking, it is human aspiration to make mathematical decisions in a multiple choice situation. In scientific terms, it aims to develop analytical and numerical methods that consider multiple criteria and multiple alternatives.

An MCDMA problem can be briefly expressed in matrix format, where $a_i = \{a_1, a_2, \dots, a_i\}$ are possible alternatives that decision makers have to choose, $g_j = \{g_1, g_2, \dots, g_j\}$ is the criteria by which alternative performance is measured, x_{ij} is the degree of alternative x_i according to the criterion g_j and $\omega_j = (\omega_1, \omega_2, \dots, \omega_j)$ is the weight of the g_j criterion.

In classical MCMD methods, the ratings and weights of the criteria are precisely known [28, 60]. The technique for order performance by similarity to ideal solution (TOPSIS) method was first developed to solve a MCDMA problem [28]. TOPSIS is one of the numerical methods of multiple criteria decision making. This is a widely applicable method with a simple mathematical model. It is based on the concept that the chosen alternative should be the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). In the TOPSIS process, performance ratings and weights of criteria are given as exact values. In many circumstances, crisp data is insufficient to model real-life situations.

Human judgments, including preferences, are often vague and cannot predict their preference with a precise numerical value. A more realistic approach would be to use linguistic evaluations instead of numerical values, i.e. to assume that the ratings and weights of the criteria in the problem are evaluated through linguistic variables [58, 61, 62, 63, 64,65]. In this paper, the concept of TOPSIS is further expanded to develop a methodology for solving multiple criteria decision making problems in fuzzy environment.

Given the fuzziness in the decision data and the group decision-making process, linguistic variables are used to evaluate the weights of all criteria and the ratings of each alternative against each criterion. After the fuzzy ratings of the decision makers are aggregated, the decision matrix can be transformed into a fuzzy decision matrix, the entropic criteria weights are calculated, and a weighted normalized fuzzy decision matrix can be created.

According to the TOPSIS concept, fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) are defined. Later, in this paper, a vertex method is proposed to calculate the distance between two triangular fuzzy ratings. Using the vertex method, the distance of each alternative to FPIS and FNIS can be calculated, respectively. Finally, a closeness coefficient of each alternative is defined to determine the ranking order of all alternatives. A higher value of the closeness coefficient indicates that an alternative is simultaneously closer to the FPIS and farther from the FNIS. Fuzzy entropic decision technique is applied to select the stealth fighter aircraft to demonstrate its feasibility and effectiveness in MCDMA problem.

To develop the fuzzy linguistic TOPSIS method, the paper is organized as follows. Section 2 introduces the basic definitions and representations of fuzzy number and linguistic variables. Section 3 presents the fuzzy linguistic TOPSIS method in group decision making and selection. Next, the proposed method is illustrated with an example. Finally, in Section 4, some conclusions are pointed out at the end of this paper.

II. METHODOLOGY

A. Fuzzy Uncertainty Theory

Let X be a space of points (objects), with a generic element of X denoted by x as $X = \{x\}$. A fuzzy set (class) A in X is characterized by a membership (characteristic) function $\mu_A(x)$ which associates with each point in X a real number in the interval $[0, 1]$, with the value of $\mu_A(x)$ at x representing the "grade of membership" of x in A . Thus, the nearer the value of $\mu_A(x)$ to unity, the higher the grade of membership of x in A . When A is a set in the ordinary sense of the term, its membership function can take on only two values 0 and 1, with $\mu_A(x) = 1$ or 0 according as x does or does not belong to A . Thus, in this case $\mu_A(x)$ reduces to the familiar characteristic function of a set A .

Definitions for fuzzy sets [57, 58, 64, 65]:

Definition 1. If X is a collection of objects denoted generically by x then a fuzzy set A in X is a set of ordered pairs:

$$A = \{(x, \mu_A(x)) \mid \mu_A(x) \in [0,1] \forall x \in X\} \quad (1)$$

where $\mu_A(x)$ is the membership function that maps X to the membership space M and $\mu_A(x)$ is the grade of membership (also degree of compatibility or degree of truth) of x in A .

