

# Identifying Chaotic Architecture: Origins of Nonlinear Design Theory

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**Abstract**—Through the emergence of modern architecture, an aggressive desire for new design theories appeared through the works of architects and critics. The discourse of complexity and volumetric composition happened to be an important and controversial issue in the discipline of architecture which was discussed through a general point of view in Robert Venturi and Denise Scott Brown's book "Complexity and contradiction in architecture" in 1966, this paper attempts to identify chaos theory as a scientific model of complexity and its relation to architecture design theory by conducting a qualitative analysis and multidisciplinary critical approach through architecture and basic sciences resources. Accordingly, we identify chaotic architecture as the correlation between chaos theory and the discipline of architecture, and as an independent nonlinear design theory with specific characteristics and properties.

**Keywords**—Architecture complexity, chaos theory, fractals, nonlinear dynamic systems, nonlinear ontology.

## I. INTRODUCTION

THE desires of architectural complexity in recent years had been an undeniable and progressive discourse in the discipline of architecture. Since Robert Venturi and Denis Scott Brown's important theoretical book "Complexity and Contradiction in Architecture" in 1966, it has been crucial for architects to define and theorize the aspects and origins of architectural complexity [1]. This desire was clearly expressed in Johnson and Wigley's "Deconstructivist Architecture" exhibition at MoMA (Museum of Modern Art) in 1988 [1]. Since then, different approaches and movements appeared in the context of post-modernist movements which could be recognized as two major tendencies among designers and theorists. The first was the shift from linguistic and representational focus, mostly driven from deconstructivist philosophers to the spatial models [1]. The second tendency was driven from a scientific model of complexity by the introduction of the new theory in basic sciences famously marked as chaos and complexity.

In the middle of the 20<sup>th</sup> century, chaos theory was discovered in meteorology and rapidly transferred to other sciences, eventually recognized as an independent term of knowledge and also as the most important theory of the 20<sup>th</sup> century alongside relativity and quantum mechanics. Chaos theory is the science of nonlinear dynamical systems which describes complexity, geometry, mathematics, structure, order and behavior of chaotic systems of nature and creatures including humans and their behavior [2]. Architecture as an

interdisciplinary phenomenon, notwithstanding its interdependency, has a contiguous relation with a variety of disciplines instantiate philosophy, theology, anthropology, mathematics, statics, chemistry, meteorology, sociology etc. [3]. Due to this interdisciplinary character, the manifestation of a revolutionary theory in the opposite direction from the basis of modern science with extravagant potentials could influence and shift the principles of architecture. Humankind as part of nature, undeniably and inevitably was inspired and influenced by nature in his inventions, revolutions and developments through history and specifically architecture as a human achievement has always been in an inevitable interaction and relation with nature through its entire eras [4]. Chaos theory is relatively a new and fresh development in sciences, researchers and writers in the discipline of architecture have not adequately addressed this issue. Interdisciplinary research in architecture and relevant disciplines is necessary to recognize and define "chaotic architecture". Chaos theory had been described in the Oxford dictionary as: the study of a group of connected things that are very sensitive, so that small changes in conditions affect them very much, the science of nonlinear topics [5]. The term "chaos theory" is established based on the disordered appearance and form of the systems which the theory attempts to describe, although chaos theory pursues the underlying and governing order in apparently random and disordered data. Since chaos theory is described as the science of nonlinear topics, its substantial potential and application in basic sciences, applied sciences, engineering and different fields of knowledge had been addressed by various scientists and researchers [6]. Edward Lorenz (1917-2008) was the first and initial experimenter of chaos in the field of meteorology regarding his research on weather prediction in 1960 [2]. During his research in 1961, Lorenz discovered the property of sensitivity of the climate system to its initial condition and unpredictable effect of initial data on the system which he described as the butterfly effect. This term was used due to the significantly small amount of difference between the starting points of two curves. His famous statement describing the butterfly effect stated that "The flapping of a single butterfly's wing today produces a tornado in the future". He describes further that the flapping of a butterfly wing will produce a diminutive change in the state of the atmosphere which over a period of time would cause divergence. In a larger period of time, for example, a month, a tornado that was not going to happen would devastate the Indonesian coast, or maybe one that was going to happen, did not [7]. Diminutive change of initial conditions could drastically affect the behavior of a

