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Spherical Harmonic Based Monostatic Anisotropic Point Scatterer Model for RADAR Applications

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Abstract—High performance computing (HPC) based emulators can be used to model the scattering from multiple stationary and moving targets for RADAR applications. These emulators rely on the RADAR Cross Section (RCS) of the targets being available in complex scenarios. Representing the RCS using tables generated from EM simulations is often times cumbersome leading to large storage requirement. In this paper, we proposed a spherical harmonic based anisotropic scatterer model to represent the RCS of complex targets. The problem of finding the locations and reflection profiles of all scatterers can be formulated as a linear least square problem with a special sparsity constraint. We solve this problem using a modified Orthogonal Matching Pursuit algorithm. The results show that the spherical harmonic based scatterer model can effectively represent the RCS data of complex targets.

Keywords—RADAR, RCS, high performance computing, point scatterer model.

I. Introduction

ELECTROMAGNETIC (EM) wave propagation has long been studied in the RADAR communities. The need to simulate complex EM wave interactions for multiple RADAR targets, transmitters, and receivers to better study the performance of RADAR systems, antenna designs, and/or stealth technologies has grown over time. Specifically, RADAR targets are illuminated by EM waves from the transmitters. In a monostatic RADAR scenario, the receiver receives modulated signals from the collocated transmitters. The complex geometric configuration and the material of a RADAR target determine the modulation of the backscattered signals and affects the propagation channel. The reflection profile of a RADAR target is represented in the form of a RADAR Cross Section (RCS), which is a complex numbered function of aspect angles, signal frequency, and polarization.

In high performance computing (HPC) based RADAR emulators, the complex numbers of the monostatic RCS response from RADAR targets are required to be stored as a table. The needed storage size increases dramatically with a denser aspect angle sampling. The large quantities of data needed often times exceeds the storage and limits manipulations. The RCS data can be compressed with point scatterer model transformation. A scatterer model includes the point scatterer locations and the reflection profile of each scatterer. A commonly used technique is to use an inverse synthetic aperture RADAR (ISAR) image to identify the facets with high reflection and then define the point scatterers [1] [2]. Yet, the required number of scatterers is large.This paper presents an alternate innovative approach to construct the 3D point anisotropic scatterer model that an HPC RADAR EM emulator can use as part of the computations.

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II. Preliminaries

In this section, the isotropic and anisotropic point scatterer models used in the HPC EM emulators are addressed. We then illustrate the monostatic RCS data generation and the math foundation of spherical harmonics. With these approaches, the problem of finding the locations and reflection profiles of each scatterers can be well defined.

A. Scatterer Model

The reflection profile of a RADAR target can be represented as a number of isotropic or anisotropic scatterers. The aspect angle is represented in elevation and azimuth using spherical coordinates (Figure 1) and are denoted by $\mathbf{\Psi} = [\theta, \phi]$. We denote the unit vectors of the aspect angle direction in Cartesian coordinates $\boldsymbol{d}$. The scatterer model possesses a local coordinate origin in the 3D space. In this coordinate, each scatterer p has a position $\boldsymbol{x}_p$. τ_p is the time delay of a signal transmitted to scatterer p from the aspect angle relative to a signal transmitted to the origin from the aspect angle as described in equation (1):

$$\tau_p = \frac{-2\left\langle \boldsymbol{x}_p, \boldsymbol{d} \right\rangle}{c} \tag{1}$$

where c represents the speed of light and $\langle \cdot, \cdot \rangle$ is the standard inner product in R^3.

Therefore, the overall reflection profile of the point scatterer model is a frequency response as shown in equation (2). It is important to note that equation (2) accounts for the two-way propagation:

$$G(f; \mathbf{\Psi}) = \sum_{p=1}^{K} \sigma_p e^{-i2\pi f \tau_p} = \sum_{p=1}^{K} \sigma_p e^{i2\pi f 2\langle \boldsymbol{x}_p, \boldsymbol{d} \rangle / c} \tag{2}$$

where K is the total number of point scatterers, σ_p is the reflection gain of the p^{th} scatterer and f is the signal frequency. With an isotropic scatterer, σ_p is identical across all aspect angles, while with an anisotropic scatterer, σ_p depends on $\mathbf{\Psi}$.

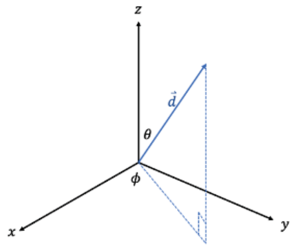

Fig. 1: Spherical coordinate

B. RCS Data Generation

The monostatic reflection frequency response is a complex-valued function and denotes the magnitude and phase changes of the far-field single-frequency EM wave signal in the spherical coordinate system after the wave interacts with the target. The data is stored as a 3-dimensional matrix where each combination of θ,ϕ, and frequency corresponds to a complex value. The RCS of a target can be either measured from the real object or approximately generated using an EM field simulation software such as CST Studio Suite, which uses physical optics-based methods [3] [4].