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in X \\ 0 & \text{if } x \notin X \end{cases} \quad (2)$$

In the theory of fuzzy sets, the degree of membership is determined by generalizing the characteristic function and is called the membership function. Instead of the set $\{0,1\}$, the interval $[0,1]$ is used and the membership function is expressed as $\mu_A(x): x \rightarrow [0,1]$ or $0 \leq \mu_A(x) \leq 1$. $\mu_A(x) = 0$ indicates that x is not a member of A , and $\mu_A(x) = 1$ indicates that x is a full member of A . Fuzzy sets can be either discrete or continuous.

Discrete fuzzy sets are defined as

$$A = \sum_{i=1}^l \mu_{(A)}(x_i) / x_i \quad (3)$$

Continuous fuzzy sets can be defined as

$$A = \int_{x \in X} \mu_{(A)}(x) / x_i \quad (4)$$

Definition 2. A fuzzy set is *empty* if and only if its membership function is identically zero on X .

Two fuzzy sets A and B are equal, written as $A = B$, if and only if $\mu_A(x) = \mu_B(x)$ for all x in X .

Definition 3. The *complement* of a fuzzy set A is denoted by A' and is defined by

$$\mu_{A'}(x) = 1 - \mu_A(x) \quad (5)$$

Definition 4. *Containment.* A is contained in B (or, equivalently, A is a subset of B , or A is smaller than or equal to B) if and only if $\mu_A(x) \leq \mu_B(x)$. In symbols

$$A \subset B \Leftrightarrow \mu_A(x) \leq \mu_B(x) \quad (6)$$

Definition 5. *Union.* The union of two fuzzy sets A and B with respective membership functions $\mu_A(x)$ and $\mu_B(x)$ is a fuzzy set C , written as $C = A \cup B$, whose membership function is related to those of A and B by

$$\mu_C(x) = \max[\mu_A(x), \mu_B(x)], x \in X \quad (7)$$

or, in abbreviated form

$$\mu_C(x) = \mu_A(x) \vee \mu_B(x) \quad (8)$$

Note that \cup has the associative property, that is,

$$A \cup (B \cup C) = (A \cup B) \cup C \quad (9)$$

The union of A and B is the smallest fuzzy set containing both A and B . More precisely, if D is any fuzzy set which contains both A and B , then it also contains the union of A and B .

To show that this definition is equivalent to (3), we note, first, that C as defined by (3) contains both A and B , since

$$\max[\mu_A(x), \mu_B(x)] \geq \mu_A(x) \quad (10)$$

and

$$\max[\mu_A(x), \mu_B(x)] \geq \mu_B(x) \quad (11)$$

Furthermore, if D is any fuzzy set containing both A and B , then

$$\mu_D(x) \geq \mu_A(x) \quad (12)$$

$$\mu_D(x) \geq \mu_B(x) \quad (13)$$

and hence

$$\mu_D(x) \geq \max[\mu_A(x), \mu_B(x)] = \mu_C(x) \quad (14)$$

which implies that $C \subset D$. The notion of an intersection of fuzzy sets can be defined in an analogous manner. Specifically:

Definition 6. *Intersection.* The intersection of two fuzzy sets A and B with respective membership functions $\mu_A(x)$ and $\mu_B(x)$ is a fuzzy set C , written as $C = A \cap B$, whose membership function is related to those of A and B by

$$\mu_C(x) = \min[\mu_A(x), \mu_B(x)], x \in X \quad (15)$$

or, in abbreviated form

$$\mu_C(x) = \mu_A(x) \wedge \mu_B(x) \quad (16)$$

As in the case of the union, it is easy to show that the intersection of A and B is the *largest* fuzzy set which is contained in both A and B . As in the case of ordinary sets, A and B are *disjoint* if $A \cap B$ is empty. Note that \cap , like \cup , has the associative property.