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system in a large period of time. Due to this property, Lorenz concluded that the exact and accurate prediction of weather condition is impossible. Although, the discovery of this phenomenon, also known as “sensitive dependence to initial conditions” as one of the fundamental properties of chaos theory, eventually led Lorenz to other aspects of this theory. Weather patterns are an example of non-linear dynamical system which exhibit chaotic behavior and are highly sensitive to initial conditions [6]. Lorenz, by conducting experiments and formulating the weather information and variable behaviors, using them as an input to compute the diagram of weather changes, discovers the pattern behind the chaotic behavior of them, known as the “Lorenz attractor” and the paradigm of chaos [8].

## II. METHODOLOGY

A qualitative analysis research through library studies and academic resources with a multidisciplinary approach had been conducted for this research. By using the context analysis method, with a critical approach and study of existing sources and works of contemporary architecture as well as related experiences and articles, an attempt was made to conduct the research. The main area of the research includes chaos theory, fractal geometry, architecture history and complexity in architecture.

## III. RESULTS

### A. Nonlinear Dynamical Systems

Evolution of modern science navigates based on the legacy of Isaac Newton and the concepts of causality (every effect has an antecedent, proximate cause), predictability (correct forecast of a system’s state can be made either qualitatively or quantitatively) and linear system (a system in which the whole is exactly equal to the sum of its components) [9]. The most important contribution of Newton to the modern science is the concept of dynamical systems. A dynamical system defines two characteristics: 1) set of parameters which states define by their values at a specific point of time and 2) set of mathematically specific rules defining the changes and behavior of the state of the system in time [10]. The classical physics assumption is the possibility of focusing upon a conceptually isolated system with no external forces from the surroundings in a well-defined situation [11]. These rules are generally known as differential equations, determining the rate of change of each parameter in the system as the function of the whole system. As stated, modern science evolved based on a linear system, for instance, in physics almost any system under study, whether planetary orbits, semiconductor electronics, or the Earth’s atmosphere, are considered dynamical linear systems, same as many systems in biology, medicine, economics, social sciences etc. Nonlinear systems, on the other hand, have none-continuous behavior. Dynamism of these systems does not follow an integrated curvature, and their diagram is geometrically irregular. The changes of output are disproportionate to changes in input. In other words, changes in multiple parameters need neither be additive nor

proportional to conclude in extreme changes [11]. In terms of the relationship between components, a nonlinear system is a set of interacting elements with communication and interactive effect between them. For instance, let’s consider a basketball team a system, it is consisting of elements in action and mutual reaction. If the central element of the system is injured in a game, the rest of the components covers his shortage. One of the members (central element) affects all components and instead, their engagement allows the central member the opportunity to recover. In contrast, from linear systems, nonlinear systems show a disproportionate relationship between cause and effect in a way that extremely little changes may affect the system unpredictably, especially over a long period of time.

### B. Characteristics of Chaotic System

Chaotic systems have four principal properties that could be considered as the main characteristics of each chaotic system which would be discussed and explained separately in the following titles:

#### 1. Sensitive Dependence on Initial Conditions

Sensitivity to initial conditions means that every little change in the system’s initial data dramatically influences the whole system through irritation. Sensitivity to initial conditions, often referred to as the butterfly effect, mentioned in a lecture by Edward Lorenz in 1972, claims that the final output and the result of the system will be vastly different for very small changes in the initial conditions, or in other words: “The flap of a butterfly’s wings in Brazil set off a tornado in Texas” [6].