C. Spherical Harmonics

Real-valued functions on a 3D unit sphere is represented in elevation $0 \leq \theta \leq \pi$ and azimuth $0 \leq \phi < \pi$ using spherical coordinates. These spherical functions form a Hilbert space with a valid inner product:

$$\langle x, y\rangle = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} x(\theta,\phi)y(\theta,\phi)\sin\theta\, d\phi\, d\theta \tag{3}$$

where x and y are two spherical functions. Spherical harmonic functions are well studied in mathematics and physical science [5] [6]. Real numbered spherical harmonic functions $Y_l^m(\theta,\phi)$ of all degrees $l \geq 0$ and orders $-l \leq m < l$ form a complete orthonormal sequence of the spherical vector space [7]. Any spherical function can be written as the sum of the orthonormal basis

$$f(\theta,\phi) = \sum_{l=0}^{\inf}\sum_{m=-l}^{l} c_l^m Y_l^m(\theta,\phi) \tag{4}$$

where c_l^m is the coefficient of the corresponding $Y_l^m(\theta,\phi)$. We define $H = (1+L)^2$ as the total number of spherical harmonics functions of degree L.

III. Constructing the Scatterer Model

Finding the locations and reflection profiles of each scatterers can be formulated as a least squares linear problem with a special sparsity constraint. We solve this problem with a modified orthogonal matching pursuit (OMP) algorithm [8]. In addition, we demonstrate how to solve real-numbered reflection profiles.

A. Least Squares Linear Problem

Two assumptions are made: (1) The angular dependent reflection profile of each anisotropic scatterer is a linear combination of $Y_l^m(\theta,\phi)$ of degree L, and (2) $\bar{K}$ finite possible scatterer locations are known. For example, we can assume that all scatterers locate on a $10m \times 10m \times 10m$ grid in the local coordinate system. That is, the components of $\boldsymbol{x}_p$ of all scatterers are integers between 0 and 10. With these assumptions, finding the locations of K scatterers and the spherical harmonics coefficients becomes a linear least squares problem with a sparsity constraint. The linear least squares problem is shown as

$$S\boldsymbol{\alpha} = \boldsymbol{r} \tag{5}$$

$$\boldsymbol{\alpha} = \begin{bmatrix}\boldsymbol{a_1} & \boldsymbol{a_2} & \cdots & \boldsymbol{a_{\bar{K}}}\end{bmatrix}^T \tag{6}$$

$$\boldsymbol{a_k} = \begin{bmatrix}a_k^1 & a_k^2 & \cdots & a_k^H\end{bmatrix}^T \tag{7}$$

a_k^h represents the coefficient of the h spherical harmonics function of the k scatterer. $\boldsymbol{r}$ represents the vector of the monostatic RCS data from the EM simulations.The sizes of the matrix and vectors are $S \in C^{FM\times H\bar{K}}$, $\boldsymbol{\alpha} \in C^{H\bar{K}}$, and $R \in C^{FM}$, where F and M is the number of RCS signal frequency samples and that of RCS aspect angle samples respectively.

If the optimal locations are known, meaning $K = \bar{K}$, solving the complex inverse problem with pseudo inverse gives us the complex spherical harmonics coefficients for all scatterers. With these coefficients, the spherical function of reflection profile of each scatterer can be constructed and the modeled RCS of the target can be constructed as well. Yet, $\bar{K}$ is often greater than K. Therefore, choosing the locations of the scatterers is also required. This can be formulated as a sparsity constraint.

B. Sparsity Constraint

In addition to solving the system of linear equations, a special sparsity constraint is used to limit the number of scatterers by choosing K optimal scatterer locations among the finite possible scatterer locations. We call all H coefficients of one scatterer as a "group". Our goal is to solve α with K groups consisting of nonzero coefficients, while $\bar{K}-K$ groups consist of zero coefficients. We propose a modified Orthogonal Matching Pursuit (OMP) method to solve this problem.

OMP is a sparse approximation algorithm that solves system of linear equations with the specified number of nonzero elements [8]. The basic algorithm iteratively greedily finds the entry in the solution which corresponds to the column in the matrix that gives the highest correlation with the residual. Instead of finding the column in the matrix that gives the highest correlation with the residual, the proposed algorithm finds which group of column space does.