Definition 7. *Convexity.* A fuzzy set A is convex if and only if the sets defined by

$$\Gamma_\alpha = \{x \mid \mu_A(x) \geq \alpha\} \quad (17)$$

are convex for all α in the interval (0, 1].

An alternative and more direct definition of convexity is the following: A is convex if and only if

$$\mu_A(x)[\lambda x_1 + (1-\lambda)x_2] \geq \min[\mu_A(x_1), \mu_B(x_2)] \quad (18)$$

for all x_1 and x_2 in X and all λ in [0, 1]. Note that this definition does not imply that $\mu_{A^c}(x)$ must be a convex function of x . In other words, the set A is convex if its values are monotonically increasing or decreasing for increasing values of A , or first monotonically increasing and then monotonically decreasing.

Definition 8. A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal.

Definition 9. Normality. A fuzzy set that takes the membership value of "1" for at least one element of X , that is, satisfies $\max_{x \in X} \mu_A(x) = 1$ the equality, is normal.

Definition 10. Fuzzy number. A fuzzy set that is normal and convex is called a fuzzy number. Fuzzy arithmetic may be considered as an extension of classical real numbers and its arithmetic. There exist different types of fuzzy numbers, Triangular, Trapezoidal, and Gaussian fuzzy numbers based on the membership functions.

Definition 11. Triangular fuzzy numbers. A triangular fuzzy number A can be a tripled defined by a triplet $A = (l, m, u)$. The membership Function

$$\mu_A(x) = \begin{cases} 0, & x < l \\ \frac{x-l}{m-l}, & l \leq x \leq m \\ \frac{u-x}{u-m}, & m \leq x \leq u \\ 0, & x > u \end{cases} \quad (19)$$

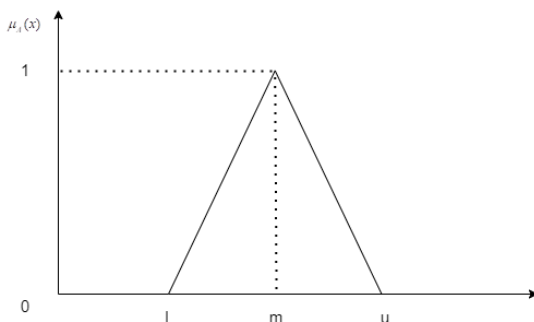


Fig. 1 A triangular fuzzy number

Let A_1 and A_2 be two triangular fuzzy numbers defined by triplets $A_1 = (l_1, m_1, u_1)$ and $A_2 = (l_2, m_2, u_2)$ respectively, then the operational laws of these two triangular fuzzy numbers are given as follows:

1. Addition operation (+) of two triangular fuzzy numbers A_1 and A_2 :

$$A_1(+)A_2 = (l_1, m_1, u_1)(+)(l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$$

2. Subtraction operation (-) of two triangular fuzzy numbers A_1 and A_2 :

$$A_1(-)A_2 = (l_1, m_1, u_1)(-)(l_2, m_2, u_2) = (l_1 - u_2, m_1 - m_2, u_1 - l_2)$$

3. Multiplication operation (\times) of two triangular fuzzy numbers A_1 and A_2 :

$$A_1(x)A_2 = (l_1, m_1, u_1)(x)(l_2, m_2, u_2) = (l_1 l_2, m_1 m_2, u_1 u_2)$$

4. Division operation (\div) of two triangular fuzzy numbers A_1 and A_2 :

$$A_1(\div)A_2 = (l_1, m_1, u_1)(\div)(l_2, m_2, u_2) = \left(\frac{l_1}{u_2}, \frac{m_1}{m_2}, \frac{u_1}{l_2} \right), l_2, m_2, u_2 \neq 0$$

5. Scalar multiplication in triangular fuzzy numbers:

$$\lambda A = (\lambda l_1, \lambda m_1, \lambda u_1), \text{ for any real constant } \lambda > 0$$

6. Scalar summation in triangular fuzzy numbers:

$$\lambda(+)A = (\lambda + l_1, \lambda + m_1, \lambda + u_1)$$

Definition 12. Aggregation of fuzzy numbers. Assume that a decision group has K respondents, then the importance of the criteria and the assessment of alternatives (objects) with respect to each criterion, and the aggregated matrix can be calculated using the following aggregation methods.

