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Synthesis of a Control System of a Deterministic Chaotic Process in the Class of Two-Parameter Structurally Stable Mappings

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Abstract**—**In this paper, the problem of unstable and deterministic chaotic processes in control systems is considered. The synthesis of a control system in the class of two-parameter structurally stable mappings is demonstrated. This is realized via the gradient-velocity method of Lyapunov vector functions. It is shown that the gradient-velocity method of Lyapunov vector functions allows generating an aperiodic robust stable system with the desired characteristics. A simple solution to the problem of synthesis of control systems for unstable and deterministic chaotic processes is obtained. Moreover, it is applicable for complex systems.

Keywords**—**Control system synthesis, deterministic chaotic processes, Lyapunov vector function, robust stability, structurally stable mappings.

I. Introduction

STUDIES in the last century have revealed a wide variety of dynamics of nonlinear systems and led to one of the most important discoveries of the 20th century in nonlinear dynamical systems - deterministic chaos and "strange attractor" [1], [2].

It is now generally accepted that real dynamical systems are nonlinear and deterministic chaos and instabilities are intrinsic properties of any deterministic dynamical system. In nonlinear dynamical systems, when deterministic chaos is generated, the trajectories of the system are globally limited and locally unstable inside the "strange attractor". When nonlinear dynamical systems are linearized, instabilities can be generated in the linear dynamical system.

Deterministic chaos manifests itself in mechanical systems in the form of vibrations, in technical and technological systems in the form of “runaway”, which leads to accidents, in economic systems in the form of short-term fluctuations and fluctuations that provoke a “crisis”.

Methods for controlling chaotic processes are developing in several directions [3]-[5], stabilization of unstable periodic oscillations [3], [5], [6], chaotization [3]-[6], controlled synchronization [3]-[7], modification of attractors [5], [8], [9], etc. A new, especially relevant direction is the systems of complete suppression of the regime of deterministic chaos and instability [10]-[14].

Chaotic and unstable systems represent a class of uncertainty models. Uncertainty may be due to ignorance of the true values of the parameters of the control system at the design stage and their unpredictable change during operation. The ability of a control system to maintain stability under uncertainty is understood as robust stability [15], [16]. Thus, if the robustness conditions are violated, i.e., when uncertain parameters go beyond the boundaries of robust stability, a regime of deterministic chaos and instability is generated in the system [11], [14].

In conditions of significant uncertainty, an increase in the potential of robust stability [11]-[13] by synthesizing a control system in the class of two-parameter structurally stable mappings [17] is the main factor that guarantees the control system protection from the regime of deterministic chaos and instability.

The task of the synthesis of automatic control systems for given quality indicators is the choice of parameters and structure of the system with a known dynamic description of the control object in order to ensure the necessary values of quality indicators [18].

The problem of synthesizing a control system for unstable and deterministic chaotic processes in the class of two-parameter structurally stable mappings is solved by the gradient-velocity method of the Lyapunov vector function [11], [19], [20], taking into account such quality indicators as: stability, robustness, the desired type of transient processes, the absence overshoot, no oscillations, speed, static accuracy of the control system, etc. In general, the gradient-velocity method of the Lyapunov vector function allows to construct an aperiodic robust stable system with the desired characteristics.

II. Problem Description

A. The Control System for Unstable and Deterministic Chaotic Processes

The control system is described by:

$$\dot{x} = Ax + Bu,\ x(t) \epsilon R^n, \tag{1}$$

where

$$A = \begin{Vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \dots & & \dots & & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{Vmatrix},\ B = \begin{Vmatrix} b_{11} & 0 & 0 & \dots & 0 \\ 0 & b_{22} & 0 & \dots & 0 \\ \dots & & \dots & & \dots \\ 0 & 0 & 0 & \dots & b_{nn} \end{Vmatrix}$$

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$$x(t)=\begin{Vmatrix}x_1(t)\\x_2(t)\\\dots\\x_n(t)\end{Vmatrix},\ u(t)=\begin{Vmatrix}u_1(t)\\u_2(t)\\\dots\\u_m(t)\end{Vmatrix}$$