#### 2. The Chaotic (Strange) Attractor

Non-chaotic attractors are simply points, cycles or smooth surfaces and their geometry is regular. Generally, the diagram of a phase space of a system is called an absorption layer or the “attractor”, because if we show the phase of a system as points in its phase space, the absorption layer determines the range and confine of these points. Predictability in non-chaotic attractors because of their non-sensibility to initial conditions is meaningful and useful. Meanwhile, due to the characteristics of chaotic systems, chaotic attractors are unpredictable. In chaotic systems, chaotic refers to the trajectory dynamics and motion of the attractor which is divergence and unpredictable. Such attractors have fractal properties. Meanwhile, it is still the set of the points which the system settles down to in phase space. Although a chaotic attractor is reproducible and shows the zones of revolving behavior in the form of orderly periodicity [12], a trajectory in a chaotic system is more complex than a simple loop or regular diagram. It usually has a complex, multi-layer inner structure and the external appearance is elaborate and more variable compared to loops and smooth surfaces. Its dimension is not necessarily an integer and usually has a fractal behavior.

#### 3. Order within Chaos

Chaos seems erratic, but it is not just a mass system of disorder. Despite its name and appearance, chaos has a great

deal of regularity and order. Chaotic order exists behind the strange attractor fractal structure, self-organization and complexity, addictiveness and flexibility, optimization [12]. Phase transition and creating and destroying information are properties in chaotic systems which have their own order and regulation in each system.

### C. Fractal Structure

Fractals are discussed separately and in detail in the body of this research. In this part we mention fundamental descriptions due to its importance in chaotic systems. A fractal is a self-similar pattern that repeats on itself. Each piece of a fractal appears as the same figure we repeatedly magnify [12]. Fractals are all around us from large landscapes, the cosmos, smoke, cracks on a surface to the structure of a snowflake, all are fractal geometry. Although fractals are a class of geometric forms and chaos is a class of dynamical behavior, they are closely intertwined and often occur together. Chaos characteristics and elements may have fractal properties, for instance, the chaos attractor or other chaos-related geometric objects. Because of this relation, fractals may often help to recognize and detect the chaos.

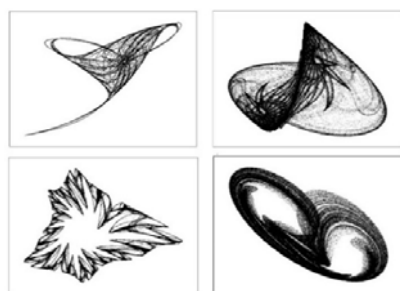


Fig. 1 Chaotic attractor or strange attractor formulated and drawn by Julien Clinton Sprott in 2000, published in the American Journal of Physics [13]

Benoit B. Mandelbrot (1924-2010), known as the father of fractals, believes in the necessity of a new geometry for describing the nature which represents a rough, non-circular, smooth and non-linear world [16]. Fractal is described as the geometry of holes, wrinkles and twists. Mandelbrot mathematically defines fractals as a rough and fragmented geometric shape which could be subdivided in parts and each of which is a scaled copy of the whole. In other words, a fractal is a self-similar pattern which repeats itself. The properties of a fractal structure could be described in four major characters:

- 1) Self-similarity over scale: Fractal structure is a self-similar phenomenon with a ritual geometry over the scale. This means that it consists of a geometry in a large scale which repeats itself in a smaller scale in the same phenomenon [14].
- 2) Detailed structure at small scale: A fractal structure is perfectly detailed and in small scales it has the same complexity as at the larger scale. This is due to the repetitive mathematical formula of its geometry which produces the same detailed geometry in different scales.

- 3) Fractal dimension: Mandelbrot in his article “How Long is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension” discusses the length of the British coast with the assumption that the length of the coast depends on the measuring tool thickness and the detail of the stick to measure it. This means that, the more detailed the stick of measurement gets, the larger and more precise the coast length would be [14].
- 4) Irregularities that cannot be described by Euclidian geometry principles: Because of the fundamental and mathematical discipline of fractal geometry, in the fractal structure, irregularities would emerge which are not justifiable by classic geometry. Rough edges, drastic curves and unpredictable fluctuations in the shape of a fractal structure are almost impossible to be calculated or described by the principles of Euclidian geometry. Living creatures are a phenomenon describable by fractals. Self-similarity property is one of the characteristics of nature. From researches and observations in biology, it turns out that neither in the animal nor vegetative world’s structure fragmentation of cells, tissues, and organs reproduce with different levels of complexity and by the characteristics of fractal systems [15]. In fact, the natural system of the world from micro biological structure to the shape of clouds and massive mountains are structured by fractal geometry. This phenomenon exists in humans and living organisms, blood vessels from the veins to the capillaries shape a kind of chain from fractal nature in their branching. Biologists have discovered systems that control the structures throughout the body, urinary system, heart pulse system, vessel branching system and all of the organs in the human body follow fractal geometry, the same as snowflakes, tree branches and leaves, fish scales, the physical shape of the brain, earthquakes and natural phenomenon.