AM1. Arithmetic mean defined by

$$x_{ij} = \frac{1}{K} \sum_{k=1}^K x_{ij}^k = \left(\frac{1}{K} \sum_{k=1}^K l_{x_{ij}}^k, \frac{1}{K} \sum_{k=1}^K m_{x_{ij}}^k, \frac{1}{K} \sum_{k=1}^K u_{x_{ij}}^k \right) \quad (20)$$

AM2. Geometric mean defined by

$$x_{ij} = \left(\prod_{k=1}^K x_{ij}^k \right)^{\frac{1}{K}} = \left(\left(\prod_{k=1}^K l_{x_{ij}}^k \right)^{\frac{1}{K}}, \left(\prod_{k=1}^K m_{x_{ij}}^k \right)^{\frac{1}{K}}, \left(\prod_{k=1}^K u_{x_{ij}}^k \right)^{\frac{1}{K}} \right) \quad (21)$$

AM3. Modified arithmetic mean defined by

$$x_{ij} = \left(\min_k l_{x_{ij}}^k, \frac{1}{K} \sum_{k=1}^K m_{x_{ij}}^k, \max_k u_{x_{ij}}^k \right) \quad (22)$$

AM4. Modified arithmetic mean defined by

$$x_{ij} = \left(\min_k l_{x_{ij}^k}, \left(\prod_{k=1}^K m_{x_{ij}^k} \right)^{\frac{1}{K}}, \max_k u_{x_{ij}^k} \right) \quad (23)$$

where $x_{ij}^K = (l_{ij}^K, m_{ij}^K, u_{ij}^K)$ and $\omega_j^K = (l_{(\omega)ij}^K, m_{(\omega)ij}^K, u_{(\omega)ij}^K)$ are the assessment and importance weight of the K -th respondent in the form of triangular fuzzy numbers. Similarly, the vector (ω_j) of the criteria weights can be calculated using the aggregation methods.

Definition 13. *Fuzzy matrix.* A matrix whose at least one element is a fuzzy number is called a fuzzy matrix. A fuzzy multiple criteria group decision making problem can be expressed in fuzzy decision matrix as

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1j} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} \end{pmatrix} \quad (i=1, \dots, I)(j=1, \dots, J) \quad (24)$$

$$\omega_j = (\omega_1, \omega_2, \dots, \omega_j) \quad (25)$$

where $x_{ij} = (l_{ij}, m_{ij}, u_{ij})$, $\omega_j = (l_{(\omega)j}, m_{(\omega)j}, u_{(\omega)j})$.

Ranking of objects with these assumptions is possible, among others, through the application of fuzzy TOPSIS.

Definition 14. Let $A_1 = (l_1, m_1, u_1)$ and $A_2 = (l_2, m_2, u_2)$ be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them as

$$d(A_1, A_2) = \sqrt{\frac{1}{3} [(l_1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2]} \quad (26)$$

Definition 15. *Fuzzy linguistic variables.* In the assessment process, the respondents tend to state their preferences in natural language expressions. Fuzzy linguistic variables reflect different aspects of human language or artificial language. The variables that define a human term can be divided into numerous linguistic criteria, i.e., an 11-point scale is proposed for the importance of attributes and rating candidates, as shown in Table 1.