We first initialize the algorithm by setting the residual as the monostatic RCS data $\boldsymbol{r}$. Let S_j be the j^{th} group of columns in S. With singular value decomposition (SVD), the orthonormal basis of S_j can be computed and defined as $S_{orth,j}$. We can then use the standard complex-domain inner product of $S_{orth,j}$ and the residual as the correlation measurement. That is, we find the vector space:

$$\lambda = \arg\max_{S_j} \|\hat{\boldsymbol{e_j}}\|_2 = \arg\max_{S_j} \| \langle S_{orth,j}, \boldsymbol{e} \rangle \|_2 \qquad (8)$$

where $\boldsymbol{e}$ is the residual. The rest of the proposed algorithm follows the standard OMP algorithm and the details are illustrated in [8]. The modified algorithm is shown in Algorithm 1.

Algorithm 1: Modified OMP

Result: $\boldsymbol{\alpha}$
$\boldsymbol{e_1} = \boldsymbol{r}$, $\boldsymbol{\Lambda_0} = \emptyset$;
Compute $S_{orth,j}$ for all j;
for $n = 1 \rightarrow K$ **do**
    $\boldsymbol{\lambda_n} = \arg\max_{S_j} \| \langle S_{orth,j}, \boldsymbol{res_n} \rangle \|_2$
    $\boldsymbol{\Lambda_n} = \boldsymbol{\Lambda_{n-1}} \cup \{\boldsymbol{\lambda_n}\}$
    $\boldsymbol{\alpha_n} = \arg\min_{\boldsymbol{\alpha}} \|S_{\boldsymbol{\Lambda_n}} \boldsymbol{\alpha} - \boldsymbol{r}\|_2$
    $\hat{\boldsymbol{r}}_{\boldsymbol{n}} = S_{\boldsymbol{\Lambda_n}} \boldsymbol{\alpha}$
    $\boldsymbol{res_{n+1}} = \boldsymbol{r} - \hat{\boldsymbol{r}}_{\boldsymbol{n}}$
end

C. Real-Numbered Scatterer Model

Real spherical harmonics coefficients construct real-numbered RCS profiles. In some HPC RADAR EM emulators, the RCS profiles are restricted to real numbers. The problem with this restriction can be solved by stacking the real parts and imaginary parts of the linear system of equations and solving the 2-times larger real system of equations. The reason is as follows:

Let v_c be a complex vector, and v be the vector with the real and imaginary numbers of v_c stacked vertically. Since $\|v_c\| = \sqrt{\Sigma |v_{c,i}|^2} = \sqrt{\Sigma \left(Re\,(v_{c,i})^2 + Im\,(v_{c,i})^2\right)} = \sqrt{\Sigma |v_j|^2} = \|v\|$, the optimal solution we obtain from this approach is the same as in the complex domain sense. The size of the system becomes $S \in C^{2FM \times H\bar{K}}$, $\boldsymbol{\alpha} \in C^{H\bar{K}}$, and $R \in C^{2FM}$. The modified OMP algorithm is still applicable.

IV. Simulation Result

We demonstrate the feasibility of the proposed approach using the RCS data of an aircraft. The aircraft geometry STL file is obtained from [9] and is shown in Figure 2. The RCS data is generated from the EM simulator CST Studio Suite with horizontal polarization at the 5 frequencies evenly distributed between 1GHz and 2GHz. That is, $F = 5$. The angle sampling increment is 10 degrees for both θ and ϕ which means $M = 18 * 36 = 3240$. We used spherical harmonics of degree 13 and therefore $H = (1 + 13)^2 = 196$. Our goal is to find 16 scatterers i.e. $K = 16$ and all scatterer locate on a $10m \times 10m \times 10m$ grid in the local coordinate i.e. $\bar{K} = 1000$. Figure 3 shows the constructed complex-numbered scatterer model. The brightness indicates the absolute value of the reflection gain at the corresponding aspect angle. Figure 4∼8 presents the image form comparison between the RCS data from CST Studio Suite and the modeled RCS at frequency 1, 1.25, 1.5, 1.75, and 2GHz, respectively.

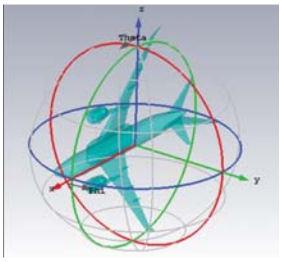

Fig. 2: Aircraft geometry

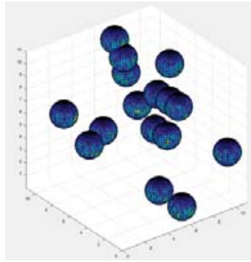

Fig. 3: Point scatterer model

V. Conclusion

The proposed spherical harmonic based anisotropic scatterer model can be used to represent the RCS of complex targets. This model can be used in RADAR HPC EM emulators. We formulate the problem of finding the scatterer locations and reflection profiles as a linear least square problem with a special sparsity constraint. The problem is solved using a modified OMP algorithm. The results show that this scatterer model can effectively represent the RCS data of complex targets.