After Mandelbrot's introduction of fractals, scientists from different branches and disciplines investigate the rout and application of fractal geometry and its principles in different disciplines. These researches and investigations show that not only the nature and environment, natural phenomena and climate, but the human organs, human brain, vain distribution and human body systems adopt fractal structure and behavior [16]. These studies and investigations through the fractals in nature and humans are vastly conducted world-wide in large and small scales, in climate behavior, biology, pathology, malacology and many other realms of science. Results and outcomes prove the fractal component and structure of natural phenomena and the reality that the world and nature is justifiable by fractal geometry. Tracing the fractals in architecture history goes back to ancient civilizations from ancient African settlements, architecture of south Asia and Europe cathedrals [17].

In a general view, we can categorize fractal geometry in three areas in our research through architecture:

- 1) Fractal geometry: The mathematical formulation of fractals and non-integer numeric property of them, which can be found on the proportions and numeric evaluation

of architecture. This geometric principle had been one of the common geometric tools in pre-modern architecture which could be traced in different architectural cultures and eras throughout history

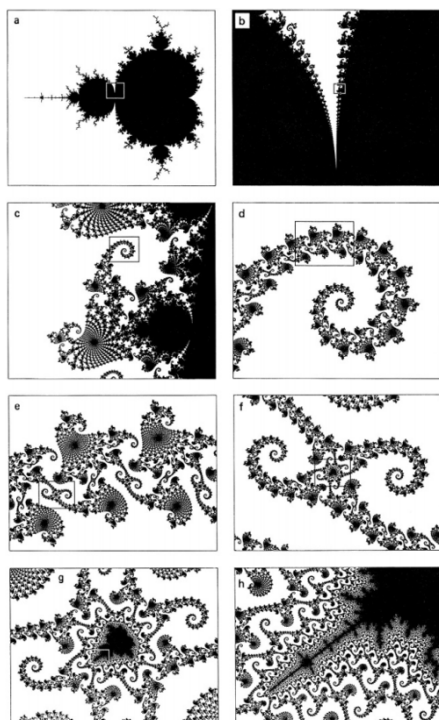


Fig. 2 Mandelbrot walk: Zoom into the Mandelbrot set – Properties of self-similarity and scaling of the fractals [16]

- 2) Fractal forms: The outcome form of fractal formulations which shows itself as Mandelbrot figure, Lorenz attractor form and natural fractal forms in nature and universe which can be found in ornaments, tiling, mosaics, windows, motifs and different elements of architecture worldwide.
- 3) Fractal mass: The spatial possibilities of fractals which appear as architecture forms, spaces and the structure of buildings. To investigate these categories, we generally adapt the visual analysis in architectural elements and buildings to find fractal properties such as the fractal pattern or self-similar elements which repeat themselves in different scales. This analysis was conducted through plans, sections, ornaments and different architectural components of a building. Through the time, researchers develop complicated methods to analyze the fractals in natural phenomena and architecture buildings such as the method of fractal dimension analysis which uses the box counting dimension method to analyze the mathematical relation between the dimensions in a figure and a phenomenon [18].

Fractals have been applied to many aspects and components of ancient and classic architecture, from ornaments to windows, decoration and elevations of structural elements and design. In fact, the presence of fractals in architecture goes back centuries before Mandelbrot theory, in ancient

architecture and especially in spiritual and sacred spaces in different civilizations and different religions from cathedrals to mosques in ornaments, structure and configuration of the spaces [19]. Interestingly, classical architecture, both in the west and east, used the famous Euclidean geometry formulas, but none practiced it. In fact, it was the modern architecture style that replaced Euclidean geometry with classical geometry and reduced spaces to a composition of Euclidean volumes [20].