The control law is given in the form of two-parameter structurally stable mappings [17]:

$$u_i(t)=-x_i^4-k_i^1x_i^2+k_i^2x_i,\quad i=1,\dots,n; \qquad (2)$$

The control system (1), taking into account (2), is represented in expanded form as:

$$\begin{cases}\dot{x}_1=-b_{11}x_1^4-b_{11}k_1^1x_1^2+(a_{11}+b_{11}k_1^2)x_1+\\ \quad +a_{12}x_2+\dots+a_{1n}x_n\\ \dot{x}_2=a_{21}x_1-b_{22}x_2^4-b_{22}k_2^1x_2^2+\\ \quad +(a_{22}+b_{22}k_2^2)x_2+\dots+a_{2n}x_n\\ \dots\dots\dots\\ \dot{x}_n=a_{n1}x_1+a_{n2}x_2+\dots-b_{nn}x_n^4-\\ \quad -b_{nn}k_n^1x_n^2+(a_{nn}+b_{nn}k_n^2)x_n\end{cases} \qquad (3)$$

System (3) has a stationary state [11], [12]:

$$x_{1s}^1=0,x_{2s}^1=0,\dots,x_{ns}^1=0 \qquad (4)$$

Other stationary states will be determined by a solution of the form:

$$k_i^1=3\left(\frac{a_{ii}+b_{ii}k_i^2}{2b_{ii}}\right)^{2/3},\ x_{is}^2=\pm\sqrt[3]{\frac{a_{ii}+b_{ii}k_i^2}{2b_{ii}}},\ i=1,\dots,n. \qquad (5)$$

First, we investigate the robust stability of the stationary state (4) of system (3) using the gradient-velocity method, of the Lyapunov vector function [11], [19], [20].

From (3), the components of the gradient vector of the Lyapunov vector function are determined $V(x)=(V_1(x),\dots,V_n(x))$:

$$\begin{cases}\frac{\partial V_1(x)}{\partial x_1}=b_{11}x_1^4+b_{11}k_1^1x_1^2-(a_{11}+b_{11}k_1^2)x_1,\\ \frac{\partial V_1(x)}{\partial x_2}=-a_{12}x_2,\frac{\partial V_1(x)}{\partial x_3}=-a_{13}x_3,\dots,\\ \frac{\partial V_1(x)}{\partial x_n}=-a_{1n}x_n;\ \frac{\partial V_2(x)}{\partial x_1}=-a_{21}x_1,\\ \frac{\partial V_2(x)}{\partial x_2}=b_{22}x_2^4+b_{22}k_2^1x_2^2-(a_{22}+b_{22}k_2^2)x_2,\\ \frac{\partial V_2(x)}{\partial x_3}=-a_{23}x_3,\dots,\frac{\partial V_2(x)}{\partial x_n}=-a_{2n}x_n;\\ \dots\dots\dots\\ \frac{\partial V_n(x)}{\partial x_1}=-a_{n1}x_1,\frac{\partial V_n(x)}{\partial x_2}=-a_{n2}x_2,\dots,\\ \frac{\partial V_n(x)}{\partial x_n}=b_{nn}x_n^4+b_{nn}k_n^1x_n^2-(a_{nn}+b_{nn}k_n^2)x_n\end{cases} \qquad (6)$$

Using the components of the gradient vectors of the Lyapunov vector function (6), we obtain the Lyapunov vector function in scalar form:

$$V(x)=\frac{1}{5}b_{11}x_1^5+\frac{1}{3}b_{11}k_1^1x_1^3-\frac{1}{2}(a_{11}+b_{11}k_1^2)x_1^2-\frac{1}{2}a_{12}x_2^2-\frac{1}{2}a_{13}x_3^2-\dots-\frac{1}{2}a_{1n}x_n^2-\frac{1}{2}a_{21}x_1^2+\frac{1}{5}b_{22}x_2^5+\frac{1}{3}b_{22}k_2^1x_2^3-\frac{1}{2}(a_{22}+b_{22}k_2^2)x_2^2-\frac{1}{2}a_{23}x_3^2-\dots-\frac{1}{2}a_{2n}x_n^2-\dots-\frac{1}{2}a_{n1}x_1^2-\frac{1}{2}a_{n2}x_2^2-\frac{1}{2}a_{n3}x_3^2-\dots+\frac{1}{5}b_{nn}x_n^5+\frac{1}{3}b_{nn}k_n^1x_n^3-\frac{1}{2}(a_{nn}+b_{nn}k_n^2)x_n^2 \qquad (7)$$

The conditions for the positive definiteness of the function $V(x)$ from (7) are not obvious; therefore, we can use the Morse lemma from catastrophe theory [17].