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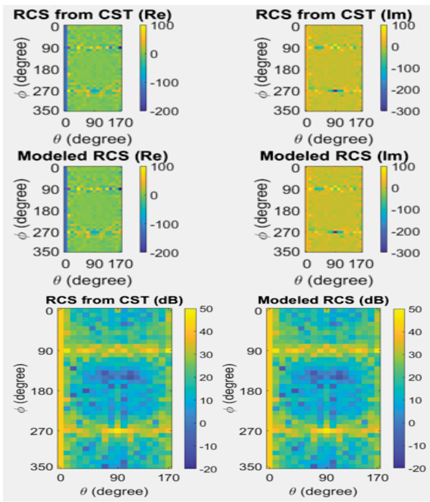

Fig. 4: Images of the RCS data at frequency 1GHz

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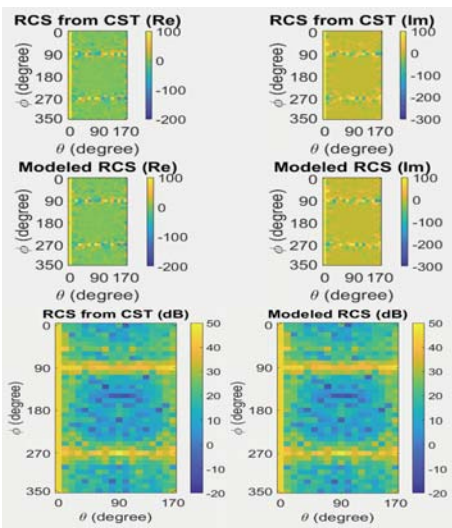

Fig. 5: Images of the RCS data at frequency 1.25GHz

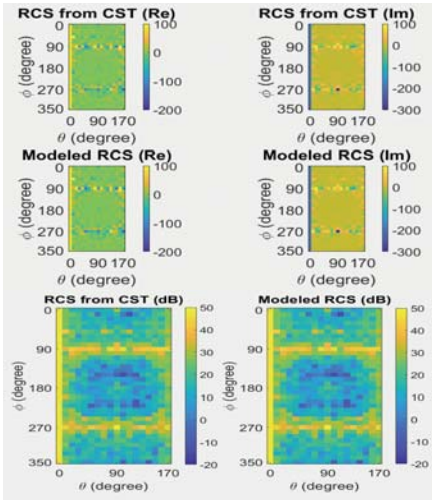

Fig. 6: Images of the RCS data at frequency 1.5GHz

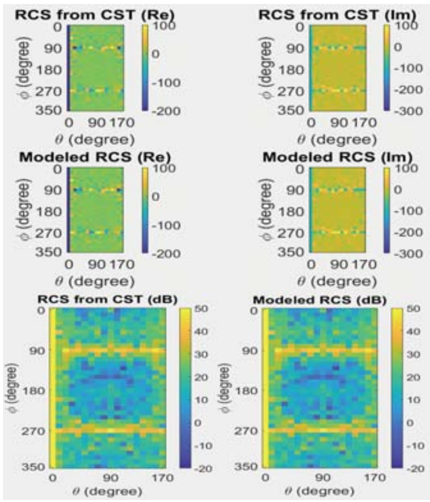

Fig. 7: Images of the RCS data at frequency 1.75GHz

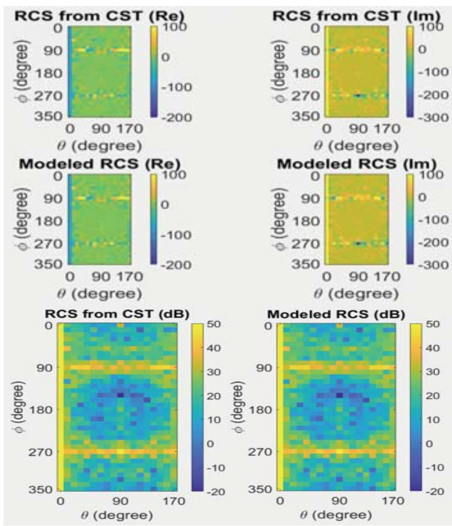

Fig. 8: Images of the RCS data at frequency 2GHz