Fig. 3 Museo della Cattedrale di Anagni Tiling, Italy, 1250 Source: Matthew Balkey Pintrest gallery

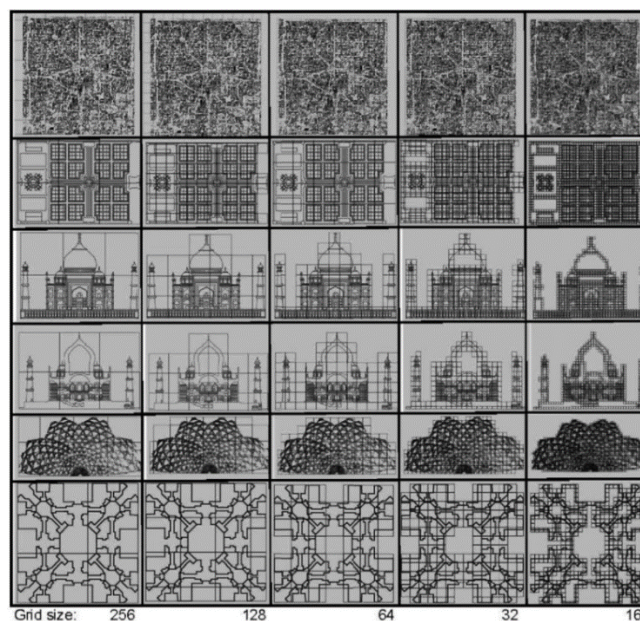


Fig. 4 Study of fractal pattern of Taj Mahal [18]

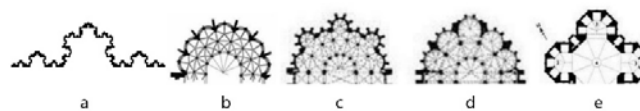


Fig. 5 (a) Koch curve, plans of (b) St Denis, (c) Charters, (d) Reims and (e) Florence cathedrals [21]

#### D. Fractals in Architecture

##### 1. Small-Scale Analysis

In small-scale analysis of fractals in architecture, we analyze the buildings and elements of fractals in buildings. The examples of fractals in architecture history are varied all over the world. We could find the self-similarity property of

fractals in cathedrals of France and Italy back in the 12<sup>th</sup> century [22]. These fractal examples are not limited to cathedrals. Castel del Monte of Italy from the 13<sup>th</sup> century has the fractal property in its planning and space configuration, which in comparison with a Mandelbrot set, these similarities and properties would be more obvious. In gothic style, the fractal geometry is embedded with Euclidian geometry, Nelly Ramzy in his research with the title of: “The Dual Language of Geometry in Gothic Architecture: The Symbolic Message of Euclidian Geometry versus the Visual Dialogue of Fractal Geometry” studied the fractal principles in the gothic cathedrals and showed that the plans, elevations and ornaments of gothic style adapt fractal geometries characteristics of self-similarity, scaling and pattern [21]. In the gothic style, apart from the textures and tiling which are the graphical representation of fractals and external form of the roof which follows the similarity in different scales, architects adapt the fractal geometric parameters to generate and design the roughness and proportions of the spaces and arcs [23]. In gothic architecture, fractal geometry emerged in different elements of architecture, from vaults to windows, planning, sections and structural elements. Examples of these studies vary by the emergence of chaos theory; researchers look at the architecture throughout history through a new perspective. The study of fractals in the Taj Mahal building in India (Fig. 4) shows the property of pattern and self-similarity of fractals and the characteristics of fractal geometry and proportions as well [18].

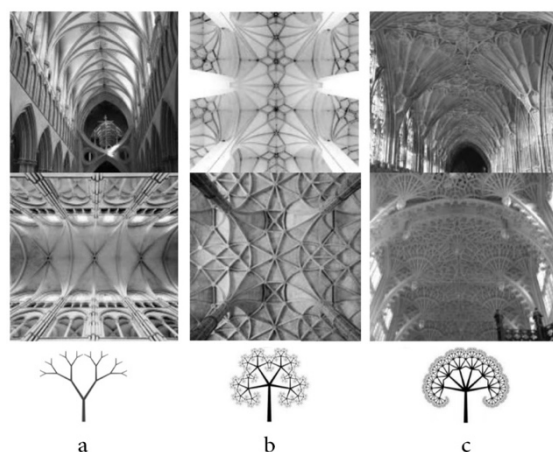


Fig. 6 Gothic cathedrals vaults fractal patterns: (a) Wales Cathedral, England and Laon. (b): Lierne ribs: Church of Hieronymite Monastery, Portugal and Frauenkirche, Munich, Germany (c) Gloucester Cathedral, England and Chapel of Henry VII, Westminster, England Cathedral, France [21]