It follows that the Lyapunov function (7) in the vicinity of the stationary state (4) can be represented as a quadratic form

$$V(x)=-(a_{11}+b_{11}k_1^2+a_{21}+a_{31}+\dots+a_{n1})x_1^2-(a_{12}+a_{22}+b_{22}k_2^2+a_{32}+\dots+a_{n2})x_2^2-(a_{13}+a_{23}+a_{33}+b_{33}k_3^2+\dots+a_{n3})x_3^2-\dots-(a_{1n}+a_{2n}+a_{3n}+\dots+a_{nn}+b_{nn}k_n^2)x_n^2 \qquad (8)$$

Conditions for the positive definiteness of the quadratic form (8), that is, of aperiodic robust stability of the stationary state (4) are determined by the inequalities

$$\begin{cases}-(a_{11}+b_{11}k_1^2+a_{21}+a_{31}+\dots+a_{n1})>0\\ -(a_{12}+a_{22}+b_{22}k_2^2+a_{32}+\dots+a_{n2})>0\\ -(a_{13}+a_{23}+a_{33}+b_{33}k_3^2+\dots+a_{n3})>0\\ \dots\dots\dots\\ -(a_{1n}+a_{2n}+a_{3n}+\dots+a_{nn}+b_{nn}k_n^2)>0\end{cases} \qquad (9)$$

Thus, the stability region of the steady state (4) is determined by the system of inequalities (9).

Let us investigate the stability of the stationary state (5) by the gradient-velocity method the vector of the Lyapunov function. We represent the equations of state (3) in deviations from the stationary state (5) [11]:

$$\begin{cases}\dot{x}_1=-b_{11}x_1^4-4b_{11}\sqrt[3]{\frac{a_{11}+b_{11}k_1^2}{2b_{11}}}x_1^3-\\ -3b_{11}\sqrt[3]{\left(\frac{a_{11}+b_{11}k_1^2}{2b_{11}}\right)^2}x_1^2-(a_{11}+b_{11}k_1^2)x_1+\\ \quad +a_{12}x_2+a_{13}x_3+\dots+a_{1n}x_n\\ \dot{x}_2=a_{22}x_1-b_{22}x_2^4-4b_{22}\sqrt[3]{\frac{a_{22}+b_{22}k_2^2}{2b_{22}}}x_2^3-\\ -3b_{22}\sqrt[3]{\left(\frac{a_{22}+b_{22}k_2^2}{2b_{22}}\right)^2}x_2^2-(a_{22}+b_{22}k_2^2)x_2+\\ \quad +a_{23}x_3+\dots+a_{2n}x_n\\ \dots\dots\dots\\ \dot{x}_n=a_{n1}x_1+a_{n2}x_2+\dots-b_{nn}x_n^4-\\ -4b_{nn}\sqrt[3]{\frac{a_{nn}+b_{nn}k_n^2}{2b_{nn}}}x_n^3-3b_{nn}\sqrt[3]{\left(\frac{a_{nn}+b_{nn}k_n^2}{2b_{nn}}\right)^2}x_n^2-\\ \quad -(a_{nn}+b_{nn}k_n^2)x_n\end{cases} \qquad (10)$$

From (10) we determine the components of the gradient vector for the Lyapunov vector function $V(x)=(V_1(x),\dots,V_n(x))$:

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_1} = b_{11}x_1^4 + 4b_{11}\sqrt[3]{\frac{a_{11}+b_{11}k_1^2}{2b_{11}}}x_1^3 + \\ +3b_{11}\sqrt[3]{\left(\frac{a_{11}+b_{11}k_1^2}{2b_{11}}\right)^2}x_1^2 + (a_{11}+b_{11}k_1^2)x_1, \\ \frac{\partial V_1(x)}{\partial x_2} = -a_{12}x_2, \frac{\partial V_1(x)}{\partial x_3} = -a_{13}x_3, \dots, \\ \frac{\partial V_1(x)}{\partial x_n} = -a_{1n}x_n;\ \frac{\partial V_2(x)}{\partial x_1} = -a_{21}x_1, \\ \frac{\partial V_2(x)}{\partial x_2} = b_{22}x_2^4 + 4b_{22}\sqrt[3]{\frac{a_{22}+b_{22}k_2^2}{2b_{22}}}x_2^3 + \\ +3b_{22}\sqrt[3]{\left(\frac{a_{22}+b_{22}k_2^2}{2b_{22}}\right)^2}x_2^2 + (a_{22}+b_{22}k_2^2)x_2, \\ \frac{\partial V_2(x)}{\partial x_3} = -a_{23}x_3, \dots, \frac{\partial V_2(x)}{\partial x_n} = -a_{2n}x_n; \dots; \\ \frac{\partial V_n(x)}{\partial x_1} = -a_{n1}x_1, \frac{\partial V_n(x)}{\partial x_2} = -a_{n2}x_2, \dots, \\ \frac{\partial V_n(x)}{\partial x_n} = b_{nn}x_n^4 + 4b_{nn}\sqrt[3]{\frac{a_{nn}+b_{nn}k_n^2}{2b_{nn}}}x_n^3 + \\ +3b_{nn}\sqrt[3]{\left(\frac{a_{nn}+b_{nn}k_n^2}{2b_{nn}}\right)^2}x_n^2 + (a_{nn}+b_{nn}k_n^2)x_n \end{cases} \quad (11)$$