Definition 16. *Defuzzification.* Pascal's triangle is a triangular array of the binomial coefficients that arise in probability theory, combinatorics, and algebra. Pascal's triangle graded mean takes the coefficients of fuzzy variables from Pascal's triangle numbers.

$$PTGM = \frac{l + 2m + u}{4} \quad (27)$$

The graded mean information representation of the generalized triangular fuzzy number $A = (l, m, u)$ is given by [66]

$$GMIR = \frac{l + 4m + u}{6} \quad (28)$$

Table 1. Fuzzy linguistic variables and triangular fuzzy numbers

Fuzzy Linguistic Variables	Triangular fuzzy number			PTGM	GMIR
	l	m	u		
Absolutely high (AH)	1	1	1	1	1
Very high (VH)	0.8	0.9	1	0.9	0.9
High (H)	0.7	0.8	0.9	0.8	0.8
Fairly high (FH)	0.6	0.7	0.8	0.7	0.7
Medium high (MH)	0.5	0.6	0.7	0.6	0.6
Fair (F)	0.4	0.5	0.6	0.5	0.5
Medium low (ML)	0.3	0.4	0.5	0.4	0.4
Fairly low (FL)	0.2	0.3	0.4	0.3	0.3
Low (L)	0.1	0.2	0.3	0.2	0.2
Very low (VL)	0	0.1	0.2	0.1	0.1
Absolutely low (AL)	0	0	0	0	0

Definition 17. *Fuzzy distance function.* The distance function is used to calculate the distance between any two fuzzy numbers.

$$d(X, Y) = \left(\frac{1}{J} \sum_{j=1}^J \omega_j (x_j - y_j)^q \right)^{1/q} \quad (29)$$

where $q \geq 1$, J is the number of attributes, and ω_j is the attribute weights vector.

B. Fuzzy TOPSIS

Classical MCDMA methods assume that criteria and weights are expressed in crisp values. However, in many real situations, the assessments of criteria are often expressed qualitatively or using linguistic expressions [62]. In such a case, the theory of fuzzy sets applies. The fuzzy set theory, combined with the appropriate fuzzy modifications of the MCDMA methods, allows analyzing imprecise and fuzzy information.

The fuzzy TOPSIS compromise decision analysis assumes that the evaluation of criteria and their weights can be expressed as triangular fuzzy numbers. The primary concept of TOPSIS compromise decision analysis approach is that the most preferred alternative should have the shortest distance from the fuzzy positive ideal solution (FPIS), but also have the farthest distance from the fuzzy negative ideal solution (FNIS). Application of fuzzy TOPSIS method requires the accomplishment of the following fundamental steps:

Step 1. Construction of normalized fuzzy decision matrix $Y = [y_{ij}]_{m \times n}$

$$Y = \begin{pmatrix} y_{11} & \cdots & y_{1j} \\ \vdots & \ddots & \vdots \\ y_{i1} & \cdots & y_{ij} \end{pmatrix} \quad (30)$$

This stage requires an indication of benefit (B) and cost criteria (C). The normalization formulas for benefit and cost criteria have the form, respectively

$$y_{ij} = \left(\frac{l_{ij}^-}{u_j^+}, \frac{m_{ij}^-}{m_j^+}, \frac{u_{ij}^-}{u_j^+} \right) j \in B \quad (31)$$

$$y_{ij} = \left(\frac{l_{ij}^-}{u_j^+}, \frac{l_{ij}^-}{m_j^+}, \frac{l_{ij}^-}{l_j^+} \right) j \in C \quad (32)$$

where $u_j^+ = \max_i u_{ij}$ if $j \in B$ and $l_j^- = \min_i l_{ij}$ if $j \in C$.