In modern architecture, Peter Eisenman in 1978, less than two years after the publication of the fractal geometry paper by Mandelbrot with a philosophical approach, adopted the fractal scaling and self-similarity construction to generate the concept of “discontinuity” which refers to the metaphysics of presence and aesthetic object. Eisenman proposes the concept of: discontinuity, recursivity and self-similarity of fractal

geometry in his process of design by an initial L shape which was originally a square divided to four equal squares, and one square was deducted [24]. In contemporary architecture, famous figures such as Zvi Hecker, Greg Lynn and Bernard Tschumi in their experience through complexity in the form of deconstruction style adapt different properties of fractals in their process of design rather than represent it in the form of decoration or ornaments [25].

## 2. Large-Scale Analysis

This analysis will take place on an urban scale and study the pattern of growth of urban structure and morphology of fractals in large scales. Theoretically, the nature of growth is happening with a fractal property [28]. By analyzing the urban morphology of cities and urban areas, we will discover the characteristics and principles of fractals, self-similarity, pattern of growth and scaling in their geometry [29].

### E. Chaos and Architecture

By reviewing and studying the chaos and characteristics of chaos theory, we attempt to define “Chaotic Architecture” and distinguish the borders and boundaries of chaos language in architecture. Forces are essential for emergence of chaos, for a chaotic architecture to be shaped in a context these forces are included. A chaotic architecture is a reaction to its context forces and will shape and grow by these forces. In other words, a chaotic architecture system is not an imposition on the context, but a growth of that context [28].

- a) Compatibility and flexibility: Architecture begins with identifying an issue and a demand. An architectural chaotic system emerges from a context and needs congruency to its environment for survival.
- b) Pattern Making: Architecture system begins with a demand, and by the definition of chaos, it should make a pattern.
- c) Optimization: The purpose of optimization in chaotic architecture is to find the best answer to the forces in a strategic way. A chaotic system has the property of optimized behavior and reaction to its external forces, and it is in a strategic state with its environment and soundings.
- d) Determination and unpredictability: These are property of chaotic architecture which emerges in the process of growth. Determination refers to environment scales that are over the human control and unpredictability refers to controllable spaces and phenomenon by humans. These two characteristics are one of the most important principles of chaotic systems and cause of the beauty and pleasantness in natural systems. Humans in relation to a phenomenon start to define their situation and comprehend the phenomenon from general to details by instinct. This process will happen in relation to this property in chaotic architecture buildings.