The Lyapunov function from (11) can be represented in scalar form:

$$V(x) = \frac{1}{5}b_{11}x_1^5 + b_{11}\sqrt[3]{\frac{a_{11}+b_{11}k_1^2}{2b_{11}}}x_1^4 + b_{11}\sqrt[3]{\left(\frac{a_{11}+b_{11}k_1^2}{2b_{11}}\right)^2}x_1^3 + \frac{1}{2}(a_{11}+b_{11}k_1^2)x_1^2 - \frac{1}{2}a_{12}x_2^2 - \frac{1}{2}a_{13}x_3^2 - \dots - \frac{1}{2}a_{1n}x_n^2 - \frac{1}{2}a_{21}x_1^2 - - \dots + \frac{1}{5}b_{22}x_2^5 + b_{22}\sqrt[3]{\frac{a_{22}+b_{22}k_1^2}{2b_{22}}}x_2^4 + b_{22}\sqrt[3]{\left(\frac{a_{22}+b_{22}k_2^2}{2b_{22}}\right)^2}x_2^3 + \frac{1}{2}(a_{22}+b_{22}k_2^2)x_2^2 - \frac{1}{2}a_{23}x_3^2 - \dots - \frac{1}{2}a_{2n}x_n^2 - \dots - \frac{1}{2}a_{n1}x_1^2 - \frac{1}{2}a_{n2}x_2^2 - \dots + \frac{1}{5}b_{nn}x_n^5 + b_{nn}\sqrt[3]{\frac{a_{nn}+b_{nn}k_n^2}{2b_{nn}}}x_n^4 + b_{nn}\sqrt[3]{\left(\frac{a_{nn}+b_{nn}k_n^2}{2b_{nn}}\right)^2}x_n^3 + \frac{1}{2}(a_{nn}+b_{nn}k_n^2)x_n^2 \quad (12)$$

Potential function (12) can be reduced to the square form [17] by Morse lemma:

$$V(x) \approx \frac{1}{2}(a_{11}+b_{11}k_1^2 - a_{21} - \dots - a_{n1})x_1^2 + \frac{1}{2}(a_{22}+b_{22}k_2^2 - a_{12} - \dots - a_{n2})x_2^2 + \dots + \frac{1}{2}(a_{nn}+b_{nn}k_n^2 - a_{1n} - a_{2n} - \dots - a_{n,n-1})x_n^2 \quad (13)$$

From (13), we obtain the conditions for the existence of the Lyapunov vector function in the form:

$$\begin{cases} a_{11}+b_{11}k_1^2 - a_{21} - a_{31} - \dots - a_{n1} > 0 \\ a_{22}+b_{22}k_2^2 - a_{12} - a_{32} - \dots - a_{n2} > 0 \\ a_{33}+b_{33}k_3^2 - a_{13} - a_{23} - \dots - a_{n3} > 0 \\ \dots\dots\dots \\ a_{nn}+b_{nn}k_n^2 - a_{1n} - a_{2n} - \dots - a_{n,n-1} > 0 \end{cases} \quad (14)$$

The control system (3), built in the class of two-parameter structurally stable mappings, will be stable in an infinitely wide range of indefinite parameters of the control object k_i^2 and $a_{ii}(i = 1,2,\dots,n)$. The stationary state (18) exists and is stable when the uncertain parameters of the object change in the region (14), and the stationary state (5) appear when the state (4) becomes unstable and they do not exist simultaneously. Stationary state (5) is a periodically robustly stable when the system of inequalities (14) is satisfied, i.e. system (3) is a control system with an increased potential for robust stability [11]-[13].