Step 2. Calculation of the vector of criteria weights

In information theory, the definition of information entropy is expressed in terms of a discrete set of probabilities $\{p_i\}$, so that

$$H_j = -\sum_{i=1}^I p_i \log p_i \quad (33)$$

Entropy is the measure of the amount of missing information before reception. Entropy is simply the amount of information in a variable [56]. Given a discrete set of probabilities $\{p_i\}$ with the condition $\sum_{i=1}^I p_i = 1$, the entropy of a discrete set of probabilities is defined as

$$H_j = \sum_{i=1}^I p_i e^{(1-p_i)} \quad (34)$$

$$\omega_j = \frac{1-H_j}{\sum_{j=1}^J (1-H_j)}, j=1, \dots, J \quad (35)$$

where ω_j is the criterion weights vector, $\sum_{j=1}^J \omega_j = 1$. In this work, the amount of information is calculated as the entropy H_j for each variable in the decision matrix,

Step 3. Construction of weighted normalized fuzzy decision matrix $V = [v_{ij}]_{m \times n}$

$$V = \begin{pmatrix} v_{11} & \cdots & v_{1j} \\ \vdots & \ddots & \vdots \\ v_{i1} & \cdots & v_{ij} \end{pmatrix} \quad (36)$$

where $v_{ij} = y_{ij} \omega_j$.

Step 4. Determining fuzzy positive ideal solution S^+ and fuzzy negative ideal solution S^- , respectively:

$$S^+ = (s_1^+, \dots, s_j^+) j \in B \quad (37)$$

$$S^- = (s_1^-, \dots, s_j^-) j \in C \quad (38)$$

where $s_j^+ = \max_i v_{ij} j \in B | s_j^+ = (1,1,1) j \in B$ and $s_j^- = \min_i v_{ij} j \in C | s_j^- = (0,0,0) j \in C$

Step 5. Calculation of the distance of each object from fuzzy positive ideal solution S^+ and fuzzy negative ideal solution S^- , respectively:

$$d_i^+ = \sum_{j=1}^J d(v_{ij}, v_j^+) \quad (39)$$

$$d_i^- = \sum_{j=1}^J d(v_{ij}, v_j^-) \quad (40)$$

where d is the distance between two positive triangular fuzzy numbers $A_1 = (l_1, m_1, u_1)$ and $A_2 = (l_2, m_2, u_2)$.

Step 6. Calculation of the closeness coefficient CC_i for each object

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (41)$$

CC_i values are normalized in an interval $[0,1]$. The smaller the distance of an object is from a positive ideal solution, and the bigger from a negative ideal solution, the closer the value of a closeness coefficient is to 1.

Step 7. Establishing the objects ranking. The best object owns the biggest value of a closeness coefficient CC_i .

III. APPLICATION

This section presents the fuzzy TOPSIS approach on a numerical example for selecting stealth fighter aircraft. Consider a fuzzy MCDMA problem for group decision making, consisting of the set of five feasible alternatives (a_i) rated with respect to the set of one cost criterion and four benefit criteria (g_j) by a group of three decision makers (DMs) with the vector of criteria weights (ω_j), derived from the fuzzy decision matrix using the entropy weighting method.

Each stealth fighter aircraft is judged on the following five criteria: (1) g_1 is operating cost; (2) g_2 is aircraft speed ; (3)

g_3 is payload, (4) g_4 is maneuverability, (5) g_5 is survivability. where (g_1) is cost criterion, and ($g_2 - g_5$) are benefit criteria.

During the group decision-making process, DMs are asked to evaluate alternatives according to criteria. In many real-life situations, fuzzy numbers can be used when DMs lack knowledge of the subject being analyzed, or when available data is incorrect, or ratings are expressed linguistically. In this case, each DM provides a decision matrix.

The DMs have used triangular fuzzy numbers to rate the alternatives with respect to the criteria and their evaluations are shown in Table 2.