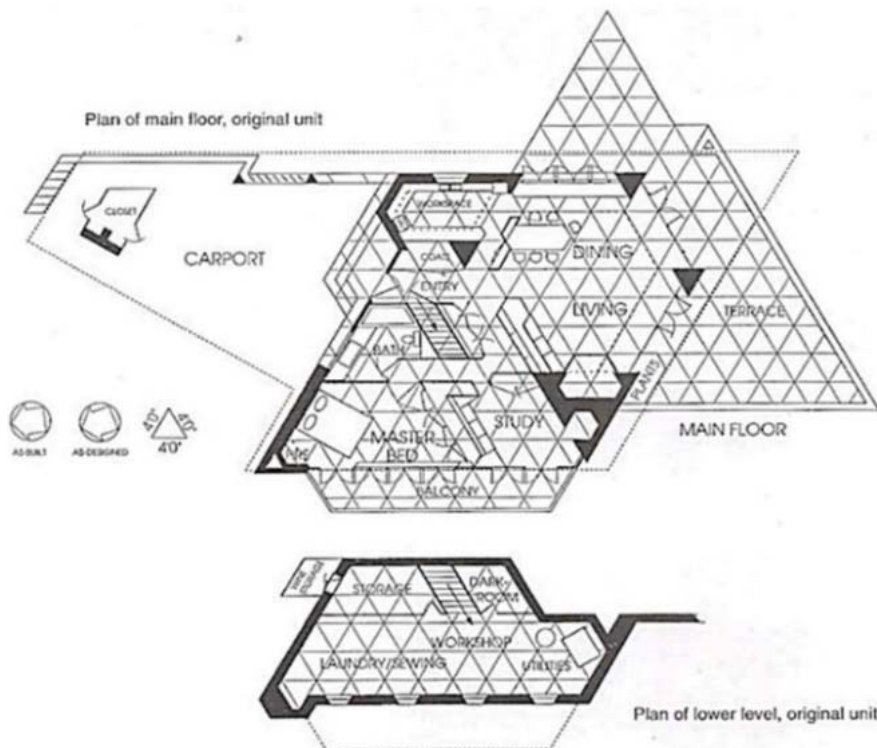


Fig. 7 Palmer House, Ann Arbor, Michigan, Frank Lloyd Wright [26]

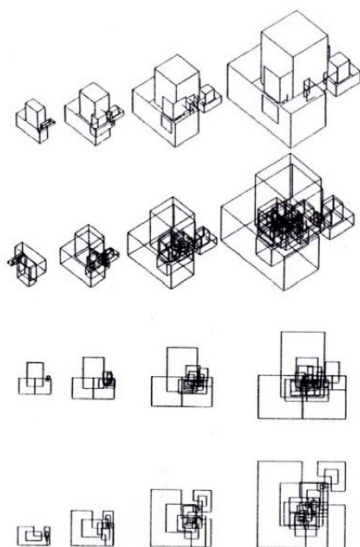


Fig. 8 Fin d'Out Hou S", Peter Eisenman [27]

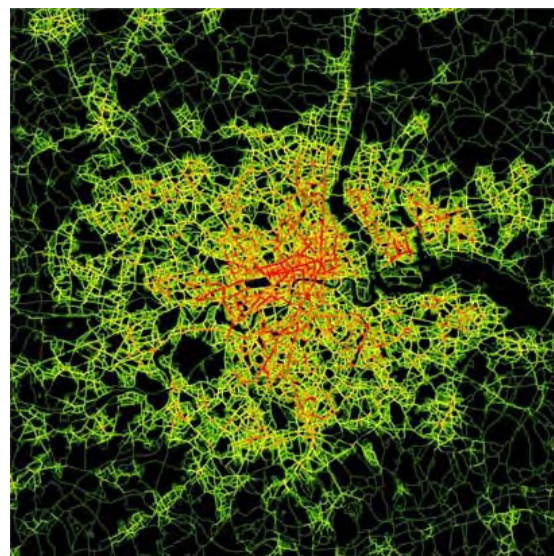


Fig. 9 Evolution of the road network in London by Michael Batty: cities develop as living systems by set of elements and components through dynamic interactions [30]

- e) Synergy of information: To create architecture similar to natural systems, architecture should try to propose new information constantly and never be linear and repetitive. This property will define different scale levels and irritation in chaotic systems.

#### IV. DISCUSSION AND CONCLUSION

Through this research, we attempt to define chaos theory and its origins and different aspects of it. We also witness evidence of chaotic figures and fractal geometries in the history of architecture but no exact definition of chaotic architecture and theorizing. Traces of chaos theory were found in decoration, ornaments and volumetric accessories as well. We also find information about structural designs based on

fractal geometry and chaos discipline in the 18<sup>th</sup>-century architecture.



Fig. 10 Kandovan, Iran. Growth pattern in the mountain zone, contexts forces shaping the architecture [31]



Fig. 11 Karbandi in Borujerdi House, Kashan, Iran [32]



Fig. 12 Alhambra castle in Granada, Spain, details and compositions are defined for any scale and distance. An endless source of information always keeps an observer busy. The building is a good estimate of the fractal object with fractional dimension at different scaling levels [33]

humans, and economics which are chaotic systems in nature, and also the relation to its sub systems such as structural system, spatial system, decoration system, geometrical system and material system within the architecture system itself. This principle in chaotic architecture makes the systematic nature of architecture an inevitable phenomenon. From this perspective, architecture phenomenon is the response and result to its super systems and an integrated system consisting of sub systems. In this definition, architecture will define a system rather than an object, the geometrical principal would be defined based on fractal geometry rather than Euclidian geometry, and non-linear principles will define the dynamic super systems, and the architecture system itself.