B. System with the Desired Transient Processes

Let us have some system with the desired transient processes, obtained on the basis of a simulation experiment on the system model:

$$\begin{cases} \dot{x}_1 = -b_{11}x_1^4 - b_{11}d_1^1x_1^2 + b_{11}d_1^2x_1 + \\ \quad +a_{12}x_2 + \dots + a_{1n}x_n \\ \dot{x}_2 = a_{21}x_1 - b_{22}x_2^4 - b_{22}d_2^1x_2^2 + \\ \quad +b_{22}d_2^2x_2 + \dots + a_{2n}x_n \\ \dots\dots\dots \\ \dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \dots - \\ \quad -b_{nn}x_n^4 - b_{nn}d_n^1x_n^2 + b_{nn}d_n^2x_n \end{cases} \quad (15)$$

The problem is to determine the coefficients of a controller with an increased potential for robust stability (elements $d_i^2, i = 1,\dots,n$) and such that the coefficients of the elements of the closed-loop system had a given value d_i^2.

Let us investigate systems with given values of the coefficients $d_i^2, i = 1,\dots,n$, using the gradient-velocity method of the Lyapunov vector function and show that system (15) is a control system with an increased potential of aperiodic robust stability.

The stationary state of system (15) is

$$x_{1s}^1 = 0, x_{2s}^1 = 0, \dots, x_{ns}^1 = 0 \quad (16)$$

Other stationary states of system (15) are:

$$x_{is}^{2,3} = \pm\sqrt[3]{\frac{d_i^2}{2}},\ i = 1,\dots,n. \quad (17)$$

The study of the stability of stationary states (16) and (17) is carried out by the gradient-velocity method of the Lyapunov vector function [11], [19].

From (15) we determine the components of the gradient vector from the Lyapunov vector function $V(x) = (V_1(x_1,\dots,x_n),\dots,V_n(x_1,\dots,x_n))$:

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_1} = b_{11}x_1^4 + b_{11}d_1^1x_1^2 - b_{11}d_1^2x_1, \\ \frac{\partial V_1(x)}{\partial x_2} = -a_{12}x_2, \dots, \frac{\partial V_1(x)}{\partial x_n} = -a_{1n}x_n; \\ \frac{\partial V_2(x)}{\partial x_2} = b_{22}x_2^4 + b_{22}d_2^1x_2^2 - b_{22}d_2^2x_2, \\ \frac{\partial V_2(x)}{\partial x_1} = -a_{21}x_1, \dots, \frac{\partial V_2(x)}{\partial x_n} = -a_{2n}x_n; \\ \dots\dots\dots \\ \frac{\partial V_n(x)}{\partial x_1} = -a_{n1}x_1, \frac{\partial V_n(x)}{\partial x_2} = -a_{n2}x_2, \dots, \\ \frac{\partial V_n(x)}{\partial x_n} = b_{nn}x_n^4 + b_{nn}d_n^1x_n^2 - b_{nn}d_n^2x_n \end{cases} \quad (18)$$

From the components of the gradient vector, of the Lyapunov vector function (18), we can construct the Lyapunov vector function in scalar form [11].

$$V(x) = \frac{1}{5}b_{11}x_1^5 + \frac{1}{3}b_{11}d_1^1x_1^3 - \frac{1}{2}b_{11}d_1^2x_1^2 - \frac{1}{2}a_{12}x_2^2 - \frac{1}{2}a_{13}x_3^2 - \dots -$$

$$\frac{1}{2}a_{1n}x_n^2 - \frac{1}{2}a_{21}x_1^2 + \frac{1}{5}b_{22}x_2^5 + \frac{1}{3}b_{22}d_2^1x_2^3 - \frac{1}{2}b_{22}d_2^2x_2^2 - \frac{1}{2}a_{23}x_3^2 - \cdots - \frac{1}{2}a_{2n}x_n^2 - \cdots - \frac{1}{2}a_{n1}x_1^2 - \frac{1}{2}a_{n2}x_2^2 - \frac{1}{2}a_{n3}x_3^2 - \cdots + \frac{1}{5}b_{nn}x_n^5 + \frac{1}{3}b_{nn}d_n^1x_n^3 - \frac{1}{2}b_{nn}d_n^2x_n^2 \quad (19)$$