Table 2. Initial fuzzy decision matrix

		g_1	g_2	g_3	g_4	g_5
a_i	DM	min	max	max	max	max
	DM1	L	MH	F	ML	MH
a_1	DM2	ML	H	F	L	F
	DM3	ML	MH	F	F	F
a_2	DM1	F	VL	F	F	F
	DM2	ML	ML	F	L	MH
a_3	DM3	F	VL	F	F	H
	DM1	H	VL	H	H	H
a_4	DM2	MH	L	MH	MH	H
	DM3	H	VL	H	H	H
a_5	DM1	F	L	F	ML	ML
	DM2	F	ML	F	ML	ML
a_5	DM3	F	ML	F	ML	ML
	DM1	FH	MH	L	F	VL
a_5	DM2	VL	H	H	H	F
	DM3	VL	F	MH	VH	H

Next, in order to ensure comparability of criteria, the fuzzy decision matrix is normalized. Using the equations (31) and (32), the decision matrices are normalized, the integrated weights of objective criteria (Table 3) can be computed according to the equations (34) and (35) and using the vector (ω_j) of criteria weights, the weighted normalized fuzzy decision matrix is calculated (Table 4).

Table 3. Objective criteria weighting vector

g_j	g_1	g_2	g_3	g_4	g_5
ω_j	0,182	0,167	0,223	0,207	0,221

Table 4. Weighted normalized fuzzy decision matrix

	g_1	g_2	g_3	g_4	g_5
a_i	min	max	max	max	max
a_1	0,115	0,145	0,134	0,091	0,141
a_2	0,080	0,043	0,134	0,091	0,168
a_3	0,050	0,029	0,197	0,182	0,194
a_4	0,074	0,072	0,134	0,099	0,106
a_5	0,129	0,138	0,143	0,182	0,124

Using the weighted normalized fuzzy decision matrix, the positive ideal solution FPIS, S^+ , and the negative ideal solution, FNIS, S^- , are determined.

$$S_j^+ = [(1,1,1), (1,1,1), (1,1,1), (1,1,1), (1,1,1)]$$

$$S_j^- = [(0,0,0), (0,0,0), (0,0,0), (0,0,0), (0,0,0)]$$

Finally, the distances of each alternative from the positive ideal solution d_i^+ and from the negative ideal solution d_i^- are calculated (Table 5). This allows to calculate the relative closeness coefficient CC_i and the rank order R_i of the alternatives (where \prec means “inferior to”):

$$a_4 \prec a_2 \prec a_1 \prec a_5 \prec a_3$$

Hence, the stealth fighter aircraft alternative (a_3) should be selected as the best candidate.

Table 5. TOPSIS distance measures (FPIS d_i^+ , NPIS d_i^-), relative closeness coefficient (CC_i) and normalized rankings (J_i) of the stealth fighter aircraft alternatives

a_i	d_i^+	R_i	d_i^-	R_i	CC_i	R_i	J_i
a_1	1,957	3	0,284	3	0,127	3	0,201
a_2	2,008	2	0,250	4	0,111	4	0,177
a_3	1,952	4	0,336	1	0,147	1	0,234
a_4	2,019	1	0,223	5	0,100	5	0,158
a_5	1,917	5	0,323	2	0,144	2	0,230

In fuzzy TOPSIS analysis, Table 5 shows the distance of each alternative (a_i) from the positive ideal solution (d_i^+) and the negative ideal solution (d_i^-), as well as the relative closeness coefficients CC_i and rank order (R_i) the alternatives using the arithmetic mean aggregation method. The last column, denoted by (J_i), consists of the normalized values of the relative closeness coefficients of each alternative to the ideal solution, which allows to highlight the differences between the final scores of the alternatives.

Validation Analysis: Fuzzy TOPSIS model was compared with the weighted sum method (WSM) and weighted product

method (WPM) for feasibility and effectiveness of the proposed methodology.

WSM (Weighted Sum Method)

$$P_i = \sum_{j=2}^J \omega_j v_{ij} \quad (42)$$

WPM (Weighted Product Method)

$$Q_i = \prod_{j=1}^J v_{ij}^{\omega_j} \quad (43)$$

The comparative ranking results for the TOPSIS, WSM and WPM models are shown in Table 6.