- Although Chaos Theory was proposed in the 20<sup>th</sup> century and in the last decades of the century recognized as a term of science, but its roots and origins go back to early centuries of human architecture, and by analyzing the evidence and information collected from all over the world, we could come to the conclusion that it is the experimental quality of vernacular and local architecture.
- Chaos has a qualitative entity in architecture. This qualitative entity means that chaos is a quality which could emerge in architecture phenomenon and reveals itself in different aspects and forms of architecture.
- Fig. 13 demonstrates the chaotic architecture based on the findings and discussions in research in which its most important characteristics are the systematic behavior of chaotic systems and their relation to each other.
- Architecture as a system interacts with its super-systems and is born, developed and influenced as a part of these systems. Regarding the aesthetic quality of chaos, it would appear to interact with the aesthetics of its context, so chaotic architecture at any point in time and place would be a unique phenomenon in its context, both internally and externally. Chaotic architecture reveals itself in different contexts and eras, and it is not a style or language, rather quality in architecture. We could trace and find chaotic quality mostly in local and vernacular buildings, cathedral, mosques, palaces, religious and sacred context because of the importance of chaotic geometry and its relation to humans.
- Architecture buildings and elements could be chaotic in different scales. Both in general and individual, chaos can express itself in different languages. This systematic perspective is not limited to visual elements and embedded the abstract elements of architecture too. In general, the architectural system consists of sub-systems with chaotic quality and as a general chaotic system in dialogue with independent and external chaotic super-systems. Chaos is of initial quality of architecture, and chaotic architecture could be recognized and represent itself in architecture practice. This qualitative property could represent itself in different dimensions and aspects of the architectural system.
- Due to the properties of chaos theory, a chaotic

architecture is the creature of its context and belongs to the specific characteristics of that context.

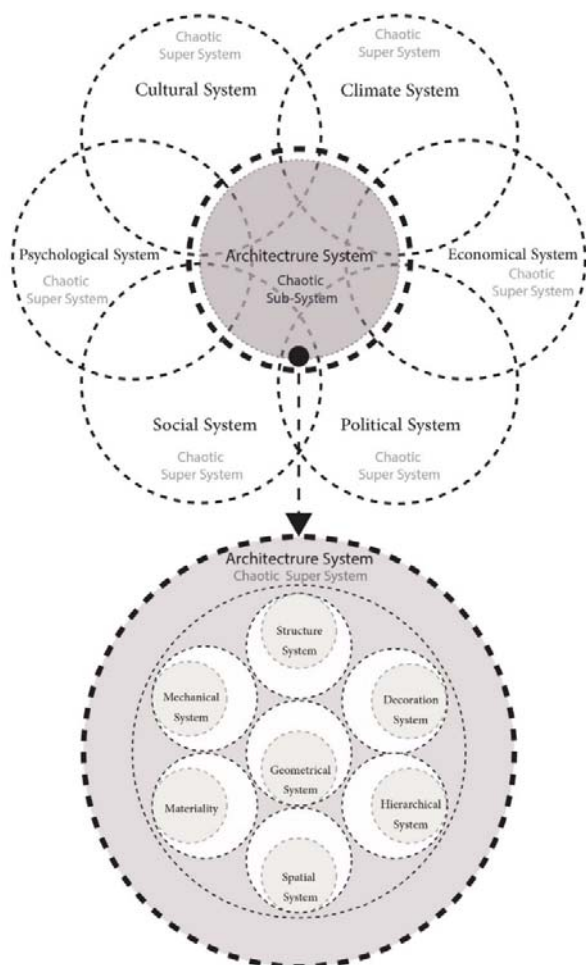


Fig. 13 Descriptive Diagram of chaotic architecture

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