The conditions for the positive definiteness of the function $V(x)$ from (19) are not obvious; therefore, we use the Morse lemma from catastrophe theory [17], in the vicinity of the stationary state (17) can be represented in the form of a quadratic form:

$$V(x) \approx -\frac{1}{2}(b_{11}d_1^2 + a_{21} + \cdots + a_{n1})x_1^2 - \frac{1}{2}(a_{12} + b_{22}d_2^2 + \cdots + a_{n2})x_2^2 - \cdots - \frac{1}{2}(a_{1n} + a_{2n} + \cdots + b_{nn}d_n^2)x_n^2 \quad (20)$$

Conditions for the existence of the Lyapunov vector function, i.e. the positive definiteness of the quadratic form (20) is determined by the inequalities:

$$\begin{cases} -(b_{11}d_1^2 + a_{21} + a_{31} + \cdots + a_{n1}) > 0 \\ -(a_{12} + b_{22}d_2^2 + a_{23} + \cdots + a_{n2}) > 0 \\ -(a_{13} + a_{23} + b_{33}d_3^2 + \cdots + a_{n3}) > 0 \\ \cdots\cdots\cdots \\ -(a_{1n} + a_{2n} + a_{3n} + \cdots + b_{nn}d_n^2) > 0 \end{cases} \quad (21)$$

The region of aperiodic robust stability of the stationary state (16) of system (15) is determined by the system of inequalities (21).

The stability of the stationary state (17) of system (15) is investigated by the gradient-velocity methods of the Lyapunov vector function [11], [19]-[21]. For this equation of state (15) is represented in deviations from the steady state (17) [11]:

$$\begin{cases} \dot{x}_1 = -b_{11}x_1^4 - 4b_{11}\sqrt[3]{\frac{d_1^2}{2}}x_1^3 - 3b_{11}\sqrt[3]{\left(\frac{d_1^2}{2}\right)^2}x_1^2 - \\ \quad -b_{11}d_1^2x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n \\ \dot{x}_2 = -b_{22}x_2^4 - 4b_{22}\sqrt[3]{\frac{d_2^2}{2}}x_2^3 - 3b_{22}\sqrt[3]{\left(\frac{d_2^2}{2}\right)^2}x_2^2 - \\ \quad -b_{22}d_2^2x_2 + a_{21}x_1 + a_{23}x_3 + \cdots + a_{2n}x_n \\ \cdots\cdots\cdots \\ \dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \cdots - b_{nn}x_n^4 - \\ \quad -4b_{nn}\sqrt[3]{\frac{d_n^2}{2}}x_n^3 - 3b_{nn}\sqrt[3]{\left(\frac{d_n^2}{2}\right)^2}x_n^2 - b_{nn}d_n^2x_n \end{cases} \quad (22)$$

The components of the gradient vector for the Lyapunov vector function are determined from (22):

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_1} = b_{11}x_1^4 + 4b_{11}\sqrt[3]{\frac{d_1^2}{2}}x_1^3 + \\ \quad +3b_{11}\sqrt[3]{\left(\frac{d_1^2}{2}\right)^2}x_1^2 + b_{11}d_1^2x_1, \\ \frac{\partial V_1(x)}{\partial x_2} = -a_{12}x_2, \ldots, \frac{\partial V_1(x)}{\partial x_n} = -a_{1n}x_n; \\ \frac{\partial V_2(x)}{\partial x_2} = b_{22}x_2^4 + 4b_{22}\sqrt[3]{\frac{d_2^2}{2}}x_2^3 + \\ \quad +3b_{22}\sqrt[3]{\left(\frac{d_2^2}{2}\right)^2}x_2^2 + b_{11}d_1^2x_1, \\ \frac{\partial V_2(x)}{\partial x_1} = -a_{21}x_1, \ldots, \frac{\partial V_2(x)}{\partial x_n} = -a_{2n}x_n; \\ \cdots\cdots\cdots \\ \frac{\partial V_n(x)}{\partial x_1} = -a_{n1}x_1, \frac{\partial V_n(x)}{\partial x_2} = -a_{n2}x_2, \ldots, \\ \frac{\partial V_n(x)}{\partial x_n} = b_{nn}x_n^4 + 4b_{nn}\sqrt[3]{\frac{d_n^2}{2}}x_n^3 + \\ \quad +3b_{nn}\sqrt[3]{\left(\frac{d_n^2}{2}\right)^2}x_n^2 + b_{nn}d_n^2x_n \end{cases} \quad (23)$$