Table 6. Comparison of ranking results for selection problem of stealth fighter alternatives using TOPSIS (CC_i), WSM (P_i) and WPM (Q_i) models

a_i	CC_i	R_i	P_i	R_i	Q_i	R_i
a_1	0,127	3	0,626	3	0,783	2
a_2	0,111	4	0,516	4	0,698	4
a_3	0,147	1	0,652	2	0,737	3
a_4	0,100	5	0,486	5	0,694	5
a_5	0,144	2	0,715	1	0,840	1

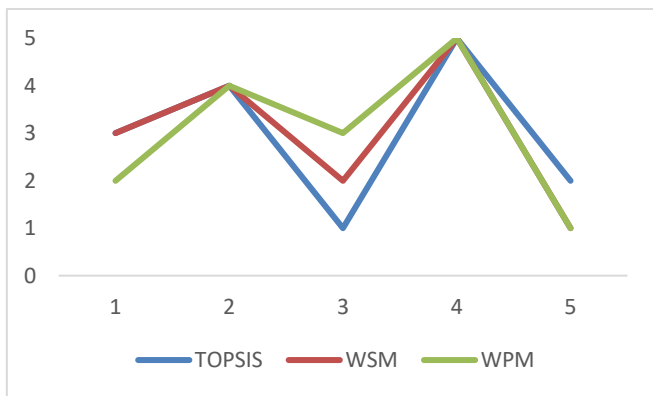


Fig. 2 The ranking of alternatives based on TOPSIS, WSM and WPM

Correlation analysis of ranking of alternatives based on TOPSIS, WSM and WPM was performed as shown in Table 7.

$$\rho_{x,y} = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y} = \frac{E[(x - \mu_x)(y - \mu_y)]}{\sigma_x \sigma_y} \quad (44)$$

Table 7. Correlation analysis of ranking results for selection problem of stealth fighter alternatives using TOPSIS, WSM and WPM models

	TOPSIS	WSM	WPM
TOPSIS	1		
WSM	0,9	1	
WPM	0,7	0,9	1

The ranking correlation coefficient between TOPSIS and WPM is 0.7, while it is 0.9 between TOPSIS and WSM. Also, the research confirms that different MCDMA methods can yield different ranking results when using the same data.

IV. CONCLUSION

Multiple criteria decision making problems depend on uncertain and imprecise data, and fuzzy set theory is sufficient to deal with this complexity. In this paper, a linguistic decision process is proposed to solve the multiple criteria decision making problem in a entropic fuzzy environment. In the decision-making process, evaluation of alternatives according to criteria and importance weights is appropriate to use linguistic variables instead of numerical values.

In this paper, an entropic fuzzy TOPSIS method based on fuzzy numbers is presented for group decision making problems. The decision matrices provided by the DMs are aggregated into an aggregated decision matrix, which is the starting point for ranking alternatives or selecting the best one using the arithmetic mean, geometric mean, or their modifications.

In this study, an entropic MCDMA model is proposed in which the attributes of the alternatives are represented by fuzzy sets. In information theory, entropy is related to the average amount of information of a resource. Starting from the principle, objective criteria weights can be obtained with the proposed entropy weighting model.

The vertex method, which is an efficient and simple method to measure the distance between two triangular fuzzy numbers and extend the TOPSIS procedure to fuzzy medium is used in the solution process of MCDMA problem. In fact, the vertex method can be easily applied to calculate the distance between any two fuzzy numbers whose membership functions are linear.

In the group decision making process, different aggregation functions are used to aggregate the fuzzy ratings of the decision makers. Here, the arithmetic mean aggregation function was used to aggregate the fuzzy ratings of the decision makers. Although the proposed method presented in this paper is illustrated with a stealth fighter aircraft selection problem, it can also be applied to problems such as information project selection, material selection, and many other technical, economic and management decision problems.

The proposed method is applied successfully to select the most preferable stealth fighter aircraft for imprecise data. Also, this model provides the ideal choice for stealth fighter aircraft after effectively avoiding vague and ambiguous judgments.

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