From (23) we obtain the Lyapunov vector function in scalar form:

$$V(x) = \frac{1}{5}b_{11}x_1^5 + b_{11}\sqrt[3]{\frac{d_1^2}{2}}x_1^4 + b_{11}\sqrt[3]{\left(\frac{d_1^2}{2}\right)^2}x_1^3 + \frac{1}{2}b_{11}d_1^2x_1^2 - \frac{1}{2}a_{12}x_2^2 - \frac{1}{2}a_{13}x_3^2 - \cdots - \frac{1}{2}a_{1n}x_n^2 - \frac{1}{2}a_{21}x_1^2 + \frac{1}{5}b_{22}x_2^5 + b_{22}\sqrt[3]{\frac{d_2^2}{2}}x_2^4 + b_{22}\sqrt[3]{\left(\frac{d_2^2}{2}\right)^2}x_2^3 + \frac{1}{2}b_{22}d_2^2x_2^2 - \frac{1}{2}a_{23}x_3^2 - \cdots - \frac{1}{2}a_{2n}x_n^2 - \cdots - \frac{1}{2}a_{n1}x_1^2 - \frac{1}{2}a_{n2}x_2^2 - \cdots + \frac{1}{5}b_{nn}x_n^5 + b_{nn}\sqrt[3]{\frac{d_n^2}{2}}x_n^4 + b_{nn}\sqrt[3]{\left(\frac{d_n^2}{2}\right)^2}x_n^3 + \frac{1}{2}b_{nn}d_n^2x_n^2 \quad (24)$$

From (24) the positive or negative definiteness of the Lyapunov function is not obvious; therefore, we use the Morse lemma from catastrophe theories [17] and obtain

$$V(x) \approx \frac{1}{2}(b_{11}d_1^2 - a_{21} - \cdots - a_{n1})x_1^2 + \frac{1}{2}(-a_{12} + b_{22}d_2^2 - \cdots - a_{n2})x_2^2 + \cdots + \frac{1}{2}(-a_{1n} - a_{2n} - \cdots + b_{nn}d_n^2)x_n^2 \quad (25)$$

The positive definiteness conditions for the quadratic form (25) are determined by the system of inequalities:

$$\begin{cases} b_{11}d_1^2 - a_{21} - a_{31} - \cdots - a_{n1} > 0 \\ b_{22}d_2^2 - a_{12} - a_{23} - \cdots - a_{n2} > 0 \\ b_{33}d_3^2 - a_{13} - a_{23} - \cdots - a_{n3} > 0 \\ \cdots\cdots\cdots \\ b_{nn}d_n^2 - a_{1n} - a_{2n} - \cdots - a_{n-1,n} > 0 \end{cases} \quad (26)$$

System (22) will be aperiodically robust stable within an infinitely wide range of variation of parameters d_i^2, $i=1, \ldots, n$, i.e. is a system with an increased potential for robust stability.

Comparing the left-hand sides of inequalities (9) and (21) or (14) and (26), we obtain

$$\begin{cases} k_1^2 = d_1^2 - \frac{a_{11}}{b_{11}}, \\ k_2^2 = d_2^2 - \frac{a_{22}}{b_{22}}, \\ \cdots\cdots\cdots \\ k_n^2 = d_n^2 - \frac{a_{nn}}{b_{nn}}. \end{cases} \quad (27)$$

Thus, for a completely controllable linear plant with a control law in the class of two-parameter structurally stable mappings, a simple solution to the problem of synthesis of control systems for unstable and deterministic chaotic processes is obtained.

III. Conclusion

Chaotic and unstable systems usually represent a class of uncertainty models. Stability under uncertainty is understood as robust stability. When the conditions of robust stability are violated, a regime of deterministic chaos and instability is generated in the system. Under conditions of significant uncertainty, the synthesis of a control system in the class of two-parameter structurally stable mappings is one of the main factors that guarantee the control system protection from the regime of deterministic chaos and instability.

The existing methods of model control and frequency methods solve the problem of synthesizing only linear control systems of low order and small dimension. This requires complex and ambiguous calculations of the eigenvalues and eigenfunctions of the control object, as well as direct and inverse canonical transformations.

The gradient-velocity method of the Lyapunov vector function allows to solve the problem of synthesizing an aperiodic robust stable nonlinear system of the high-order.

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